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Understanding the relationship between points on a line and its slope is fundamental in coordinate geometry. When a line passes through two points, the slope provides insight into its steepness and direction. In this article, we explore a specific problem: given that a line with a slope of -3 passes through the points (-8, P) and (2, 3P), how do we determine the value of P? We will analyze the problem step-by-step, review relevant concepts, and provide a comprehensive solution.

Understanding the Problem

Before diving into the calculations, it's essential to interpret the problem correctly:

- The line has a slope of -3.
- It passes through two points: (-8, P) and (2, 3P).
- The goal: find the value of P that satisfies these conditions.

This problem involves understanding the relationship between the points on the line and the slope. Since P appears in both coordinate points, the problem translates into an algebraic equation involving P, which we will solve systematically.

Key Concepts and Definitions

1. Slope of a Line

The slope (m) of a line passing through two points $((x_1, y_1))$ and $((x_2, y_2))$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This formula measures the rate of change of y with respect to x, indicating whether the line rises or falls as x increases.

2. Points on the Line

The points in the problem are:

- $(-8, P)$
- $(2, 3P)$

Note that P is an unknown, and our task involves determining its value.

3. Using the Slope to Find P

Since the line passes through these points with slope -3 , substituting the points into the slope formula will yield an equation involving P , which can be solved to find its value.

Step-by-Step Solution

Step 1: Write the Slope Formula with Given Points

Applying the slope formula:

$$\begin{aligned} & \backslash \\ -3 &= \frac{3P - P}{2 - (-8)} \\ & \backslash \end{aligned}$$

Note that:

- $y_2 = 3P$
- $y_1 = P$
- $x_2 = 2$
- $x_1 = -8$

Simplify numerator:

$$\begin{aligned} & \backslash \\ 3P - P &= 2P \\ & \backslash \end{aligned}$$

Simplify denominator:

$$\begin{aligned} & \backslash \\ 2 - (-8) &= 2 + 8 = 10 \\ & \backslash \end{aligned}$$

Therefore, the equation becomes:

$$\begin{aligned} & -3 = \frac{2P}{10} \\ & \end{aligned}$$

Step 2: Solve for P

Multiply both sides of the equation by 10:

$$\begin{aligned} & -3 \times 10 = 2P \\ & \end{aligned}$$

$$\begin{aligned} & -30 = 2P \\ & \end{aligned}$$

Divide both sides by 2:

$$\begin{aligned} & P = \frac{-30}{2} = -15 \\ & \end{aligned}$$

Thus, the value of P is (-15) .

Verification of the Solution

To ensure our solution is correct, verify that the points with $(P = -15)$ satisfy the slope condition.

- First point: $((-8, -15))$
- Second point: $((2, 3 \times -15) = (2, -45))$

Calculate the slope:

$$\begin{aligned} & \frac{-45 - (-15)}{2 - (-8)} = \frac{-45 + 15}{2 + 8} = \frac{-30}{10} = -3 \\ & \end{aligned}$$

Since the calculated slope matches the given slope of -3, our solution is confirmed.

Additional Insights and Applications

1. Real-World Contexts

Understanding how to find unknown variables like P in coordinate geometry has practical applications:

- Physics: Calculating variables related to motion along a line, such as velocity or acceleration.
- Engineering: Designing structures where components align along particular slopes.
- Economics: Modeling relationships between variables, such as cost and demand, where slopes indicate rates of change.

2. Extending the Concept

This problem exemplifies a common approach:

- Using the slope formula with two points.
- Setting the expression equal to a known slope.
- Solving for unknown variables.

These steps are foundational in many areas of mathematics and science.

Summary and Key Takeaways

- When a line passes through two points, the slope can be calculated using the difference in their y-coordinates divided by the difference in their x-coordinates.
- If the slope and coordinates involve variables, algebraic equations can be set up and solved for the unknowns.
- In this problem, substituting the known points into the slope formula and solving for P yielded $(P = -15)$.
- Verifying the solution by plugging back into the points ensures accuracy.

