Evaluate C Yz Dx + Xz Dy + Xy Dz Over The Line Segment From (1, 1, 1) To (5, 2, 0).

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Evaluate C Yz Dx + Xz Dy + Xy Dz Over The Line Segment From (1, 1, 1) To (5, 2, 0) is a problem involving the calculation of a line integral of a vector field over a specified path. This type of problem is fundamental in vector calculus, with applications spanning physics, engineering, and mathematics, particularly in the context of work done by a force field, flux calculations, and more. To solve this problem systematically, we need to understand the components involved, parameterize the line segment, and then evaluate the integral accordingly.

Understanding the Vector Field and the Line Segment

The Vector Field Components

Given the integral involves the expression C Yz Dx + Xz Dy + Xy Dz, it suggests a vector field of the form:

- **$F(x, y, z) = (F_1, F_2, F_3)$**
- where F_1 = C Yz, F_2 = Xz, and F_3 = Xy

However, the notation C suggests a constant coefficient or parameter. For clarity, we will interpret the vector field as:

```
F(x, y, z) = ( C y z, x z, x y )
```

This is a common form for such problems, where each component involves products of variables and possibly a constant.

The Line Segment Parametrization

The line segment from point $P_0 = (1, 1, 1)$ to point $P_1 = (5, 2, 0)$ can be parametrized as:

```
r(t) = P₀ + t (P₁ - P₀), for t in [0, 1]
```

1. Calculate the difference vector:

2. $P_1 - P_0 = (5 - 1, 2 - 1, 0 - 1) = (4, 1, -1)$

3. Define the parametrization:

```
r(t) = (1, 1, 1) + t (4, 1, -1) = (1 + 4t, 1 + t, 1 - t)
```

for t in [0, 1]

Calculating the Line Integral

Step 1: Find r(t) and r'(t)

The parametrization is:

```
x(t) = 1 + 4t
y(t) = 1 + t
z(t) = 1 - t
```

The derivative r'(t) = (dx/dt, dy/dt, dz/dt) is:

```
r'(t) = (4, 1, -1)
```

Step 2: Substitute into the vector field F

Evaluate each component at r(t):

- $F_1 = C\, y\, z = C\, (1 + t)(1 - t) = C\, (1 - t^2)$
- $F_2 = x\, z = (1 + 4t)(1 - t) = (1 + 4t)(1 - t)$
- $F_3 = x\, y = (1 + 4t)(1 + t) = (1 + 4t)(1 + t)$

Step 3: Write the line integral

The line integral of a vector field F along a curve r(t), t in [0, 1], is:

$$\int_0^1 F(r(t)) \cdot r'(t)\, dt$$

which expands to:

$$\int_0^1 [F_1(x(t), y(t), z(t))\, dx/dt + F_2(x(t), y(t), z(t))\, dy/dt + F_3(x(t), y(t),$$

```
z(t)) dz/dt] dt
```

Performing the Integration

Step 4: Compute each term

- **Term 1:** F_1 dx/dt = C (1 - t^2) 4 = 4C (1 - t^2)
- **Term 2:** F_2 dy/dt = (1 + 4t)(1 - t) 1 = (1 + 4t)(1 - t)
- **Term 3:** F_3 dz/dt = (1 + 4t)(1 + t) (-1) = - (1 + 4t)(1 + t)

Step 5: Simplify each integral component

Evaluating each term separately:

1. Integral of 4C (1 - t^2) dt from 0 to 1:
2. Integral of (1 + 4t)(1 - t) dt from 0 to 1:
3. Integral of - (1 + 4t)(1 + t) dt from 0 to 1:

Step 6: Calculate each integral

Term 1: 4C $\int_0^1$ (1 - t^2) dt

```
∫₀¹ (1 - t^2) dt = [t - (t^3)/3]_0^1 = (1 - 1/3) - 0 = 2/3
```

So, the first term is:

```
4C (2/3) = (8C)/3
```

Term 2: $\int_0^1$ (1 + 4t)(1 - t) dt

```
Expand numerator:
(1 + 4t)(1 - t) = 1 - t + 4t - 4t^2 = 1 + 3t - 4t^2
Integral:
```

$\int_0^1 (1 + 3t - 4t^2)\,dt = [t + (3/2) t^2 - (4/3) t^3]_0^1$
$= (1 + (3/2) - (4/3)) - 0$

Calculate:

$1 + 1.5 - 1.333... \approx 1 + 1.5 - 1.333 = 1.167$

Expressed as fractions:

$1 + (3/2) - (4/3) = (6/6) + (9/6) - (8/6) = (6 + 9 - 8)/6 = 7/6$

So, the second term is:

$7/6$

Term 3: $- \int_0^1 (1 + 4t)(1 + t)\,dt$

Expand numerator:
$(1 + 4t)(1 + t) = 1 + t + 4t + 4t^2 = 1 + 5t + 4t^2$
Integral:
$- [t + (5/2) t^2 + (4/3) t^3]_0^1$
$= - (1 + (5/2) - (4/3))$

Calculate inside:

$1 + 2.5 - 1.333... \approx 2.166...$

Expressed as fractions:

$1 + (5/2) - (4/3) = (6/6) + (15/6) - (8/6) = (6 + 15 - 8)/6 = 13/6$

Therefore, the third term is:

$-13/6$

Final Calculation of the Line Integral

Adding all the terms together:

Line integral $= (8C)/3 + 7/6 - 13/6 = (8C)/3 + (7 - 13)/6 = (8C)/3 - 6/6 = (8C)/3 - 1$

Expressed as a single simplified expression, the value of the line integral is:

$\int_K F \cdot dr = (8C)/3 - 1$

Frequently Asked Questions

What is the line segment over which the vector field C Yz Dx + Xz Dy + Xy Dz is being evaluated?

The line segment is from the point (1, 1, 1) to the point (5, 2, 0).

How do you parametrize the line segment from (1, 1, 1) to (5, 2, 0)?

A suitable parametrization is $r(t) = (1 + 4t, 1 + t, 1 - t)$, where t ranges from 0 to 1.

What are the derivatives of the parametric equations for the line segment?

The derivatives are $dr/dt = (4, 1, -1)$.

How do you substitute the parametric equations into the vector field to evaluate the line integral?

Replace x, y, z in the vector field with the parametric expressions to get the integrand in terms of t, then compute the dot product with dr/dt.

What is the expression for the vector field C Yz Dx + Xz Dy + Xy Dz in terms of x, y, z?

The vector field is (C y z, x z, x y).

How do you evaluate the line integral of the vector field over the given line segment?

Calculate the integral of the dot product of the vector field (after substitution) and dr/dt over t from 0 to 1.

What is the value of the line integral if C is a constant?

The value depends on the specific constant C; the integral involves substituting the parametric equations and integrating accordingly.

Are there any special cases or simplifications when C equals zero?

When C = 0, the vector field reduces to (0, x z, x y), simplifying the integral accordingly.

What are the steps to compute the line integral explicitly?

Parametrize the line, substitute into the vector field, compute the dot product with dr/dt, and integrate with respect to t from 0 to 1.

What is the significance of evaluating this line integral in vector calculus?

It helps in understanding flux, work done, or circulation of the vector field along a specified path, which are fundamental concepts in vector calculus.

Additional Resources

Evaluate C Yz Dx + Xz Dy + Xy Dz Over The Line Segment From (1, 1, 1) To (5, 2, 0).

Introduction: Understanding the Problem

In the realm of vector calculus, line integrals are fundamental tools used to analyze how vector fields interact with paths in space. The problem at hand involves evaluating a particular line integral:

\[

\int_{C} (C\,Yz\,Dx + Xz\,Dy + Xy\,Dz)
\]

over a line segment stretching from the point \((1, 1, 1)\) to \((5, 2, 0)\). This integral combines multiple components of a vector field, each involving different variables and their differentials, prompting a comprehensive approach that blends parametrization, substitution, and calculus techniques.

This article aims to unpack this problem carefully and systematically, providing a clear pathway for understanding, setting up, and computing the integral. Whether you're a student brushing up on vector calculus concepts or a curious reader interested in the mechanics of such integrals, you'll find a detailed, step-by-step explanation that demystifies the process.

Understanding the Vector Field and the Line Segment

Before diving into calculations, it’s crucial to interpret the integral's structure and the path over which it’s evaluated.

The Vector Field

The integrand appears as:

\[
C\,Yz\,Dx + Xz\,Dy + Xy\,Dz
\]

which can be interpreted as the dot product of a vector field \(\mathbf{F}\) with an infinitesimal displacement \(d\mathbf{r}\):

\[
\mathbf{F} = (X y, \quad C\,Yz, \quad X z)
\]

but in the original expression, the terms are presented in a form reminiscent of differential forms or component-wise expressions.

Important note: The notation suggests variables \(X, Y, Z\) are functions of position, with the differential notation \(Dx, Dy, Dz\) representing differentials along those axes. The constants and variables seem to be mixed, so clarity requires us to interpret the integral as a line integral of a vector field.

The Path: From \((1, 1, 1)\) to \((5, 2, 0)\)

The path C is a straight line segment connecting these two points. This is the simplest path to assume unless otherwise specified.

Parameterizing the Line Segment

To evaluate a line integral, the first step is to find an appropriate parametrization of the path C.

