Determine If The Following Vectors Are Collinear: = (3,5, −2) And B = [12, −20,8]

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Understanding whether two vectors are collinear is a fundamental concept in vector algebra and plays a crucial role in various fields such as physics, engineering, computer graphics, and mathematics. When vectors are collinear, they lie along the same line, meaning one vector is a scalar multiple of the other. In this article, we will explore how to determine if the vectors A = (3, 5, -2) and B = (12, -20, 8) are collinear, discussing the theoretical background, step-by-step methods, and practical applications.

What Does It Mean for Vectors to Be Collinear?

Definition of Collinearity in Vectors

- Two vectors are said to be collinear if they lie on the same straight line in space.
- Mathematically, vectors A and B are collinear if one is a scalar multiple of the other, i.e., there exists a scalar k such that:

$$
\mathbf{A} = k \mathbf{B}
$$

or equivalently,

$$
\mathbf{B} = k \mathbf{A}
$$

- This property implies that the vectors point in the same or exactly opposite directions.

Why Is Determining Collinearity Important?

- Simplification: Recognizing collinearity simplifies problems involving lines and directions.
- Collision and Path Analysis: In physics and engineering, it helps analyze forces, trajectories, and movement paths.
- Mathematical Applications: It is essential in vector projection, calculating angles, and solving systems of equations.

Mathematical Approach to Check Collinearity

To determine whether vectors A = (3, 5, -2) and B = (12, -20, 8) are collinear, we need to verify if one vector is a scalar multiple of the other. Several methods can be employed, including:

- Comparing component ratios
- Using the concept of scalar multiplication
- Cross product method (more common in 3D space)

Below, we will delve into each method, starting with the simplest approach.

Method 1: Comparing Component Ratios

The most straightforward way to check for collinearity in 3D vectors is to compare the ratios of their corresponding components:

$$
\frac{A_x}{B_x}, \quad \frac{A_y}{B_y}, \quad \frac{A_z}{B_z}
$$

Step-by-step process:

1. Identify components:

- A: (3, 5, -2)
- B: (12, -20, 8)

2. Calculate ratios where possible:

- $\frac{3}{12} = \frac{1}{4}$
- $\frac{5}{-20} = -\frac{1}{4}$
- $-2/8 = -\frac{1}{4}$

3. Analyze the ratios:

- First ratio: $\frac{1}{4}$
- Second ratio: $-\frac{1}{4}$
- Third ratio: $-\frac{1}{4}$

Since the ratios are not all the same ($\frac{1}{4}$ vs. $-\frac{1}{4}$), the vectors are not scalar multiples of each other in the same direction, but they could be negatives of each other, indicating opposite directions.

Observation:

- The ratios of the x-component and y-component have opposite signs, suggesting a potential scalar multiple with a negative scalar.

Method 2: Checking for a Uniform Scalar Multiple

- For vectors to be collinear, all component ratios should be equal (or equal in magnitude but opposite in sign, indicating opposite directions).

- Based on the previous calculations:

$$
\frac{A_x}{B_x} = \frac{1}{4}
$$

$$
\frac{A_y}{B_y} = -\frac{1}{4}
$$

$$
\frac{A_z}{B_z} = -\frac{1}{4}
$$

- Since the ratios are not all equal in sign, but two are equal in magnitude and one differs, this suggests that the vectors are scalar multiples but with a negative scalar.

- Check if $A = -0.25 B$:

$$
-0.25 \times (12, -20, 8) = (-3, 5, -2)
$$

- Comparing this to $A = (3, 5, -2)$:

$$
A = -1 \times (-3, 5, -2)
$$

$$
A = (3, 5, -2)
$$

- We observe that $A = -1 \times (-3, 5, -2)$, which matches A exactly.

- Therefore:

$$
A = -1 \times (-3, 5, -2)
$$

But since:

$$
-3 = -0.25 \times 12
$$

and similarly for other components, this confirms that:

$$
A = -1 \times B
$$

Conclusion from Method 2:

- The vectors are scalar multiples of each other, with scalar $k = -1$.

- This means:

$$
\mathbf{A} = -1 \times \mathbf{B}
$$

- Since the scalar is real, and the relation holds for all components, the vectors are collinear.

Method 3: Using the Cross Product (Optional in 3D)

- The cross product of two vectors in 3D space results in a vector perpendicular to both.

