E Find The Equation Of The Tangent Line To The Curve Liten 15x) En El Punto X = Ya 1 5

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Understanding how to find the equation of a tangent line to a curve at a specific point is a fundamental concept in calculus. This process involves calculating the derivative of the function to determine the slope at the given point and then using point-slope form to derive the equation. In this comprehensive guide, we will explore the steps involved in finding the tangent line to the curve defined by the function "Liten 15x" at the point where \(x = 1.5 \). Whether you are a student preparing for exams or a math enthusiast seeking clarity, this article will provide detailed explanations, examples, and tips to master this essential skill.

Understanding the Concept of a Tangent Line

What Is a Tangent Line?

A tangent line to a curve at a particular point is a straight line that touches the curve exactly at that point without crossing it locally. It represents the instantaneous direction of the curve at that point, effectively "matching" the curve's slope exactly at the point of contact.

Why Is Finding the Tangent Line Important?

- It helps in understanding the behavior of the function at specific points.
- It is fundamental in calculus for analyzing rates of change.
- It provides linear approximations for complex functions near the point of tangency.
- It is used in physics to determine instantaneous velocity and acceleration.

Step-by-Step Process to Find the Equation of the Tangent Line

1. Identify the Function and the Point of Tangency

Suppose the function is \(y = f(x) \). The problem specifies the point \(x = a \), often given as \(x = 1.5 \).

Example:
Let’s assume the function is \(y = Liten(15x) \). (Note: Since "Liten" appears to be a typo or placeholder, we will interpret it as a function notation, possibly intended as \(\tan \) or similar. For this guide, we will consider a general function \(y = f(x) \).)

Given:
- Function: $y = f(x)$
- Point of tangency: $x = a$

Determine:
- $y_a = f(a)$, the y-coordinate at $x = a$.

2. Compute the Derivative $f'(x)$

The derivative provides the slope of the tangent line at any point x. Use differentiation rules appropriate for the function.

Common differentiation rules:
- Power rule
- Chain rule
- Product rule
- Quotient rule

Example:
If $y = \tan(15x)$,
then,
$$
f'(x) = \frac{d}{dx} \tan(15x) = 15 \sec^2(15x)
$$

3. Find the Slope at $x = a$

Calculate the derivative at the specific point:
$$
m = f'(a)
$$
This is the slope of the tangent line at $(a, f(a))$.

Example:
$$
m = 15 \sec^2(15 \times 1.5)
$$

4. Compute the Coordinates of the Point

Calculate the y-coordinate:
$$
y_a = f(a)
$$

Example:
$$
y_a = \tan(15 \times 1.5)

\]

5. Write the Equation of the Tangent Line

Using the point-slope form of the line:

$$
y - y_a = m (x - a)
$$

This formula gives the tangent line passing through (a, y_a) with slope m.

Example:

$$
y - \tan(22.5) = 15 \sec^2(22.5) (x - 1.5)
$$

Applying the Process to a Specific Function

Suppose the actual function is:

$$
f(x) = \tan(15x)
$$

and we need to find the tangent line at $x = 1.5$.

Step 1: Compute $f(1.5)$

$$
f(1.5) = \tan(15 \times 1.5) = \tan(22.5)
$$

Step 2: Compute $f'(x)$

$$
f'(x) = 15 \sec^2(15x)
$$

Step 3: Find the slope at $x = 1.5$

$$
m = 15 \sec^2(22.5)
$$

Use a calculator or tables to compute $\sec^2(22.5)$:
- $\sec(22.5^\circ) = \frac{1}{\cos(22.5^\circ)}$
- $\cos(22.5^\circ) \approx 0.9239$
- $\sec(22.5^\circ) \approx 1.0828$

- $\sec^2(22.5^\circ) \approx 1.172$

Therefore,

$$
m \approx 15 \times 1.172 = 17.58
$$

Step 4: Write the tangent line equation

$$
y - \tan(22.5) = 17.58 (x - 1.5)
$$

Calculate y_a:

$$
y_a = \tan(22.5^\circ) \approx 0.4142
$$

Final tangent line:

$$
y - 0.4142 = 17.58 (x - 1.5)
$$

Or, in slope-intercept form:

$$
y = 17.58 (x - 1.5) + 0.4142
$$

Additional Tips and Common Mistakes

Tips for Accurate Calculation

- Always double-check your differentiation, especially when applying the chain rule.
- Use calculator functions carefully, ensuring the correct mode (degrees vs. radians).
- When approximating, use sufficient decimal places for better accuracy.

Common Mistakes to Avoid

- Confusing the notation or function definitions.
- Forgetting to evaluate the derivative at the specific point.
- Using the wrong differentiation rule for complex functions.
- Mixing degrees and radians in trigonometric functions.

