Evaluate The Integral X 2 (X 3 3) 12 D X By Making The Substitution U = X 3 3 .

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This integral problem is a classic example of how substitution techniques can simplify seemingly complex calculus expressions. By strategically choosing a substitution, we can transform the integral into a more manageable form, making the process of integration straightforward. In this article, we will explore in detail how to evaluate the integral of the form $\int X^2 (X^3 3)^{12} \, dX$ by making the substitution $U = X^3 3$. We'll cover the step-by-step process, the underlying principles, and tips for applying similar techniques to other integrals, all while emphasizing the importance of substitution in calculus.

Understanding the Integral and Its Components

Before diving into the substitution process, it’s essential to understand the structure of the integral:

$$\int X^2 (X^3 3)^{12} \, dX$$

Breaking Down the Integral

- The polynomial X^2: A simple quadratic term that appears as a multiplicative factor.
- The expression $(X^3 3)^{12}$: A composite function involving a power of a cubic term.
- The differential dX: Indicates integration with respect to X.

Recognizing patterns in the integrand is crucial. Notice that the inner part $X^3 3$ is raised to the 12th power, suggesting that a substitution involving $X^3 3$ could simplify the integral.

Choosing the Appropriate Substitution

Why Make the Substitution $U = X^3 3$?

The substitution $U = X^3 3$ leverages the inner function inside the power, transforming the integral into a form that involves U and its differential dU. This approach simplifies the complex algebraic expression and makes the integration process more manageable.

Key Points for Substitution

- The inner function $X^3 3$ is a natural candidate for substitution because it appears as the base of a power.
- The derivative of U with respect to X should relate to terms in the integrand, enabling cancellation or straightforward replacement.
- The substitution reduces the original integral into a standard power function integral.

Step-by-Step Solution

Step 1: Define the Substitution

Let:

$$U = X^3 3$$

Step 2: Differentiate U with respect to X

Calculate dU/dX:

$$\frac{dU}{dX} = 3 \times X^2 \times 3$$

Note: The derivative of $X^3 3$ with respect to X involves applying the chain rule:

$$\frac{d}{dX} (X^3 3) = 3 \times X^2 \times 3$$

which simplifies to:

$$\frac{dU}{dX} = 3 \times 3 \times X^2 = 9 X^2$$

Step 3: Express dX in terms of dU

Rearranged:

$$dU = 9 X^2 \, dX$$

Thus,

$$X^2 \, dX = \frac{dU}{9}$$

Step 4: Rewrite the integral in terms of U

The original integral:

$$\int X^2 (X^3 3)^{12} \, dX$$

becomes:

$$\int X^2 \times U^{12} \, dX$$

Using the substitution:

$$X^2 \, dX = \frac{dU}{9}$$

The integral is now:

$$\int U^{12} \times \frac{dU}{9}$$

which simplifies to:

$$\frac{1}{9} \int U^{12} \, dU$$

Step 5: Integrate with respect to U

The integral of U^{12} is straightforward:

$$\int U^{12} \, dU = \frac{U^{13}}{13} + C$$

Therefore:

$$\frac{1}{9} \times \frac{U^{13}}{13} + C = \frac{U^{13}}{117} + C$$

Step 6: Back-substitute to X

Recall $U = X^3 3$, so:

$$\boxed{\int X^2 (X^3 3)^{12} \, dX = \frac{(X^3 3)^{13}}{117} + C}$$

Final Result and Interpretation

The evaluated integral simplifies elegantly to:

$$\boxed{\frac{(X^3 3)^{13}}{117} + C}$$

This result demonstrates how substitution transforms a complex integral into a simple power function, allowing for straightforward integration.

Additional Tips for Integration by Substitution

To maximize your understanding and efficiency in solving integrals via substitution, consider the following tips:

1. Recognize Inner Functions

Identify functions nested inside powers, roots, or compositions that can serve as substitution candidates.

2. Compute Derivatives Carefully

Ensure accurate differentiation of the substitution candidate to relate dU and dX.

3. Simplify Carefully

Always rewrite the integral entirely in terms of the substitution variable before integrating.

4. Back-Substitute at the End

After integrating, replace the substitution variable with the original expression to get the final answer.

5. Practice with Variations

Apply similar substitution techniques to different integrals, such as those involving exponential, logarithmic, or trigonometric functions.

Common Mistakes to Avoid

- Incorrect differentiation: Double-check derivatives, especially with chain rule applications.
- Forgetting to adjust the differential: Remember to express dX in terms of dU to avoid errors.
- Missing constants: Always include the constant of integration C.
- Overlooking the limits: When dealing with definite integrals, change limits accordingly.

Conclusion

Evaluating the integral $\int X^2 (X^3 3)^{12} \, dX$ becomes straightforward with a strategic substitution $U = X^3 3$. This approach exemplifies the power of substitution in calculus, transforming complex expressions into manageable forms. Mastering this technique enables you to tackle a wide range of integrals involving composite functions effectively. Remember, the key is to identify the inner function, differentiate it correctly, and rewrite the integral accordingly. With practice, substitution becomes an intuitive and invaluable tool in your calculus toolkit.

Summary of Key Steps

1. Identify the inner function $U = X^3 3$.
2. Find dU/dX and express dX in terms of dU.
3. Rewrite the integral entirely in terms of U.
4. Perform the integration in U.
5. Substitute back $U = X^3 3$ to obtain the final answer.

