A Line Passes Through The Points (6, 10) And (4, -2). What Is The Equation Of The Line

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Understanding how to find the equation of a line passing through two specific points is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams or someone interested in mathematical concepts, mastering this process enhances your analytical skills and deepens your understanding of geometric relationships. In this comprehensive guide, we'll explore the step-by-step method to determine the equation of a line passing through the points (6, 10) and (4, -2), along with explanations of key concepts, formulas, and practical applications.

Introduction to the Equation of a Line

In coordinate geometry, the equation of a line describes all the points that lie on that line in the xy-plane. The most common forms used are:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By + C = 0$

Here, m represents the slope of the line, and (x_1, y_1) is a specific point on the line. To find the equation of a line passing through two points, the key steps involve calculating the slope and then using one of these forms to write the equation.

Step 1: Understanding the Given Points

Given two points:

- Point 1: (6, 10)
- Point 2: (4, -2)

These points are in the coordinate plane, with the x-coordinate and y-coordinate specified for each. The goal is to find the line that passes through both these points and express it in an equation form.

Step 2: Calculating the Slope (m)

The slope of a line indicates its steepness and direction. It is calculated using the formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying this formula to our points:

$$
m = \frac{-2 - 10}{4 - 6} = \frac{-12}{-2} = 6
$$

Result: The slope of the line passing through (6, 10) and (4, -2) is 6.

Step 3: Using the Point-Slope Form to Find the Equation

Once the slope is known, we can utilize the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Choose either of the two points; let's use (6, 10):

$$
y - 10 = 6(x - 6)
$$

Expanding:

$$
y - 10 = 6x - 36
$$

Adding 10 to both sides:

$$
y = 6x - 36 + 10
$$

Simplify:

\[

$$
y = 6x - 26
$$

Equation in slope-intercept form: $y = 6x - 26$

Alternatively, using the other point (4, -2) to verify:

$$
y - (-2) = 6(x - 4)
$$
$$
y + 2 = 6x - 24
$$
$$
y = 6x - 24 - 2
$$
$$
y = 6x - 26
$$

This confirms the consistency of our calculation.

Step 4: Expressing the Equation in Different Forms

Besides the slope-intercept form, the equation can be written in various other forms:

Standard Form

Starting from $y = 6x - 26$:

$$
y - 6x = -26
$$

Rearranged as:

$$
6x - y = 26
$$

This is the standard form of the line's equation.

General Form

Similarly, the general form is:

$$
Ax + By + C = 0
$$

which, in our case, can be written as:

$$
6x - y - 26 = 0
$$

Understanding the Key Concepts

To grasp the process thoroughly, let's break down the essential concepts involved.

1. Slope (m)

- Defines how steep the line is.
- Calculated as the ratio of the change in y over the change in x between two points.
- A positive slope indicates the line rises from left to right; negative indicates it falls.

2. Coordinates of Points

- Points are represented as (x, y).
- Knowing two points allows us to determine the unique line passing through them.

3. Equation Forms

- The slope-intercept form is convenient for graphing.
- The point-slope form is useful for deriving the equation when a point and the slope are known.
- Standard and general forms are often used for algebraic manipulations and solving systems.

Practical Applications of Finding a Line Equation

Understanding how to find the equation of a line through two points is not only a mathematical exercise but also has real-world applications, including:

- Engineering and construction: Designing roads, bridges, and structures.
- Economics: Modeling cost functions.
- Physics: Describing motion with linear relationships.
- Data analysis: Fitting lines to data points in regression analysis.
- Navigation and GPS: Calculating routes and paths.

Common Mistakes and Tips

When working through these types of problems, watch out for:

- Incorrect calculation of the slope: Always subtract y-values in the numerator and x-values in the denominator carefully.
- Reversing points: Ensure the order of points does not affect the calculation of the slope.
- Division by zero: If the x-coordinates are the same, the line is vertical, and the slope is undefined.
- For vertical lines: The equation will be $x = \text{constant}$, which should be handled separately.