- If the vectors are collinear, their cross product should be the zero vector:

$$
\mathbf{A} \times \mathbf{B} = \mathbf{0}
$$

- Calculate the cross product:

$$
\mathbf{A} \times \mathbf{B} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & 5 & -2 \\
12 & -20 & 8
\end{vmatrix}
$$

- Expanding determinant:

$$
\mathbf{i} (5 \times 8 - (-2) \times (-20)) - \mathbf{j} (3 \times 8 - (-2) \times 12) + \mathbf{k} (3 \times (-20) - 5 \times 12)
$$

- Calculations:

- $\mathbf{i}$ component: $5 \times 8 - (-2) \times (-20) = 40 - 40 = 0$

- $\mathbf{j}$ component: $3 \times 8 - (-2) \times 12 = 24 - (-24) = 24 + 24 = 48$

- $\mathbf{k}$ component: $3 \times (-20) - 5 \times 12 = -60 - 60 = -120$

- Resultant cross product:

$$
\mathbf{A} \times \mathbf{B} = (0, -48, -120)
$$

- Since this is not the zero vector, the vectors are not collinear by the cross product criterion.

- Note: This appears to contradict the earlier conclusion. The reason is that the cross product is zero only if the vectors are scalar multiples. Our calculation indicates they are not scalar multiples in the same or opposite directions unless there is an error.

- Let's verify the scalar multiple directly:

- Earlier, we found A = -1 B is not consistent with the cross product result.

- Wait, this suggests our previous conclusion might need revisiting.

Re-examining the scalar multiple:

- From the component ratios:

$$
\frac{3}{12} = \frac{1}{4}
$$

$$
\frac{5}{-20} = -\frac{1}{4}
$$

$$
\frac{-2}{8} = -\frac{1}{4}
$$

- The ratios for the y and z components are both $-\frac{1}{4}$, but the x ratio is $\frac{1}{4}$. This inconsistency indicates that the entire vector cannot be scaled uniformly by a single scalar to produce the other.

- Therefore, the earlier assumption that A = -1 B is invalid because the ratios differ in sign.

Final determination:

- Since the ratios do not all match in magnitude and sign, the vectors are not scalar multiples.

- The cross product confirms this, as it is not zero.

Conclusion:

- The vectors A = (3, 5, -2) and B = (12

Frequently Asked Questions

How can I determine if the vectors A = (3, 5, -2) and B = (12, -20, 8) are collinear?

You can check if one vector is a scalar multiple of the other by dividing corresponding components; if all ratios are equal, the vectors are collinear.

What is the scalar multiple that relates vector A to vector B?

Dividing each component of B by the corresponding component of A gives: 12/3

= 4, -20/5 = -4, 8/(-2) = -4. Since these ratios are not all the same, the vectors are not scalar multiples.

Can the vectors (3, 5, -2) and (12, -20, 8) be considered collinear based on their ratios?

No, because the ratios are 4, -4, and -4, which are not all equal, indicating the vectors are not scalar multiples and thus not collinear.

Is there a quick method to verify collinearity of three-dimensional vectors?

Yes, one common method is to check if the cross product of the vectors is the zero vector; if it is, the vectors are collinear.

What is the cross product of vectors A = (3, 5, -2) and B = (12, -20, 8)?

Calculating the cross product yields (0, 0, 0), which suggests they are collinear; however, since the ratios differ, this indicates they are not scalar multiples, so rechecking is necessary.

Based on the ratio test, are vectors A and B collinear?

No, because the ratios of their components are not all equal, so they are not collinear.

What is the significance of vectors being scalar multiples?

Vectors are scalar multiples if one can be obtained by multiplying the other by a scalar; this indicates they are collinear and point in the same or opposite directions.

How can I confirm if vectors are parallel or anti-parallel?

Check if the vectors are scalar multiples with a positive scalar for parallel vectors, or a negative scalar for anti-parallel vectors; in this case, since ratios differ, they are neither parallel nor anti-parallel.

Additional Resources

Determine If The Following Vectors Are Collinear: = (3,5, -2) And B = [12, -20,8]

In the realm of vector mathematics, understanding the relationships between vectors is fundamental for numerous applications, from physics to computer graphics. One such critical relationship is collinearity, which determines whether two vectors lie along the same straight line in space. When vectors are collinear, they are scalar multiples of each other, meaning one can be obtained by multiplying the other by a constant scalar. This property simplifies many calculations and provides insight into the geometric structure of the vectors involved.

Today, we explore a specific example involving two vectors: A = (3, 5, -2) and B = (12, -20, 8). Our goal is to determine whether these vectors are collinear. This question may seem straightforward at first glance, but it involves a series of mathematical steps rooted in vector algebra. We will dissect this problem methodically, elucidating the concepts, methods, and reasoning processes essential for such an analysis.

Understanding the Concept of Collinearity in Vectors

What Does It Mean for Vectors to Be Collinear?

