

A Line Passes Through The Points (1, 2) And (3, 3). Find Its X- And Y-intercepts.

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Understanding how to find the equation of a line passing through two points and determining its intercepts is fundamental in coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or a math enthusiast exploring the concepts, mastering this skill is essential. In this comprehensive guide, we will walk through the step-by-step process of deriving the equation of the line passing through the points (1, 2) and (3, 3), and then calculating its x- and y-intercepts. We will also discuss related concepts, tips, and real-world applications to deepen your understanding.

Understanding the Basics: Coordinates, Lines, and Intercepts

Before diving into calculations, it's important to review some fundamental concepts:

Coordinates of Points

- A point in the Cartesian plane is specified by an ordered pair (x, y).
- For example, (1, 2) indicates $x = 1$ and $y = 2$.

Equation of a Line

- The general form of a line's equation is $y = mx + b$, where:
- m is the slope of the line.
- b is the y-intercept (the point where the line crosses the y-axis).

Intercepts of a Line

- X-intercept: The point where the line crosses the x-axis ($y=0$). It can be found by solving for x when $y=0$.
- Y-intercept: The point where the line crosses the y-axis ($x=0$). It can be found by solving for y when $x=0$.

Step 1: Find the Slope of the Line (m)

The first step in finding the equation of a line passing through two points is to compute its slope.

What is the Slope?

- The slope m measures the steepness of the line.
- It is calculated as the ratio of the change in y to the change in x between two points:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating the Slope for the Points (1, 2) and (3, 3)

Given:

- Point 1: $(x_1, y_1) = (1, 2)$
- Point 2: $(x_2, y_2) = (3, 3)$

Applying the formula:

$$m = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$

Result: The slope of the line is $m = 0.5$.

Note: A positive slope indicates the line rises as x increases.

Step 2: Find the Equation of the Line

Now that we have the slope, we can find the equation of the line using the point-slope form.

Point-Slope Form

- The point-slope form is:

$$y - y_1 = m(x - x_1)$$

- Choose either point; here, we will use $(1, 2)$.

Substituting Values

$$y - 2 = 0.5(x - 1)$$

Expand the right side:

$$y - 2 = 0.5x - 0.5$$

Add 2 to both sides to solve for y:

$$y = 0.5x - 0.5 + 2$$
$$y = 0.5x + 1.5$$

Result: The equation of the line is:

$$\boxed{y = 0.5x + 1.5}$$

Step 3: Find the X- and Y-Intercepts

With the line's equation established, we can now determine its intercepts.

Finding the Y-Intercept

- The y-intercept occurs when $x=0$.
- Substitute $x=0$ into the equation:

$$y = 0.5(0) + 1.5 = 1.5$$

Y-intercept point: $\boxed{(0, 1.5)}$

Finding the X-Intercept

- The x-intercept occurs when $y=0$.
- Set $y=0$ and solve for x:

$$0 = 0.5x + 1.5$$

Subtract 1.5 from both sides:

$$-1.5 = 0.5x$$

Divide both sides by 0.5:

$$x = \frac{-1.5}{0.5} = -3$$

X-intercept point: $(-3, 0)$

Summary of Results

Intercept Type	Coordinate Point	Calculation Details
Y-intercept	$(0, 1.5)$	Set $x=0$ in $y=0.5x+1.5$
X-intercept	$(-3, 0)$	Set $y=0$ in $y=0.5x+1.5$, solve for x

Visual Representation of the Line and Intercepts

A graph provides valuable insights into the line's behavior.

How To Sketch the Line

- Plot the two points: $(1, 2)$ and $(3, 3)$.
- Draw the line passing through these points.
- Mark the intercepts:
 - Y-intercept at $(0, 1.5)$.
 - X-intercept at $(-3, 0)$.

Visualization helps confirm the correctness of calculations and enhances understanding.

Additional Tips and Common Mistakes

Tips for Accurate Calculation

- Always double-check the slope calculation.
- Use consistent units and coordinate points.
- When solving for intercepts, carefully handle algebraic operations.