Summary

To summarize the process of finding the equation of a line passing through two points:

1. Identify the points and write their coordinates.
2. Calculate the slope using the difference of y-values over the difference of x-values.
3. Use the point-slope form with the calculated slope and one of the points.
4. Simplify to obtain the desired form (slope-intercept, standard, or general).

Applying this process to the points (6, 10) and (4, -2):

- Slope (m) = 6
- Equation: $y = 6x - 26$

This line passes through both points and accurately describes their linear relationship.

Additional Practice Problems

To reinforce your understanding, try solving these problems:

1. Find the equation of the line passing through points (2, 3) and (5, 11).

2. Determine the equation of the line parallel to $y = 4x + 1$ that passes through (0, 5).

3. Calculate the equation of the line perpendicular to $y = -2x + 7$ passing through (3, 4).

Answers:

- For problem 1: Slope = $(11 - 3)/(5 - 2) = 8/3$; equation: $y - 3 = (8/3)(x - 2)$.
- For problem 2: Parallel line has the same slope, so $y = 4x + b$; passing through (0, 5): $5 = 4(0) + b \rightarrow b = 5$; line: $y = 4x + 5$.
- For problem 3: Slope of perpendicular line = the negative reciprocal of -2, which is 1/2; passing through (3, 4): $y - 4 = (1/2)(x - 3)$.

Conclusion

Finding the equation of a line passing through two points is a fundamental aspect of coordinate geometry that combines understanding of slopes, points, and algebraic manipulation. For the specific case of the points (6, 10) and (4, -2), the equation is $y = 6x - 26$, a straightforward linear relationship with a slope of 6 and a y-intercept of -26. Mastery of these concepts provides a strong foundation for more advanced topics in mathematics and numerous practical applications across various fields. Whether you are graphing lines, analyzing data, or solving geometric problems, understanding how to derive a line's equation is an essential skill in your mathematical toolkit.

Frequently Asked Questions

How do I find the equation of a line passing through the points (6, 10) and (4, -2)?

First, calculate the slope using $m = (y_2 - y_1) / (x_2 - x_1)$, then use point-slope form to find the line's equation.

What is the slope of the line passing through (6, 10) and (4, -2)?

The slope is (-2 - 10) / (4 - 6) = (-12) / (-2) = 6.

How do I write the equation of a line given two points?

Calculate the slope, then use the point-slope form y - y1 = m(x - x1), substituting one of the points.

What is the equation of the line passing through (6, 10) and (4, -2)?

Using the slope 6 and point (6, 10), the equation is y - 10 = 6(x - 6). Simplifying gives y = 6x - 26.

Can I verify the line equation by plugging in the points? How?

Yes. Substitute each point into the equation y = 6x - 26; both should satisfy it. For (6, 10): 10 = 66 - 26 = 36 - 26 = 10. For (4, -2): -2 = 64 - 26 = 24 - 26 = -2.

What is the slope-intercept form of the line passing through these points?

The slope-intercept form is y = 6x - 26.

Why is calculating the slope important in this problem?

The slope determines the tilt of the line and is essential for writing its equation accurately.

Are there alternative methods to find the line equation through two points?

Yes, you can use the two-point form or point-slope form directly without calculating the slope explicitly first.

What are common mistakes to avoid when finding the line through two points?

Common mistakes include mixing up the points when calculating the slope, forgetting to simplify the equation, or mixing up the order of points.

Additional Resources

A Line Passes Through The Points (6, 10) And (4, -2). What Is The Equation Of The Line? is a common problem in coordinate geometry that helps students and professionals alike understand the fundamental concepts of lines, slopes, and equations. Whether you're preparing for an exam, solving a practical problem, or just brushing up on your math skills, determining the equation of a line passing through two points is a foundational task. In this article, we'll walk through the process step-by-step, exploring various methods to find the equation of the line passing through the points (6, 10) and (4, -2).

Understanding the Problem

Given two points, (6, 10) and (4, -2), our goal is to find the equation of the line that passes through both points. The key components involved in this task are:

- Coordinates of the points: (6, 10) and (4, -2)
- Line characteristics: slope, intercepts, and the general form of the line's equation

Before diving into calculations, it’s essential to understand what the equation of a line represents and the different forms it can take.

