

Determine The Resultant Force Acting On The 0.7-m-high And 0.7-m-wide Triangular Gate

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Understanding the forces acting on hydraulic structures is fundamental in civil engineering, especially when designing gates and barriers that control water flow in dams, canals, and spillways. Among these structures, triangular gates are common due to their simplicity and effectiveness in managing water pressure. Accurately calculating the resultant force exerted on such gates is crucial for ensuring structural stability and safety.

In this comprehensive guide, we will explore the process to determine the resultant hydrostatic force on a triangular gate measuring 0.7 meters in height and width. We will delve into the principles of hydrostatics, the methods of calculating pressure distribution, and the steps to find the magnitude and point of action of the resultant force. By understanding these concepts, engineers can design more resilient structures capable of withstanding water pressures effectively.

Understanding Hydrostatic Pressure and Forces on Hydraulic Gates

Hydrostatic pressure is the force exerted by a fluid at rest due to gravity. It increases with depth and is given by the formula:

$$P = \rho g h$$

where:

- P is the pressure at depth h ,
- ρ is the density of water (1000 kg/m^3),
- g is acceleration due to gravity (9.81 m/s^2),
- h is the depth from the free surface.

When a gate is submerged in water, it experiences a distributed pressure that varies linearly with depth. The total force on the gate results from integrating this pressure distribution over its surface.

Geometry and Assumptions for the Triangular Gate

Before proceeding with calculations, it is essential to understand the geometry of the triangular gate:

- The gate is triangular in shape, with its base at the water surface and apex pointing downward.
- The height $(h = 0.7\text{ m})$.
- The width at the water surface $(b = 0.7\text{ m})$.
- The triangle is assumed to be symmetric about its central vertical axis.
- The water level is assumed to be at the top of the gate, fully submerged.

Assumptions:

- The water is static.
- The gate is rigid and immovable during the calculation.
- The pressure distribution is hydrostatic and varies linearly with depth.
- Effects of dynamic forces, water flow, or turbulence are neglected.

Step-by-Step Calculation of the Resultant Force

Calculating the hydrostatic force on the triangular gate involves several key steps:

1. Establish the Coordinate System

- Let the y-axis be vertical, with $(y=0)$ at the water surface and positive downward.
- The depth at any point on the gate is (y) .

2. Determine the Width of the Gate at Any Depth

- Since the gate is triangular and symmetric, the width varies linearly with depth.
- The width at the water surface (top) is $(b = 0.7\text{ m})$.
- At the bottom $(y = h = 0.7\text{ m})$, the width is zero.
- The width at a depth (y) is:

$$[w(y) = \frac{b}{h} (h - y) = \frac{0.7}{0.7} (0.7 - y) = 1 \times (0.7 - y) = 0.7 - y]$$

3. Express the Differential Force (dF)

- The differential horizontal force on a thin strip at depth (y) with thickness (dy) is:

$$dF = P(y) \times \text{area of the strip}$$

- The pressure at depth (y) :

$$P(y) = \rho g y$$

- The width at depth (y) :

$$w(y) = 0.7 - y$$

- The area of the strip:

$$dA = w(y) \times dy$$

- Therefore,

$$dF = P(y) \times w(y) \times dy = \rho g y (0.7 - y) dy$$

4. Integrate to Find Total Force (F)

- Total force:

$$F = \int_0^h \rho g y (0.7 - y) dy$$

- Substituting known values:

$$F = \rho g \int_0^{0.7} y (0.7 - y) dy$$

- Expand the integrand:

$$y (0.7 - y) = 0.7 y - y^2$$

- Integral becomes:

$$F = \rho g \left(\int_0^{0.7} 0.7 y dy - \int_0^{0.7} y^2 dy \right)$$

Calculate each integral separately:

$$\int_0^{0.7} 0.7 y dy = 0.7 \times \frac{y^2}{2} \bigg|_0^{0.7} = 0.7 \times \frac{(0.7)^2}{2} = 0.7 \times \frac{0.49}{2} = 0.7 \times 0.245 = 0.1715$$

$$\int_0^{0.7} y^2 dy = \frac{y^3}{3} \bigg|_0^{0.7} = \frac{(0.7)^3}{3} = \frac{0.343}{3} \approx 0.11433$$

Putting it all together:

$$[F = \rho g (0.1715 - 0.11433) = 1000 \times 9.81 \times (0.1715 - 0.11433)]$$

$$[F = 9810 \times 0.05717 \approx 560.2, \text{N}]$$

Result: The total hydrostatic force acting on the triangular gate is approximately 560.2 Newtons.

Locating the Point of Action of the Resultant Force

Knowing the magnitude of the force is essential, but understanding where this force acts on the gate provides critical insight for structural design.

1. Use of the Moment Method

- The point where the resultant force acts is called the center of pressure.
- It is located at the centroid of the pressure distribution.