Common Mistakes to Avoid

- Mixing up points when calculating the slope.
- Forgetting to switch the signs when solving for intercepts.
- Confusing the coordinates of the intercepts with other points.

Real-World Applications of Line Intercepts

Understanding line intercepts is not just an academic exercise; it has practical applications in various fields:

- Physics: Analyzing motion graphs, such as velocity vs. time, where intercepts can indicate initial conditions or thresholds.
- Economics: Modeling cost-revenue relationships where intercepts represent fixed costs or starting revenues.
- Engineering: Designing systems where thresholds or limits are represented via intercepts.
- Data Analysis: Interpreting linear trends and their points of intersection in datasets.

Practice Problems for Mastery

To reinforce learning, try solving these related problems:

1. Find the equation of a line passing through (2, 5) and (4, 9). Determine its x- and y-intercepts.
2. Given the line $y = -2x + 4$, find its intercepts.
3. If a line passes through the points (0, -3) and (5, 2), what are its intercepts? Verify the line's equation.

Engaging with practice problems boosts confidence and helps solidify understanding.

Conclusion

Calculating the equation of a line passing through two points and finding its intercepts are foundational skills in coordinate geometry. Starting with the calculation of the slope provides the basis for deriving the line's equation, which then allows for straightforward determination of x- and y-intercepts. These concepts not only deepen mathematical understanding but also have practical applications across various disciplines. Remember to verify each step carefully, visualize your results, and practice regularly to master these essential skills. Whether you're solving homework problems or analyzing real-world data, proficiency in these techniques will serve you well in your mathematical journey.

Frequently Asked Questions

How do you find the equation of a line passing through the points (1, 2) and (3, 3)?

First, calculate the slope (m) using $m = (y_2 - y_1) / (x_2 - x_1) = (3 - 2) / (3 - 1) = 1/2$. Then, use point-slope form: $y - y_1 = m(x - x_1)$. Plugging in (1, 2): $y - 2 = (1/2)(x - 1)$. Simplify to get $y = (1/2)x + 3/2$.

What is the slope of the line passing through points (1, 2) and (3, 3)?

The slope is $(3 - 2) / (3 - 1) = 1/2$.

How do you find the x-intercept of the line passing through (1, 2) and (3, 3)?

Set $y = 0$ in the line's equation $y = (1/2)x + 3/2$ and solve for x : $0 = (1/2)x + 3/2 \Rightarrow (1/2)x = -3/2 \Rightarrow x = -3$.

How do you determine the y-intercept of the line passing through (1, 2) and (3, 3)?

The y-intercept is the value of y when $x = 0$. Substitute $x = 0$ into $y = (1/2)x + 3/2$: $y = (1/2)(0) + 3/2 = 3/2$.

What are the x- and y-intercepts of the line passing through (1, 2) and (3, 3)?

The x-intercept is at $(-3, 0)$, and the y-intercept is at $(0, 1.5)$.

Can you verify the intercepts by plotting the line passing through the given points?

Yes, plotting the line with the equation $y = (1/2)x + 3/2$ will pass through the points (1, 2) and (3, 3), and will cross the x-axis at (-3, 0) and y-axis at (0, 1.5).

Additional Resources

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Introduction

In the realm of coordinate geometry, the ability to determine the characteristics of a line passing through specific points is fundamental. The problem of identifying the line passing through points (1, 2) and (3, 3), along with its x- and y-intercepts, exemplifies core principles that underpin many advanced mathematical and scientific applications. This exploration not only demonstrates the methodical process of deriving the line's equation but also underscores the importance of intercepts in understanding a line's position relative to the coordinate axes.

The Significance of Determining a Line's Equation

Before diving into calculations, it is crucial to appreciate why understanding a line's equation and its intercepts matters. Intercepts offer intuitive insights into where a line crosses the axes, serving as vital clues in graph interpretation, problem-solving, and data analysis. In fields ranging from physics to engineering, accurately finding these points facilitates the modeling of real-world phenomena.