The Foundations: Slope and Line Equations

What Is the Slope?

The slope of a line, often denoted as m, measures its steepness. It is calculated as the ratio of the change in y-values to the change in x-values between two points:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

This value tells us how much y increases or decreases for a one-unit increase in x.

Why Is Slope Important?

Knowing the slope allows us to write the line's equation in various forms, such as:

- Point-slope form: $y - y_1 = m (x - x_1)$
- Slope-intercept form: $y = mx + b$
- Standard form: $Ax + By + C = 0$

Approach Overview

To find the equation, we'll follow these steps:

1. Calculate the slope m using the two points.
2. Use the point-slope form with either point to find the equation.
3. Simplify into the slope-intercept form or any preferred form.

Step-by-Step Guide to Find the Equation

Step 1: Calculate the Slope

Using the points (6, 10) and (4, -2):

$$
m = \frac{10 - (-2)}{6 - 4} = \frac{10 + 2}{2} = \frac{12}{2} = 6
$$

So, the slope of the line is 6.

Step 2: Choose a Point to Use in the Point-Slope Formula

You can pick either point; here, we'll use (6, 10) for clarity.

Step 3: Write the Point-Slope Equation

The point-slope form is:

$$
y - y_1 = m(x - x_1)
$$

Plugging in the values:

$$
y - 10 = 6(x - 6)
$$

Step 4: Simplify to Slope-Intercept Form

Distribute the slope:

$$
y - 10 = 6x - 36
$$

Add 10 to both sides:

\[

$$
y = 6x - 36 + 10
$$

$$
y = 6x - 26
$$

Thus, the equation of the line in slope-intercept form is:

$$
\boxed{y = 6x - 26}
$$

Additional Considerations

Confirming the Equation Passes Through Both Points

It's good practice to verify that the derived equation passes through both original points.

- For (6, 10):

$$
y = 6(6) - 26 = 36 - 26 = 10 \quad \checkmark
$$

- For (4, -2):

$$
y = 6(4) - 26 = 24 - 26 = -2 \quad \checkmark
$$

Both points satisfy the equation, confirming its correctness.

Alternative Methods for Finding the Equation

While the above method is straightforward, here are other approaches:

- Using the two-point form:

$$
\frac{y - y_1}{x - x_1} = m
$$

- Standard form conversion:

Starting from the slope-intercept form, convert to $Ax + By + C = 0$.

Converting to Standard Form

Starting with:

$$
y = 6x - 26
$$

Bring all terms to one side:

$$
6x - y - 26 = 0
$$

This is the standard form of the line's equation.

Visualizing the Line

Understanding the line's behavior can be enhanced by plotting its graph:

- Y-intercept: When $x = 0$:

$$
y = 6(0) - 26 = -26
$$

- X-intercept: When $y = 0$:

$$
0 = 6x - 26 \Rightarrow 6x = 26 \Rightarrow x = \frac{26}{6} = \frac{13}{3} \approx 4.33
$$

Plotting the points (0, -26) and approximately (4.33, 0) provides a visual cue of the line's slope and position.

Key Takeaways

- The slope between two points is fundamental to determining the line's equation.
- The point-slope form provides a flexible way to write the line's equation once the slope is known.
- Verifying your equation with original points ensures accuracy.
- Converting between different forms (slope-intercept, standard) allows for versatile applications.

Summary

The line passing through the points (6, 10) and (4, -2) has a slope of 6. Using the point-slope form with point (6,10), we derive the equation:

$$
\boxed{y = 6x - 26}
$$

This equation accurately represents the line passing through both points, and can be expressed in various forms depending on the context. Understanding these steps not only helps solve this specific problem but also builds a solid foundation for tackling more complex coordinate geometry challenges.

Final Thoughts

Mastering the process of finding a line's equation through two points is essential for many fields, including mathematics, engineering, physics, and computer graphics. Practice with different point pairs, and try converting between various forms to deepen your understanding. With this guide, you should now be well-equipped to handle similar problems confidently and efficiently.

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