A Line Passes Through (6,-3) And Has A Slope Of 2. What Is An Equation Of The Line?

A Line Passes Through (6, -3) And Has A Slope Of 2. What Is An Equation Of The Line?

A line passes through the point (6, -3) and has a slope of 2. What is an equation of this line? This is a common question in coordinate geometry, and understanding how to derive the equation of a line from given information is fundamental. In this article, we will explore the process step-by-step, discuss the underlying concepts, and provide various methods to arrive at the equation of the line.

Understanding the Basics of Line Equations

What Is a Line Equation?

An equation of a line describes all the points (x, y) that lie on that line. The most common form of the line's equation in a Cartesian coordinate system is the slope-intercept form:

- **y = mx + b**

where:

- **m** is the slope of the line
- **b** is the y-intercept (the point where the line crosses the y-axis)

The Significance of Slope and a Point

Given the slope of a line and a specific point through which the line passes, we can determine its equation. The slope indicates the steepness and direction of the line, while the point provides a specific location on that line. Combining these two pieces of information allows us to write the line's equation uniquely.

Using the Point-Slope Form to Find the Equation

Introduction to Point-Slope Form

The point-slope form of a line's equation is particularly useful when you know a point (x_1, y_1) on the line and the slope m. It is expressed as:

- **$y - y_1 = m(x - x_1)$**

This form is flexible and straightforward for deriving the equation when the given information fits these parameters.

Applying the Point-Slope Form to Our Problem

Given:

- Point: (6, -3)
- Slope: 2

Substituting into the point-slope form:

- $y - (-3) = 2(x - 6)$

Simplify the left side:

- $y + 3 = 2(x - 6)$

Deriving the Slope-Intercept Form

To convert this into the more familiar slope-intercept form ($y = mx + b$), expand and simplify:

1. Distribute the 2:
2. $y + 3 = 2x - 12$
3. Subtract 3 from both sides:
4. $y = 2x - 15$

Thus, the equation of the line passing through (6, -3) with a slope of 2 is:

- **y = 2x - 15**

Verifying the Equation

Check if the Point (6, -3) Lies on the Line

Substitute x = 6 into y = 2x - 15:

- y = 2(6) - 15 = 12 - 15 = -3

Since y = -3, which matches the y-coordinate of the point, the point lies on the line. This confirms our equation is correct.

Graphical Interpretation

Plotting the point and the line confirms the correctness visually. The line crosses the y-axis at -15 (the y-intercept) and has a slope of 2, meaning it rises 2 units for every 1 unit it runs horizontally.

Alternative Forms of the Equation

Standard Form of the Line Equation

The standard form is generally written as:

- **Ax + By = C**

To convert y = 2x - 15 into standard form:

1. Subtract 2x from both sides:
2. -2x + y = -15
3. Multiply through by -1 to make A positive:
4. 2x - y = 15

This is the standard form of the line's equation.

Point-Intercept Form

If you prefer, you can express the line in intercept form, but in this case, since the y-intercept is -15 and the slope is known, the slope-intercept and standard forms are more straightforward.

Special Considerations and Additional Insights

What if the Slope Was Different?

If the slope were different, the process remains the same: use the point-slope form with the given point and slope, then simplify to the desired form. The key is the initial formula:

- $y - y_1 = m(x - x_1)$

Handling Vertical and Horizontal Lines

- **Horizontal line:** slope $m = 0$. Equation: $y = y_1$
- **Vertical line:** Slope is undefined. Equation: $x = x_1$

Summary of the Key Steps

1. Identify the given point and slope.
2. Use the point-slope formula: $y - y_1 = m(x - x_1)$.
3. Substitute the known values into the formula.
4. Simplify to slope-intercept form ($y = mx + b$) or standard form as needed.
5. Verify the point lies on the derived line for confirmation.

Conclusion

Given a point and a slope, finding the equation of a line is a systematic process grounded in the fundamental formulas of coordinate geometry. For the specific problem where the line passes through (6, -3) and has a slope of 2, the equation in slope-intercept form is **$y = 2x - 15$**. This

equation accurately describes all points on that line and can be transformed into other forms depending on the context or requirements. Mastery of these methods enables you to handle a wide variety of problems involving lines in the Cartesian plane effectively.

Frequently Asked Questions

What is the slope of the line passing through (6, -3) with a slope of 2?

The slope of the line is 2, as given in the problem.

How do you write the equation of a line given a point and a slope?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope.

What is the point given in the problem for the line?

The point given is (6, -3).

How do you find the equation of the line passing through (6, -3) with slope 2?

Plug the point and slope into the point-slope form: $y - (-3) = 2(x - 6)$.

What is the simplified form of the line's equation passing through (6, -3) with slope 2?

Simplifying $y + 3 = 2x - 12$ gives $y = 2x - 15$.

What is the final equation of the line passing through (6, -3) with a slope of 2?

The equation is $y = 2x - 15$.

