Evaluate The Indefinite Integral. (use C For The Constant Of Integration.) (6 5x)^6 Dx

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When encountering the indefinite integral of a composite function such as \(\int (6 \, 5x)^6 \, dx\), it is essential to understand the fundamental techniques of integration, especially substitution. This integral appears simple at first glance but requires a systematic approach to simplify and evaluate correctly. Properly evaluating indefinite integrals is a cornerstone of calculus, pivotal in fields like physics, engineering, and mathematics, where understanding the antiderivative of a function leads to insights into area, accumulated quantities, and the behavior of dynamic systems. In this article, we will explore step-by-step methods to evaluate the indefinite integral \(\int (6 \, 5x)^6 \, dx\), including substitution techniques, the application of the constant of integration \(C\), and optimization strategies for solving similar integrals efficiently.

Understanding the Indefinite Integral

Before diving into the specific integral, let's clarify what an indefinite integral represents:

What Is an Indefinite Integral?

- An indefinite integral, denoted as \(\int f(x) \, dx\), finds the antiderivative of a function \(f(x)\).
- It represents the family of all functions \(F(x)\) such that \(F'(x) = f(x)\).
- The constant of integration \(C\) accounts for any constant value that could be added, as derivatives of constants are zero.

Importance of the Constant of Integration C

- Since derivatives of constants are zero, when integrating, we include C to represent all possible antiderivatives.
- The constant ensures the generality of the solution, especially in contexts like physics where initial conditions determine specific solutions.

Step-by-Step Evaluation of $\int (6 \, 5x)^6 \, dx$

Let's now focus on evaluating the integral:

$$
\int (6 \, 5x)^6 \, dx
$$

This integral involves a composite function raised to a power, which suggests substitution as an effective method.

Step 1: Simplify the Expression

- Recognize that $6 \, 5x = 30x$.
- Rewrite the integral as:

$$
\int (30x)^6 \, dx
$$

This simplifies the constant factors and makes substitution more straightforward.

Step 2: Apply Substitution

- Let $u = 30x$.
- Then, $du/dx = 30$ or equivalently, $du = 30 \, dx$.
- Rearranged, $dx = \frac{du}{30}$.

Step 3: Rewrite the Integral in Terms of u

- Substitute u and dx:

$$
\int u^6 \cdot \frac{du}{30} = \frac{1}{30} \int u^6 \, du
$$

This reduces the integral to a basic power rule form.

Step 4: Integrate Using Power Rule

- Recall that $\int u^n \, du = \frac{u^{n+1}}{n+1} + C$, for $n \neq -1$.
- Apply to u^6:

$$
\frac{1}{30} \times \frac{u^{7}}{7} + C = \frac{1}{210} u^{7} + C
$$

Step 5: Substitute Back to x

- Recall $u = 30x$:

$$
\frac{1}{210} (30x)^7 + C

\]

- Simplify \((30x)^7\):

\[

(30)^7 \times x^7

\]

- Therefore, the indefinite integral becomes:

\[

\boxed{\frac{(30)^7}{210} \, x^7 + C}

\]

Simplification and Final Expression

To present the integral in the most simplified form, let's compute the coefficient:

\[

\frac{(30)^7}{210}

\]

Note that:

\[

(30)^7 = (2 \times 3 \times 5)^7 = 2^7 \times 3^7 \times 5^7

\]

And the denominator:

$$
210 = 2 \times 3 \times 5 \times 7
$$

So, the coefficient simplifies as:

$$
\frac{2^7 \times 3^7 \times 5^7}{2 \times 3 \times 5 \times 7} = \frac{2^{7-1} \times 3^{7-1} \times 5^{7-1}}{7} = \frac{2^{6} \times 3^{6} \times 5^{6}}{7}
$$

Expressed explicitly:

$$
\frac{(2^6)(3^6)(5^6)}{7}
$$

Calculating each:

- $2^6 = 64$
- $3^6 = 729$
- $5^6 = 15625$

Multiplying:

$$
64 \times 729 = 46656
$$

\[

$$
46656 \times 15625 = 729000000
$$

Finally, divide by 7:

$$
\frac{729000000}{7} \approx 104142857.14
$$

Thus, the integral can be expressed as:

$$
\boxed{\frac{2^{6} \times 3^{6} \times 5^{6}}{7} \, x^7 + C
$$

or approximately:

$$
104142857.14 \, x^7 + C
$$

Key Points for Evaluating Similar Integrals

When approaching integrals like $\int (k \cdot m x)^n \, dx$, keep these key points in mind:

1. **Identify constants and simplify:** Factor out constants to simplify the integrand.