In simple terms, two vectors are said to be collinear if they lie along the same line in space, regardless of their magnitude. More formally, vectors A and B are collinear if there exists a scalar k such that:

$$
\mathbf{B} = k \mathbf{A}
$$

or equivalently,

$$
\mathbf{A} = \frac{1}{k} \mathbf{B}
$$

This scalar k is called the scalar multiple that relates the vectors. If such a scalar exists, the vectors are aligned along the same direction (or exactly opposite directions if k is negative).

Key Implication: For vectors to be collinear, their ratios of corresponding components must be equal. That is:

$$
\frac{B_x}{A_x} = \frac{B_y}{A_y} = \frac{B_z}{A_z}
$$

provided that none of the components in the denominator are zero.

Step-by-Step Approach to Determine Collinearity

To analyze whether vectors A = (3, 5, -2) and B = (12, -20, 8) are collinear, we follow a systematic process:

1. Check for Zero Components in the Vectors

Before computing ratios, it is crucial to identify if any component in vector A is zero because division by zero is undefined. In vector A:

- $A_x = 3$ (non-zero)
- $A_y = 5$ (non-zero)
- $A_z = -2$ (non-zero)

All components are non-zero, simplifying our ratio comparisons.

2. Calculate the Ratios of Corresponding Components

Next, we compare the ratios:

$$
k_x = \frac{B_x}{A_x} = \frac{12}{3} = 4
$$
$$
k_y = \frac{B_y}{A_y} = \frac{-20}{5} = -4
$$
$$
k_z = \frac{B_z}{A_z} = \frac{8}{-2} = -4
$$

3. Analyze the Ratios

The ratios are:

- $k_x = 4$
- $k_y = -4$
- $k_z = -4$

Since these ratios are not all equal, the vectors are not scalar multiples of each other. Specifically, A scaled by 4 yields (12, 20, -8), which is not the same as B = (12, -20, 8). The discrepancy in the ratios indicates that A and B are not collinear.

Alternative Method: Using the Cross Product

While comparing component ratios is straightforward, an alternative and often more robust method involves the cross product of the vectors.

1. Recall the Cross Product Definition

The cross product A × B of two vectors $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ is given by:

$$
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x)
$$

2. Compute the Cross Product for Our Vectors

Plugging in the values:

$$
A = (3, 5, -2)
$$
$$
B = (12, -20, 8)
$$

Calculate each component:

- $X = (5)(8) - (-2)(-20) = 40 - 40 = 0$
- $Y = (-2)(12) - 3(8) = -24 - 24 = -48$
- $Z = 3(-20) - 5(12) = -60 - 60 = -120$

Thus,

$$
\mathbf{A} \times \mathbf{B} = (0, -48, -120)
$$

3. Interpretation

If the vectors A and B were collinear, their cross product would be the zero vector:

$$
\mathbf{A} \times \mathbf{B} = \mathbf{0}
$$

Since our calculation yields (0, -48, -120), which is not the zero vector, A and B are not collinear.

Summary of Findings

- Component Ratio Method: The ratios of the components do not all match ($4, -4, -4$), indicating the vectors are not scalar multiples.
- Cross Product Method: The cross product yields a non-zero vector, reaffirming that the vectors are not collinear.

Conclusion: The vectors A = (3, 5, -2) and B = (12, -20, 8) are not collinear.

Why Is Determining Collinearity Important?

Understanding whether vectors are collinear has practical significance in various fields:

- Physics: To determine if forces or velocities act along the same line.
- Computer Graphics: For rendering objects aligned in space.
- Mathematics: Simplifying vector calculations and understanding geometric relationships.

- Engineering: Analyzing structural components aligned along a common axis.

In each case, recognizing collinearity helps in simplifying complex problems and making accurate predictions.

Additional Considerations and Tips

- Zero Components: Always check for zero components before calculating ratios to avoid division by zero errors.
- Use Cross Product: When in doubt, the cross product provides a reliable test for collinearity.
- Scalar Multiple Check: Confirm that the same scalar relates all components, not just some.

Final Thoughts

Determining whether two vectors are collinear is a fundamental skill in vector algebra, combining straightforward ratio comparisons and the more geometric cross product method. In our case, both methods converge on the conclusion that vectors A = (3, 5, -2) and B = (12, -20, 8) are not collinear. This insight not only enhances our understanding of their geometric relationship but also exemplifies the power of algebraic and vector calculus techniques in analyzing spatial problems.

Whether you're a student, engineer, or enthusiast, mastering these methods equips you with essential tools to navigate the multidimensional world of vectors effectively.

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