Can the equation y = 2x - 15 be used to find y when x is 0?

Yes, substituting $x = 0$ gives $y = -15$, which is the y-intercept.

What is the significance of the slope in the line's equation?

The slope indicates the rate of change of y with respect to x; here, for each increase of 1 in x, y increases by 2.

Additional Resources

A Line Passes Through (6, -3) And Has A Slope Of 2. What Is An Equation Of The Line? Understanding how to find the equation of a line given specific points and slope is a fundamental skill in algebra and coordinate geometry. When a problem states that a line passes through (6, -3) and has a slope of 2, it provides all the essential information needed to determine the line's equation. This guide will walk you through the process step-by-step, explain key concepts, and offer practical tips to master this type of problem.

Introduction to Line Equations

Before diving into the specifics of this problem, it's important to understand the basic forms of a line's equation:

- Slope-Intercept Form: $y = mx + b$
where m is the slope, and b is the y-intercept.

- Point-Slope Form: $y - y_1 = m(x - x_1)$
used when you know a point (x_1, y_1) on the line and the slope m.

- Standard Form: $Ax + By = C$
a general form often used for algebraic manipulations.

In this scenario, since a point and the slope are given, the point-slope form is the most straightforward to use.

Step-by-Step Guide to Find the Equation of the Line

Step 1: Identify the Given Data

- Point: (6, -3)
- Slope: $m = 2$

These are your starting points for calculations.

Step 2: Choose the Appropriate Formula

Since you have a point and a slope, the point-slope form is ideal:

$y - y_1 = m(x - x_1)$

where $(x_1, y_1) = (6, -3)$, and $m = 2$.

Step 3: Plug in the Known Values

Substituting into the point-slope form:

$y - (-3) = 2(x - 6)$

which simplifies to:

$y + 3 = 2(x - 6)$

Step 4: Simplify to Slope-Intercept or Standard Form

To make the equation more usable, you can rewrite it in different forms.

Convert to Slope-Intercept Form:

- Distribute the slope:

$y + 3 = 2x - 12$

- Subtract 3 from both sides:

$y = 2x - 15$

This is the slope-intercept form of the line, with slope 2 and y-intercept -15.

Convert to Standard Form:

- Starting from $y + 3 = 2x - 12$

- Rearrange to bring all variables to one side:

$2x - y - 12 - 3 = 0$

which simplifies to:

$2x - y - 15 = 0$

or

$2x - y = 15$

This is the standard form of the line.

Practical Tips and Common Variations

1. When Given Two Points Instead of Slope

If, instead of slope, you are given two points, you would first find the slope:

$m = (y_2 - y_1) / (x_2 - x_1)$

Then, use the point-slope form with either point.

2. Confirming the Correct Equation

Always verify your equation by plugging in the original point (6, -3):

- Using $y = 2x - 15$:

$y = 2(6) - 15 = 12 - 15 = -3$

which matches the y-coordinate of the point, confirming correctness.

3. Graphical Interpretation

Plotting the point (6, -3) and drawing a line with slope 2 (which means for every 1 unit increase in x, y increases by 2) provides a visual way to verify your equation.

Extended Concepts and Applications

Parallel and Perpendicular Lines

- Parallel lines have the same slope, so any other line passing through (6, -3) with slope 2 would be parallel.
- Perpendicular lines have slopes that are negative reciprocals; for example, a line perpendicular to our line would have a slope of -1/2.

Using the Equation in Word Problems

This fundamental process is applicable across various fields—physics, engineering, economics—where you need to model relationships between variables.

Summary of Key Takeaways

- The point-slope form is the most direct method when given a point and a slope.
- Converting between point-slope, slope-intercept, and standard forms helps in different contexts.
- Always verify your equation by substituting the known point.
- Understanding the geometric interpretation of the slope aids in visualization.

Final Thoughts

Finding the equation of a line passing through a specific point with a given slope is a foundational skill in algebra. By mastering the point-slope form and its conversions, students and professionals can efficiently analyze and interpret linear relationships in numerous contexts. Remember, practicing with different points and slopes will reinforce your understanding and help you solve similar problems with confidence.

In conclusion, for the problem "A line passes through (6, -3) and has a slope of 2," the most straightforward equation is:

y = 2x - 15

This equation accurately captures the line's behavior, passing through the point (6, -3) and exhibiting a slope of 2. With this approach, you can confidently handle similar questions involving linear equations in your studies or professional work.

A Line Passes Through 6 3 And Has A Slope Of 2 What Is An Equation Of The Line

A Line Passes Through 6 3 And Has A Slope Of 2 What Is An Equation Of The Line

Related Articles

- Why Does A Runner Breathe Hard For A Few Minutes After Finishing A Race? Biology.
- E Find The Equation Of The Tangent Line To The Curve Liten 15x) En El Punto X = Ya 1 5
- In The Process Of Frost Wedging, How Does Ice Cause Cracks In Rocks To Become Larger?.

Back to Home