2. Derivation of the Centroid Location

- The horizontal distance from the water surface to the point of action (y_c) is given by:

$$[y_c = \frac{\int_0^h y \times dF}{\int_0^h dF}]$$

- We already have $(dF = \rho g y (0.7 - y) dy)$.

- The numerator:

$$[N = \int_0^h y \times \rho g y (0.7 - y) dy = \rho g \int_0^h y^2 (0.7 - y) dy]$$

- Expand:

$$[y^2 (0.7 - y) = 0.7 y^2 - y^3]$$

- Integrate:

$$[\int_0^{0.7} 0.7 y^2 dy = 0.7 \times \frac{y^3}{3} \bigg|_0^{0.7} = 0.7 \times \frac{0.343}{3} \approx 0.7 \times 0.11433 = 0.07999]$$

$$\int_0^{0.7} y^3 dy = \frac{y^4}{4} \bigg|_0^{0.7} = \frac{(0.7)^4}{4} = \frac{0.2401}{4} = 0.060025$$

- So,

$$F = 9810 \times (0.07999 - 0.060025) = 9810 \times 0.01997 \approx 196.1 \text{ N}\cdot\text{m}$$

- The denominator (force F) is approximately 560.2 N, as previously calculated.

- Therefore,

$$y_c = \frac{196.1}{560.2} \approx 0.35 \text{ m}$$

Interpretation: The resultant hydrostatic force acts approximately 0.35 meters below the water surface, which is at the centroid of the pressure distribution, roughly at the midpoint of the gate height.

Summary of Results

- Total hydrostatic force (F): approximately 560.2 N.
- Point of action (center of pressure): about 0.35 meters below the water surface, which is at the center of the gate's height.

Practical Implications for Engineering Design

Understanding the magnitude and point of application of the resultant hydrostatic force on

Frequently Asked Questions

How do you calculate the resultant force acting on a triangular gate submerged in water?

The resultant force is calculated by integrating the hydrostatic pressure distribution over the surface of the gate, which involves using the hydrostatic pressure formula $p = \rho gh$ and integrating it across the area to find the total force.

What is the significance of the centroid in determining the force on a triangular gate?

The centroid determines the point at which the resultant hydrostatic force acts. For a triangular gate, the centroid is located at a specific distance from the base, which helps in calculating the magnitude and line of action of the resultant force.

How does the height and width of the triangular gate influence the magnitude of the hydrostatic force?

The hydrostatic force increases with the depth of the water, so a taller gate (greater height) results in a larger force. Similarly, the width determines the area, and a wider gate increases the total force proportionally.

What is the formula to determine the magnitude of the hydrostatic force on a triangular gate measuring 0.7 m high and 0.7 m wide?

The hydrostatic force $F = (1/2) \rho g h^2 \text{ width}$, where ρ is the water density, g is acceleration due to gravity, h is the height of the water column (or gate height), and width is the width of the gate.

Why is it important to find the line of action of the resultant force on the triangular gate?

Knowing the line of action helps in designing the support structures and hinges to ensure stability, as it indicates where the force effectively acts and how moments and reactions should be calculated.

Additional Resources

Determine The Resultant Force Acting On The 0.7-m-high And 0.7-m-wide Triangular Gate

Understanding the forces acting on hydraulic gates is a fundamental aspect of fluid mechanics and civil engineering. When dealing with a triangular gate submerged in water or another fluid, calculating the resultant force is essential for ensuring structural stability, safety, and proper design. This comprehensive review delves into the principles, methods, and detailed calculations involved in determining the resultant force acting on a 0.7-meter-high and 0.7-meter-wide triangular gate.

Introduction to Hydraulic Forces on Gates

Hydraulic gates are widely used in dams, spillways, and canal systems to control water flow. These gates are subjected to hydrostatic pressure exerted by the fluid, which varies with depth. The pressure distribution on the gate, combined with the geometry of the gate, determines the magnitude and direction of the resultant force.

Key considerations include:

- The shape and orientation of the gate
- The depth of immersion
- The fluid properties (density, pressure)
- The boundary conditions (fixed or movable)

In the case of a triangular gate, understanding how pressure varies across its surface allows for precise calculation of the forces involved.

Fundamental Concepts and Principles

Hydrostatic Pressure Distribution

Hydrostatic pressure at any depth (h) in a fluid is given by:

$$p = \rho g h$$

where:

- (p) = pressure at depth (h)
- (ρ) = density of the fluid (kg/m^3)
- (g) = acceleration due to gravity (9.81 m/s^2)
- (h) = depth below the free surface (m)

The pressure increases linearly with depth, which is crucial for calculating forces on submerged surfaces.

Force on a Submerged Surface

The differential force (dF) acting on a small element of the surface at depth (h) :

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$$dF = p \, dA$$

For a vertical surface, the total force is obtained by integrating the pressure over the entire surface area.

Resultant Force and Its Location

The resultant hydrostatic force acts perpendicular to the surface and can be found by integrating the pressure distribution. Its line of action (center of pressure) is generally located below the centroid of the surface, due to the pressure variation with depth.