2. **Use substitution:** Let $u = \text{inner function}$ to reduce to a basic power integral.

3. **Adjust dx accordingly:** Express dx in terms of du.

4. **Apply power rule:** Use $\int u^n du = \frac{u^{n+1}}{n+1} + C$.

5. **Substitute back:** Replace u with the original expression.

6. **Include the constant of integration C:** Always remember to add C at the end.

Optimization Tips for Evaluating Indefinite Integrals

To enhance efficiency and accuracy when evaluating integrals similar to $\int (6 \, 5x)^6 \, dx$, consider these tips:

- **Factor Constants Early:** Simplify constants outside the integral to reduce complexity.

- **Use substitution systematically:** Identify the inner function and its derivative carefully.

- **Remember power rule formulas:** Familiarity with $\int u^n du$ accelerates calculations.

- **Double-check algebraic simplifications:** Be meticulous with exponents and coefficients to avoid errors.

- **Express coefficients in prime factorization:** Simplifies large powers and fractions.

Applications of Evaluating Indefinite Integrals

Understanding how to evaluate indefinite integrals like $\int (6 \, 5x)^6 \, dx$ is fundamental in various real-world scenarios, such as:

Physics and Engineering

- Calculating displacement from velocity functions.
- Determining work done by variable forces.

Mathematics and Statistics

- Finding probability density functions.
- Computing areas under curves.

Economics and Biology

- Modeling growth rates and accumulated quantities.

Conclusion

Evaluating the indefinite integral $\int (6 \, 5x)^6 \, dx$ demonstrates the importance of substitution techniques, the power rule, and careful algebraic manipulation within calculus. By recognizing the structure of the integrand, simplifying constants, applying substitution, and correctly reintegrating, one can find precise antiderivatives efficiently. Always remember to include the constant of integration C to represent the family of all possible solutions. Mastery of these techniques enables solving a broad class of integrals encountered in academic and professional settings, providing a solid foundation for advanced calculus, physics, engineering, and beyond.

Keywords: indefinite integral, calculus, integration techniques, substitution, power rule, constant of integration, evaluate integral, calculus tutorial, integral of $(6 \, 5x)^6$, optimization in integration

Frequently Asked Questions

What is the indefinite integral of (6 + 5x)^6 with respect to x?

The indefinite integral of (6 + 5x)^6 dx is (1/25) (6 + 5x)^7 + C.

How do you approach integrating a function like (6 + 5x)^6 dx?

Use substitution: let u = 6 + 5x, then du = 5 dx, so dx = du/5. Integrate u^6 with respect to u, then substitute back.

What is the step-by-step solution for ∫(6 + 5x)^6 dx?

Let u = 6 + 5x, then du = 5 dx, so dx = du/5. The integral becomes (1/5) ∫ u^6 du = (1/5) (u^7 / 7) + C = (1/35) u^7 + C. Substituting u back gives (1/35) (6 + 5x)^7 + C.

Why do we include '+ C' in the indefinite integral result?

Because the indefinite integral represents a family of functions differing by a constant, the '+ C' accounts for any constant of integration.

What is the importance of substitution in solving $\int(6 + 5x)^6\,dx$?

Substitution simplifies the integral by converting a composite function into a power function, making it easier to integrate.

Can the integral of $(6 + 5x)^6$ be expressed without substitution?

While possible, substitution is the most straightforward method; directly expanding would be cumbersome and not recommended.

What is the final answer to $\int(6 + 5x)^6\,dx$ including the constant of integration?

The indefinite integral is $(1/35)\,(6 + 5x)^7 + C$.

Additional Resources

Evaluate The Indefinite Integral. (use C For The Constant Of Integration.) (6 5x)^6 Dx

When faced with the task of evaluating an indefinite integral such as ∫ (6 5x)^6 dx, understanding the process and the underlying principles of integration is crucial. This particular problem exemplifies the common scenario where a composite function raised to a power appears, and mastering its solution involves applying substitution techniques and the chain rule in reverse. In this guide, we will explore the step-by-step approach to evaluate ∫ (6 5x)^6 dx, with detailed explanations, tips, and insights suitable for learners and practitioners alike.