Choosing a Parameter

Let’s define a parameter t that varies from 0 to 1, mapping the start point at $t=0$ to the end point at $t=1$:

$$
\mathbf{r}(t) = (x(t), y(t), z(t))
$$

Deriving the Parametric Equations

Given the start point $\mathbf{r}(0) = (1, 1, 1)$ and end point $\mathbf{r}(1) = (5, 2, 0)$, the parametric equations are:

$$
x(t) = 1 + (5 - 1)t = 1 + 4t
$$
$$
y(t) = 1 + (2 - 1)t = 1 + t
$$
$$
z(t) = 1 + (0 - 1)t = 1 - t
$$

for $t \in [0, 1]$.

Computing the Differential Components

The derivatives with respect to t are:

$$
\frac{dx}{dt} = 4, \quad \frac{dy}{dt} = 1, \quad \frac{dz}{dt} = -1
$$

The differentials are:

$$
Dx = \frac{dx}{dt} dt = 4 dt

\]

$$
Dy = \frac{dy}{dt} dt = 1 dt
$$

$$
Dz = \frac{dz}{dt} dt = -1 dt
$$

This parametrization simplifies the integral into a form suitable for direct substitution.

Rewriting the Integrand for Substitution

The original integral:

$$
\int_{C} (C\,Yz\,Dx + Xz\,Dy + Xy\,Dz)
$$

can be viewed as:

$$
\int_{t=0}^{1} \left[C\,Yz\,\frac{dx}{dt} + Xz\,\frac{dy}{dt} + Xy\,\frac{dz}{dt} \right] dt
$$

Our task is to express all variables X, Y, Z and their products in terms of the parameter t.

Clarifying Variable Assignments

Given the notation, a reasonable assumption is:

- $X(t) = x(t) = 1 + 4t$
- $Y(t) = y(t) = 1 + t$
- $Z(t) = z(t) = 1 - t$

Constants like C are not specified numerically; assuming they are constants, we keep C as is.

Expressing the Products

- $Yz = Y(t) \times Z(t) = (1 + t)(1 - t) = 1 - t^2$
- $Xz = (1 + 4t)(1 - t) = (1 + 4t)(1 - t)$
- $Xy = (1 + 4t)(1 + t)$

Calculating these:

$$
Xz = (1 + 4t)(1 - t) = 1 - t + 4t - 4t^2 = 1 + 3t - 4t^2
$$
$$
Xy = (1 + 4t)(1 + t) = 1 + t + 4t + 4t^2 = 1 + 5t + 4t^2
$$

Constructing the Integral Expression

Now, substitute everything into the integral:

$$
\int_{0}^{1} \left[C (1 - t^2) \times 4 + (1 + 3t - 4t^2) \times 1 + (1 + 5t + 4t^2) \times (-1) \right] dt
$$

which simplifies to:

$$
\int_{0}^{1} \left[4C (1 - t^2) + (1 + 3t - 4t^2) - (1 + 5t + 4t^2) \right] dt
$$

Further simplification:

$$
= \int_{0}^{1} \left[4C (1 - t^2) + 1 + 3t - 4t^2 - 1 - 5t - 4t^2 \right] dt
$$

Combine like terms:

- Constants: $1 - 1 = 0$
- t terms: $3t - 5t = -2t$
- t^2 terms: $-4t^2 - 4t^2 = -8t^2$
- The term involving C: $4C(1 - t^2) = 4C - 4C t^2$

Putting it all together:

$$
\int_{0}^{1} \left[4C + (-2t) + (-8t^2) - 4C t^2 \right] dt
$$

or:

\[
\int_{0}^{1} \left[4C - 2t - 8t^2 - 4C t^2 \right] dt
\]

Computing the Integral

Now, integrate term-by-term over \(t\) from 0 to 1:

\[
\int_{0}^{1} 4C \, dt = 4C \int_{0}^{1} dt = 4C [t]_0^1 = 4C
\]

\[
\int_{0}^{1} -2t \, dt = -2 \times \frac{t^2}{2} \bigg|_0^1 = -2 \times \frac{1}{2} = -1
\]

\[
\int_{0}^{1} -8t^2 \, dt = -8 \times \frac{t^3}{3} \bigg|_0^1 = -8 \times \frac{1}{3} = -\frac{8}{3}
\]

\[
\int_{0}^{1} -4C t^2 \, dt = -4C \times \frac{t^3}{3} \bigg|_0^1 = -4C \times \frac{1}{3} = -\frac{4C}{3}
\]

Adding all these results:

\[
\boxed{
\text{Line integral} = 4C - 1 - \frac{8}{3} - \frac{4C}{3}
}
\]

which simplifies to:

\[
\

Evaluate C Yz Dx Xz Dy Xy Dz Over The Line Segment

From 1 1 1 To 5 2 0

Evaluate C Yz Dx Xz Dy Xy Dz Over The Line Segment From 1 1 1 To 5 2 0

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