Step 1: Establishing the Line's Slope

The first step involves calculating the slope (m)—a measure of the steepness of the line connecting the two given points. The slope formula is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the points (1, 2) and (3, 3):

$$m = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$

This indicates a gentle incline, with the line rising one unit vertically for every two units horizontally.

Step 2: Deriving the Equation of the Line

Having the slope, the next step involves formulating the line's equation. Applying the point-slope form:

$$y - y_1 = m(x - x_1)$$

Choosing point (1, 2):

$$y - 2 = \frac{1}{2}(x - 1)$$

Expanding and simplifying:

$$\begin{aligned} y - 2 &= \frac{1}{2}x - \frac{1}{2} \\ y &= \frac{1}{2}x - \frac{1}{2} + 2 \\ y &= \frac{1}{2}x + \frac{3}{2} \end{aligned}$$

Thus, the equation of the line in slope-intercept form is:

$$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$$

Deep Dive: Validating the Line Equation

To ensure the correctness, verify that the line passes through both points:

- For (1, 2):

$$y = \frac{1}{2}(1) + \frac{3}{2} = \frac{1}{2} + \frac{3}{2} = 2$$

- For (3, 3):

$$y = \frac{1}{2}(3) + \frac{3}{2} = \frac{3}{2} + \frac{3}{2} = 3$$

Both points satisfy the equation, confirming its validity.

Step 3: Finding the X-Intercept

The x-intercept occurs where the line crosses the x-axis, i.e., when $(y=0)$. Set $(y=0)$ in the equation:

$$0 = \frac{1}{2}x + \frac{3}{2}$$

Solve for (x) :

$$\begin{aligned} \frac{1}{2}x &= -\frac{3}{2} \\ x &= -\frac{3/2}{1/2} = -\frac{3/2}{1/2} = -3 \end{aligned}$$

X-intercept:

$$\boxed{(-3, 0)}$$

This point indicates the line crosses the x-axis three units to the left of the origin.

Step 4: Finding the Y-Intercept

The y-intercept is where the line crosses the y-axis, at $(x=0)$. Substituting $(x=0)$:

$$y = \frac{1}{2}(0) + \frac{3}{2} = 0 + \frac{3}{2} = \frac{3}{2}$$

Y-intercept:

$$\boxed{\left(0, \frac{3}{2}\right)}$$

This point shows the line crosses the y-axis 1.5 units above the origin.

Implications and Applications

Understanding the intercepts of a line is not merely an academic exercise but a practical tool with widespread applications:

- Graphical visualization: Intercepts help sketch the line quickly without plotting numerous points.
- Data analysis: When fitting linear models to data, intercepts indicate baseline levels.
- Physics: In motion problems, intercepts can signify initial positions or thresholds.
- Engineering: Design layouts often rely on intercept points for alignment and placement.

Additional Considerations: General Form and Alternative Methods

While the slope-intercept form is straightforward, sometimes lines are expressed in standard form $(Ax + By + C = 0)$. Converting the derived equation:

$$y = \frac{1}{2}x + \frac{3}{2}$$

to standard form:

$$2y = x + 3$$

$$x - 2y + 3 = 0$$

This form is particularly useful in certain computational algorithms and geometric proofs.

Challenges in Interpretation and Calculation

Despite the clarity of the process demonstrated, several challenges can arise:

- Precision in calculations: Small errors in slope computation can lead to incorrect intercepts.
- Handling vertical or horizontal lines: Special cases where the slope is undefined or zero require different approaches.
- Data inconsistencies: Real-world data may contain noise, necessitating regression analysis instead of direct calculation.

Conclusion

The exercise of determining the line passing through (1, 2) and (3, 3), along with its x- and y-intercepts, encapsulates essential principles of coordinate geometry. Starting from the basic slope calculation, progressing through the derivation of the line's equation, and culminating in the identification of intercepts, the process exemplifies analytical rigor and practical utility. Mastery of these methods enhances one's capacity to interpret spatial relationships, model physical phenomena, and solve complex problems across scientific disciplines.

Understanding this simple yet foundational problem sets the stage for tackling more intricate geometric and algebraic challenges, affirming the enduring relevance of coordinate geometry in both academic and applied contexts.

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