Understanding the Problem

Before diving into calculations, it's vital to analyze what the integral represents and identify the best method to evaluate it.

The integral at a glance

The integral in question is:

∫ (6 5x)^6 dx

- The integrand is a composite function: a polynomial inside a power.
- The expression 6 5x suggests that 6 and 5x are multiplied, or perhaps it's a typo or formatting issue.

Usually, in such contexts, the integral looks like:

∫ (6 5x)^6 dx

or

∫ (6 5x)^6 dx

Assuming it is ∫ (6 5x)^6 dx, the expression simplifies to:

∫ (30 x)^6 dx

Alternatively, if the original integral is:

∫ (6 5x)^6 dx

then the integrand is (30x)^6.

Note: The key is to clarify the expression. For the purposes of this guide, we'll assume the integral is:

∫ (6 5x)^6 dx

which simplifies to:

∫ (30 x)^6 dx

Step 1: Recognize the Structure

The integral involves a function raised to a power, with a linear function inside. When integrating expressions like (ax + b)^n, substitution is often the most straightforward approach.

Why substitution?

- The derivative of ax + b with respect to x is a, a constant.
- This simplifies the integral because the differential dx can be related to d(ax + b).

In this case, 30 x suggests substitution of u = 30 x, simplifying the integral significantly.

Step 2: Applying Substitution

Define the substitution

Let:

u = 30 x

Then, differentiate both sides:

du/dx = 30

which implies:

$du = 30\,dx$

or:

$dx = du / 30$

Rewriting the integral

Substituting into the original integral:

$\int (30x)^6\, dx = \int u^6\, (du / 30)$

This simplifies to:

$(1/30) \int u^6\, du$

Step 3: Evaluate the Simplified Integral

The integral becomes:

$(1/30) \int u^6\, du$

Recall the power rule for indefinite integrals:

$\int u^n\, du = u^{n+1} / (n+1) + C$

Applying this:

$\int u^6 du = u^{7} / 7 + C$

Therefore,

$(1/30) \int u^6 du = (1/30) (u^{7} / 7) + C$

which simplifies to:

$u^{7} / (30 7) + C$

or:

$u^{7} / 210 + C$

Step 4: Back-substitute to Original Variable

Recall that $u = 30 x$, so:

$u^{7} = (30 x)^{7}$

Thus, the integral evaluates to:

$(30 x)^{7} / 210 + C$

Final Result:

\[

\boxed{

\int (6 \times 5x)^6 dx = \frac{(30 x)^7}{210} + C

}

\]

where C is the constant of integration.

Additional Insights and Tips

1. Confirm the expression

Always double-check the original integral to clarify whether the integrand involves multiplication or addition, and to verify any formatting ambiguities.

2. Use substitution effectively

- Identify the inner function, especially when a composite function is raised to a power.
- Express dx in terms of du to simplify the integral.

3. Simplify constants

- Combining constants early simplifies calculations.
- For example, 30 appears repeatedly; handling it carefully prevents errors.

4. Remember the power rule

- The integral of u^n is u^{n+1} / (n+1), provided n ≠ -1.

5. Reintroduce the constant of integration

- Always include + C at the end of indefinite integrals to account for the family of antiderivatives.

Variations and Practice Problems

To reinforce understanding, consider practicing with similar integrals:

- ∫ (4x + 7)^5 dx
- ∫ (2 x)^4 dx
- ∫ (3x - 2)^3 dx

Applying the same substitution principles will help solidify your skills.

Summary

Evaluating ∫ (6 5x)^6 dx involves recognizing the structure of the integrand, applying substitution to simplify the integral, and then using the power rule for integration. The key steps include:

- Defining an appropriate substitution: u = 30 x
- Converting dx to du / 30
- Integrating u^6 using the power rule
- Back-substituting u = 30 x into the result
- Including the indefinite integral constant C

This method exemplifies a fundamental technique in calculus: transforming complex integrals into manageable forms via substitution. Mastery of these steps enhances your ability to evaluate a wide range of indefinite integrals confidently.

Final Note

The process demonstrated here underscores the importance of careful algebraic manipulation and understanding of substitution techniques in calculus. Whether dealing with simple polynomials or more complex composite functions, these foundational methods are essential tools for any student or professional working with integrals.

Evaluate The Indefinite Integral Use C For The Constant Of Integration 6 5x 6 Dx

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