Practice Problems for Mastery

To reinforce your understanding, try solving the following:

1. Find the tangent line to $y = \sin(2x)$ at $x = \frac{\pi}{4}$.
2. Determine the equation of the tangent line to $y = e^{3x}$ at $x = 0$.
3. Given $y = \ln(x)$, find the tangent line at $x = 1$.
4. For the function $y = \frac{1}{x}$, find the tangent line at $x = 2$.

Conclusion

Finding the equation of a tangent line to a curve at a specific point involves understanding the function, calculating its derivative to find the slope, and then applying the point-slope form of a line. This process provides valuable insights into the behavior of functions and their rates of change, essential concepts in calculus. Practice with different functions and points will help solidify this skill, making you proficient in analyzing curves and approximating functions locally. Remember, accuracy in differentiation and substitution is key to deriving correct tangent line equations.

By mastering these steps, you enhance your mathematical toolkit, enabling you to approach more complex calculus problems with confidence.

Frequently Asked Questions

How do I find the equation of the tangent line to the curve y = 15x at a specific point?

To find the tangent line, first compute the derivative of y = 15x, which is 15. Then, substitute the x-value of the point into the original function to get y. Use the point-slope form: $y - y_1 = m(x - x_1)$, where m = 15 and (x_1, y_1) is the point.

What is the tangent line to y = 15x at x = 5?

At x = 5, y = 15(5) = 75. The slope m is 15. Using point-slope form: y - 75 = 15(x - 5). Simplified, the equation is y = 15x + 0, or y = 15x.

How do I determine the slope of the tangent line to y = 15x at any point?

Since y = 15x is a linear function, its derivative is 15 everywhere, so the slope of the tangent line at any point is 15.

Is the tangent line to y = 15x at x=5 the same as the original line?

Yes, because y = 15x is a straight line, its tangent line at any point is the line itself, which is y = 15x.

Can I use the point-slope form to find the tangent line at x = 1?

Yes. At x=1, y=15(1)=15. Using slope m=15, the tangent line is y - 15 = 15(x - 1), which simplifies to y=15x.

What is the general method to find tangent lines for linear functions like y=15x?

For linear functions, the tangent line at any point is the same as the original line because the slope is constant everywhere. Therefore, the tangent line is y=15x.

How do I verify that the tangent line touches the curve at the point (5,75)?

Since y=15x is a straight line, the tangent at (5,75) is y=15x itself. To verify, plug x=5 into the tangent line: y=15(5)=75, which matches the point, confirming the tangent touches the curve at that point.

What changes if the function were non-linear, like y = 15x^2?

If the function were y=15x^2, the derivative would be y' = 30x, which varies with x. The tangent line's slope at x=a would be 30a, and the equation of the tangent line at (a, 15a^2) would be y - 15a^2 = 30a(x - a).

What is the significance of the derivative when finding tangent lines?

The derivative at a point gives the slope of the tangent line to the curve at that point, which is essential for writing the equation of the tangent line.

How can I confirm that my tangent line equation is correct at a given point?

Plug the point's x-coordinate into your tangent line equation to see if it yields the correct y-value. If it does, and the slope matches the derivative at that point, your tangent line is correct.

Additional Resources

E Find The Equation Of The Tangent Line To The Curve Liten 15x) En El Punto X = Ya 1 5

In the world of calculus, understanding the behavior of curves and their slopes at specific points is fundamental. Whether analyzing the trajectory of a projectile or optimizing a function in economics, the tangent line provides crucial insight into the curve’s instantaneous rate of change. Today, we delve into a classic problem: deriving the equation of the tangent line to a given function at a specified point. Specifically, our task is to find the tangent line to the curve defined by the function $Liten\ 15x$ at the point where the x-coordinate equals 1.5. While the notation appears unconventional, we will interpret it as a function, likely $f(x) = 15x$, a common linear function. This exploration will guide you through the steps of calculus—finding derivatives, evaluating at a point, and formulating the tangent line—presented with clarity for readers at all levels.

Understanding the Problem: Clarifying the Function and Point

Before diving into calculations, it’s essential to clarify the problem statement and interpret the given notation accurately:

- The Function: The phrase "Liten 15x" appears to indicate a function involving the term 15x. Given the context, it's reasonable to assume the function is $f(x) = 15x$. This interpretation is consistent with standard algebraic notation and typical calculus exercises involving linear functions.

- The Point: The notation "X = Ya 1 5" is somewhat ambiguous, but it likely signifies $x = 1.5$. The phrase might be a typo or formatting issue, but in the context of calculus, points are denoted as (x, y). Therefore, the point on the curve is at $x = 1.5$.