Conclusion

Finding the value of P in the context of a line with a given slope is a straightforward yet essential skill in algebra and coordinate geometry. By applying the slope formula and solving the resulting algebraic equation, we determined that $(P = -15)$. This process reinforces key mathematical concepts and demonstrates their practical utility in various real-world and academic scenarios. Mastery of these techniques enables problem-solvers to approach more complex geometric and algebraic challenges with confidence.

Additional Resources for Further Learning

- Khan Academy: Coordinate Geometry Tutorials
- Mathway: Step-by-step Math Problem Solver
- Purplemath: Algebra and Geometry Guides
- Books: "Algebra and Trigonometry" by Robert F. Blitzer

Understanding and practicing these concepts will enhance your mathematical reasoning and problem-solving skills, paving the way for success in more advanced mathematics topics.

Frequently Asked Questions

How do you find the value of P for the line passing through points (-8, P) and (2, 3P) with a slope of -3?

Use the slope formula: $\text{slope} = (Y_2 - Y_1) / (X_2 - X_1)$. Substitute the points and set it equal to -3, then solve for P.

What is the first step to determine P given the points (-8, P) and (2, 3P) on a line with slope -3?

Calculate the slope using the points: $(-3) = (3P - P) / (2 - (-8))$, then solve for P.

How does the slope formula help in solving for P in this problem?

The slope formula relates the coordinates: $(3P - P) / (2 + 8) = -3$. Simplifying this allows us to find P.

What is the simplified form of the numerator when applying the slope formula to points (-8, P) and (2, 3P)?

The numerator simplifies to 2P because $3P - P = 2P$.

How do you solve for P after setting up the equation using the slope formula?

Set $(2P) / (10) = -3$, then multiply both sides by 10 to get $2P = -30$, and finally divide by 2 to find $P = -15$.

What is the final value of P for the given points and slope?

The value of P is -15.

Why is it important to verify the value of P after solving the equation in this problem?

To ensure that the calculated P satisfies the original slope condition and both points lie on the line with slope -3.

Can the value of P be any other number besides -15 for the line to pass through the given points with slope -3?

No, P must be -15 to satisfy the slope condition for the given points; any other value would change the slope.

Additional Resources

Understanding the Relationship Between Slope and Points: A Deep Dive into the Problem of Finding P

Mathematics, often regarded as the language of the universe, offers an elegant way to describe relationships between quantities. Among its fundamental concepts, the slope of a line holds particular significance, as it indicates the rate of change between two points on a coordinate plane. When a line's slope and specific points it passes through are known, it becomes possible to determine unknown variables that define the line's precise position and orientation. This article explores a classic problem in coordinate geometry: a line with a given slope passing through two points involving an unknown variable P, and how to find the value of P based on the given information. Through detailed explanations, step-by-step solutions, and contextual insights, we aim to demystify the problem and reinforce the foundational principles involved.

Introduction to the Problem: Setting the Context

The problem under consideration is as follows:

> A line with a slope of -3 passes through two points: $((-8, P))$ and $((2, 3P))$. Find the value of (P) .

At its core, this problem combines several core concepts of coordinate geometry:

- The meaning and calculation of the slope of a line.
- The relationship between the slope and points through which the line passes.
- The role of unknown variables in coordinate points and how they relate through the slope.

Understanding this problem requires a solid grasp of the basic principles of slope calculation and the algebraic manipulation of equations. The problem's structure also demonstrates how variables can be interconnected within geometric contexts, and how algebraic techniques can be employed to uncover unknown values.

Fundamental Concepts in Slope and Coordinates

The Definition and Significance of Slope

The slope of a line, often denoted as m , measures its steepness and direction. Mathematically, the slope between two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This ratio captures the vertical change (Δy) relative to the horizontal change (Δx), effectively describing how much y increases or decreases as x moves along the line.

Key points about slope:

- A positive slope indicates the line rises from left to right.
- A negative slope indicates the line falls as it moves from left to right.
- A zero slope corresponds to a horizontal line.
- An undefined slope (division by zero) indicates a vertical line.