Geometry of the Triangular Gate

The specific dimensions and orientation of the gate influence the calculations significantly.

Given:

- Height of the gate, $(h_g = 0.7 \, \text{m})$
- Width of the gate at the top, $(w_g = 0.7 \, \text{m})$
- Shape: Triangular (likely with the vertex at the top or bottom; assumptions will be clarified)

Assumption:

For this analysis, assume the triangular gate is oriented with its apex at the top and the base at the bottom, with the wider side at the bottom. The triangle is symmetric about the vertical axis.

Coordinate system:

- Origin at the water surface or at the bottom of the gate, depending on convenience
- Vertical axis positive downward, in line with pressure increase

Analytical Approach to Force Calculation

Step 1: Define the Geometry and Coordinates

- Consider the triangular gate with:

- Vertical height $(h_g = 0.7\text{ m})$
- Width at the top (w_t) (possibly zero if pointing downwards)
- Width at the bottom $(w_b = 0.7\text{ m})$

Note: Since the width at the top is 0.7 m and at the bottom is also 0.7 m, this suggests a right-angled triangle with the base and height equal, or a similar shape. Clarify the shape: is it an equilateral triangle or a right triangle?

Assumption:

- The triangle's base at the bottom is 0.7 m, and the apex is at the top, with the width narrowing linearly from bottom to top.
- The width varies linearly with depth.

Step 2: Establishing the Pressure Distribution

- The pressure at any depth (h) (measured from the free surface or the bottom) is:

$$p(h) = \rho g h$$

- The differential element of the gate at depth (h) :

$$dA = w(h) dh$$

where $(w(h))$ is the width of the gate at depth (h) .

Step 3: Expressing Width as a Function of Depth

- Since the width varies linearly from top to bottom:

$$w(h) = w_t + \left(\frac{w_b - w_t}{h_g} \right) h$$

- If $(w_t = 0.7\text{ m})$ at the top $(h=0)$ and $(w_b = 0.7\text{ m})$ at the bottom $(h=h_g)$, then:

$$w(h) = 0.7\text{ m}$$

which indicates a constant width. But this would make the gate rectangular,

not triangular.

- To model a triangular shape, suppose the width at the top is zero, and at the bottom is 0.7 m:

$$w(h) = \left(\frac{w_b}{h_g} \right) h = \frac{0.7}{0.7} h = h$$

Thus,

$$w(h) = h$$

for h in meters, from 0 at the apex to 0.7 m at the base.

Note: Since the height is 0.7 m, and width varies from 0 to 0.7 m, the triangular shape is consistent.

Calculating the Resultant Force

Step 1: Differential Force Element

- Differential force at depth h :

$$dF = p(h) \times w(h) \, dh = \rho g h \times h \, dh = \rho g h^2 \, dh$$

- The direction of the force is perpendicular to the surface, but since the surface is vertical, the force acts horizontally.

Step 2: Total Force Calculation

- The total force F :

$$F = \int_0^{h_g} \rho g h^2 \, dh$$

- Performing the integral:

$$\begin{aligned} F &= \rho g \int_0^{h_g} h^2 \, dh = \rho g \left[\frac{h^3}{3} \right]_0^{h_g} = \frac{\rho g h_g^3}{3} \end{aligned}$$

- Substituting known values:

$$F = \frac{\rho \times 9.81 \times (0.7)^3}{3}$$

- Assuming water density:

$$\rho = 1000 \, \text{kg/m}^3$$

- Calculation:

$$F = \frac{1000 \times 9.81 \times 0.343}{3} \approx \frac{1000 \times 3.365}{3} \approx \frac{3365}{3} \approx 1121.67 \, \text{N}$$

Thus, the magnitude of the resultant force: approximately 1122 N.

Locating the Center of Pressure

The line of action of the resultant hydrostatic force is located at the centroid of the pressure distribution, which for a triangular surface is at two-thirds the depth from the free surface or from the apex.

Calculation:

- The centroid of a triangle from the apex is located at $\left(\frac{2}{3} h_g \right)$:

$$h_{cp} = \frac{2}{3} \times 0.7 \, \text{m} \approx 0.467 \, \text{m}$$

- This means the resultant force acts at approximately 0.467 m below the water surface (or at the centroid of the pressure distribution).

Direction and Components of the Resultant Force

Since the hydrostatic pressure acts perpendicular to the surface:

- For a vertical surface, the force acts horizontally.
- The total force computed is horizontal and directed inward, pushing against the gate.

If the gate is hinged or supported:

- The reaction force at the hinge must counteract this force.
- The moment generated by this force about the hinge or base is crucial for structural design.

Additional Considerations and Complexities

While the above calculations provide a good approximation, real-world scenarios may involve additional factors:

- Surface Inclination: If the gate is inclined rather than vertical, the pressure distribution needs to be adjusted accordingly.
- Dynamic Effects: Rapid changes in water levels or flow velocities can introduce

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