Summary:
- Function: $f(x) = 15x$
- Point of tangency: $x = 1.5$

Knowing this, our goal becomes clear: find the equation of the line tangent to $f(x)$ at $x = 1.5$.

Fundamentals of Tangent Lines and Derivatives

Understanding what a tangent line is forms the foundation of this problem.

What Is a Tangent Line?

A tangent line to a curve at a specific point is a straight line that touches the curve precisely at that point, matching the curve’s slope at that point. The tangent line "just grazes" the curve without cutting through it near that point, capturing the instantaneous rate of change.

The Role of Derivatives

In calculus, derivatives quantify how a function changes at any given point. The derivative of a function $f(x)$, denoted as $f'(x)$, provides the slope of the tangent line to the curve at each point x.

Key idea:
The equation of the tangent line at a point $x = a$ is given by:

$$
L(x) = f(a) + f'(a)(x - a)
$$

This formula combines the point's coordinates and the slope at that point, forming the linear approximation of the function near a.

Step-by-Step Solution: Deriving the Tangent Line Equation

With the fundamental concepts clarified, let's move into the step-by-step process.

1. Identify the Function and Point

- Function: $f(x) = 15x$
- Point of tangency: $x = 1.5$

2. Calculate the Function’s Value at $x=1.5$

Compute $f(1.5)$:

$$
f(1.5) = 15 \times 1.5 = 22.5
$$

This gives the y-coordinate for the point of tangency: $(1.5, 22.5)$.

3. Find the Derivative $f'(x)$

Since $f(x) = 15x$, which is a linear function, its derivative is straightforward:

$$
f'(x) = \frac{d}{dx}(15x) = 15
$$

The derivative (slope) is constant across all x, meaning the tangent line at any point on this line will have a slope of 15.

4. Evaluate the Derivative at $x=1.5$

$$
f'(1.5) = 15
$$

Since the derivative is constant, the slope of the tangent line at $x=1.5$ remains 15.

5. Write the Equation of the Tangent Line

Using the point-slope form:

$$
L(x) = f(1.5) + f'(1.5)(x - 1.5)
$$

Substitute the known values:

$$
L(x) = 22.5 + 15(x - 1.5)
$$

Simplify the expression:

$$
L(x) = 22.5 + 15x - 22.5 = 15x
$$

Observation: The tangent line is exactly the same as the original function, which makes sense because $f(x) = 15x$ is linear; its tangent line at any point coincides with the line itself.

6. Final Equation

$$
\boxed{
\text{Tangent line at } x=1.5: \quad y = 15x
}
$$

This result confirms the tangent line's equation is $y = 15x$, identical to the original function due to its linearity.

Implications and Additional Insights

Given this specific function, the process illustrates some important points:

- Linear functions have constant slopes everywhere, so their tangent lines are the same as the function itself.
- The derivative's constant nature simplifies tangent line calculations, emphasizing the importance of derivatives in understanding a function's behavior.

For more complex functions, such as quadratics or higher-degree polynomials, the process involves calculating derivatives that vary with x, and the tangent line will differ at different points.

Extending the Concept: Non-Linear Curves

While our example involved a linear function, the same principles apply to non-linear functions, albeit with more involved calculations:

- Calculate the derivative $f'(x)$.
- Evaluate the derivative at the point of interest $x=a$.
- Find the corresponding $f(a)$.
- Apply the point-slope formula to obtain the tangent line.

For instance, if the function were quadratic, such as $f(x) = x^2$, the derivative $f'(x) = 2x$ varies with x, and the tangent line at $x=a$ would be:

$$
L(x) = f(a) + f'(a)(x - a)
$$

which results in a line with slope $2a$ passing through (a, a^2).

Conclusion: The Power of Calculus in Geometry

Finding the tangent line to a curve at a specific point exemplifies how calculus bridges algebra and geometry, offering tools to understand instantaneous change. In our example, the simplicity of the function $f(x) = 15x$ demonstrated that the tangent line coincides with the function itself, highlighting the unique nature of linear functions.

For students and professionals alike, mastering these steps lays the groundwork for tackling more complex problems in physics, engineering, economics, and beyond. Whether analyzing the slope of a tangent to a parabola, a sine wave, or a more complicated curve, the core principles remain the same: differentiate, evaluate, and formulate.

Remember, the key takeaway is that the derivative provides the slope, and the point-slope form of a line offers an elegant way to write the tangent line. With practice, these methods become intuitive tools for exploring the dynamic world of curves and their tangent lines—fundamental concepts that underpin much of modern science and mathematics.

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