In this problem, the slope is explicitly given as -3 , indicating a line that descends at a steady rate.

Points with Unknown Coordinates and Their Role

The points provided involve an unknown variable P :

- $(-8, P)$
- $(2, 3P)$

Here, P is a real number that relates the vertical positions of the two points. Both points share the variable P , which suggests an intrinsic connection between the two, likely stemming from the same line's properties.

This setup offers an interesting challenge: by applying the slope formula to these points, we can derive an equation involving P , which can then be solved to find its value.

Deriving the Equation: Applying the Slope Formula

To solve for (P) , the first step involves applying the slope formula to the two points, considering that the line passing through them has a known slope of (-3) .

Step-by-Step Calculation

Given points:

$$(x_1, y_1) = (-8, P)$$

$$(x_2, y_2) = (2, 3P)$$

The slope (m) between these points is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Plugging in the known values:

$$-3 = \frac{3P - P}{2 - (-8)} = \frac{2P}{2 + 8} = \frac{2P}{10}$$

Simplify the fraction:

$$-3 = \frac{2P}{10}$$

Multiply both sides of the equation by 10 to eliminate the denominator:

$$-3 \times 10 = 2P$$

$$-30 = 2P$$

Finally, divide both sides by 2:

$$P = \frac{-30}{2} = -15$$

Thus, the value of P is -15 .

Verification of the Solution

To ensure the correctness, substitute $P = -15$ back into the points and verify the slope:

- First point: $(-8, -15)$
- Second point: $(2, 3 \times -15) = (2, -45)$

Calculate the slope:

$$\frac{-45 - (-15)}{2 - (-8)} = \frac{-45 + 15}{2 + 8} = \frac{-30}{10} = -3$$

This matches the given slope, confirming the solution's accuracy.

Analytical Insights and Broader Implications

Understanding the Nature of the Variable P

The solution reveals that the variable P is intrinsically linked to the line's slope and the points it passes through. The negative value indicates that the point $(-8, P)$ lies significantly below $(-8, 0)$, consistent with a downward slope.

The fact that both points involve P and $3P$ signifies a proportional relationship in the vertical coordinates, which is preserved across the line with slope -3 . This proportionality, governed by the algebraic relation, helps confirm the internal consistency of the problem.

The Geometric Interpretation

From a geometric perspective, the points:

- $(-8, -15)$
- $(2, -45)$

are located such that the line connecting them has the specified slope of -3 . The negative slope indicates the line descends as it moves from left to right, which aligns with the coordinates showing P and $3P$ decreasing proportionally.

This interpretation enhances understanding of how algebraic relationships reflect geometric realities,

solidifying the importance of slope in visualizing and analyzing lines.

Applications and Educational Significance

Problems like this serve as foundational exercises in algebra and coordinate geometry, illustrating essential skills:

- Manipulation of algebraic expressions involving variables.
- Application of the slope formula in various contexts.
- Understanding the relationship between points and the equations of lines.
- Structuring logical problem-solving approaches.

In educational settings, such problems reinforce conceptual understanding and develop problem-solving agility. They also serve as stepping stones toward more complex topics like linear equations, functions, and analytic geometry.

Conclusion: The Significance of the Result and Its Broader Context

The process of determining the value of P in this problem exemplifies the elegance and utility of algebra in solving geometric problems. By leveraging the fundamental formula for slope, we deduced that:

$$\boxed{P = -15}$$

This result not only confirms the internal consistency of the points and the slope but also highlights how variables can be interconnected within a geometric framework. The problem underscores the importance of understanding the relationships between algebraic expressions and geometric interpretations, a core principle in mathematics education and research.

Furthermore, such problems have practical relevance beyond theoretical exercises. They underpin various applications in engineering, physics, computer science, and data analysis, where relationships between variables and rates of change are crucial.

In conclusion, analyzing this problem deepens our appreciation for the synergy between algebra and geometry, fostering a comprehensive understanding of how mathematical principles operate in tandem to reveal hidden relationships and solve complex problems.

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