By following these steps, you can confidently evaluate similar integrals, enhancing your calculus proficiency and problem-solving skills.

Meta Description:
Learn how to evaluate the integral $\int X^2 (X^3 3)^{12} \, dX$ using substitution $U = X^3 3$. This detailed guide covers step-by-step solutions, tips, and common mistakes to improve your calculus skills.

Frequently Asked Questions

What is the integral to be evaluated in the given problem?

The integral is $\int x^2 (x^3 + 3)^{12} dx$.

What substitution is suggested to evaluate the integral?

The substitution $u = x^3 + 3$ is suggested.

How do you differentiate $u = x^3 + 3$ to find du?

Differentiating $u = x^3 + 3$ gives $du/dx = 3x^2$, so $du = 3x^2\,dx$.

How does the substitution simplify the integral?

Since $du = 3x^2\,dx$, the integral becomes $(1/3) \int u^{12}\,du$ after substitution.

What is the integral of u^{12} with respect to u?

The integral of $u^{12}\,du$ is $(u^{13}) / 13 + C$.

How do you express the final answer in terms of x?

Substitute back $u = x^3 + 3$ to get $(1/39)(x^3 + 3)^{13} + C$.

Why is the substitution method effective for this integral?

Because the integrand is a composition of a power function and a linear substitution, simplifying the integral significantly.

Are there alternative methods to evaluate this integral?

Yes, but substitution is the most straightforward approach here; integration by substitution simplifies the process efficiently.

Additional Resources

Evaluate The Integral X 2 (X 3 3) 12 D X By Making The Substitution U = X 3 3

Introduction

Mathematics, especially integral calculus, often presents complex problems that demand innovative approaches for their resolution. One such problem involves evaluating the integral:

$$
\int X^2 \left(X^{3 3} \right)^{12} \, dX
$$

At first glance, the notation appears unconventional or perhaps misformatted, but upon careful analysis, it reveals a classic structure amenable to substitution methods. This article aims to thoroughly dissect this integral, understand the rationale behind substitution, and explore its evaluation step-by-step.

Our focus is on the substitution $U = X^{3 3}$, which suggests a strategic attempt to simplify the integrand. As we proceed, we will clarify the notation, interpret the integral's structure, and ultimately derive the solution, providing insights into the underlying calculus principles and techniques.

Clarifying the Integral: Deciphering the Notation

Before delving into the solution, it’s critical to interpret the integral's notation accurately. The original expression:

$$
\text{Evaluate The Integral } X 2 (X 3 3) 12 \, dX
$$

may be intended as:

$$
\int X^2 \left(X^{33} \right)^{12} \, dX
$$

or perhaps:

$$
\int X^2 \left(X^{3 \times 3} \right)^{12} \, dX
$$

Given the context and common notation, the most plausible interpretation is:

$$
\int X^2 \left(X^{33} \right)^{12} \, dX
$$

which simplifies to:

$$
\int X^2 \times X^{33 \times 12} \, dX = \int X^2 \times X^{396} \, dX
$$

since $33 \times 12 = 396$.

Alternatively, if the notation was intended as X^{3^3}, that is, $X^{3^3} = X^{27}$, then:

\[

$$
\int X^2 \times (X^{27})^{12} \, dX = \int X^2 \times X^{324} \, dX
$$

which simplifies to:

$$
\int X^{2 + 324} \, dX = \int X^{326} \, dX
$$

Given the ambiguity, the most instructive and general approach is to examine the integral:

$$
\int X^2 \left(X^{27} \right)^{12} \, dX
$$

which aligns with the substitution $U = X^{27}$.

In conclusion, the integral is:

$$
\int X^2 \left(X^{27} \right)^{12} \, dX
$$

or equivalently:

$$
\int X^2 \times X^{324} \, dX
$$

which simplifies to:

$$
\int X^{326} \, dX
$$

This simplification makes the integral straightforward, but to illustrate the substitution approach, we'll proceed with the substitution $U = X^{27}$ as the core method.

The Substitution Strategy: Why and How?

The substitution $U = X^{27}$ is motivated by the desire to reduce the complexity of the integrand. When an integrand involves a power of a function, substitution often transforms the integral into a more manageable form.

Why Use Substitution?

- Simplify the variable dependence: When the integrand involves composite functions or powers, substitution reduces the integrand into a form involving a single variable, enabling straightforward integration.

- Leverage derivative relationships: The substitution is chosen based on the derivative of the inner function, ensuring that dU relates directly to dX, simplifying the integral.

- Standard technique: Substitution is a fundamental method in integral calculus, especially effective for polynomial and exponential functions.

Step-by-Step Evaluation of the Integral

Assuming the integral:

$$
I = \int X^{326} \, dX
$$

which results from the previous simplification, the direct integration is straightforward:

$$
I = \frac{X^{327}}{327} + C
$$

where C is the constant of integration.

However, to demonstrate the substitution method explicitly, consider the original integral:

$$
I = \int X^2 (X^{27})^{12} \, dX
$$

Step 1: Recognize the inner function

Define:

$$
U = X^{27}
$$

then,

$$
dU = 27 X^{26} \, dX
$$

which implies:

$$
dX = \frac{dU}{27 X^{26}}
$$

Step 2: Rewrite the integrand in terms of U

Observe that:

$$
X^{326} = X^{27 \times 12 + 2} = X^{324 + 2} = X^{326}
$$

which matches the previous expression, but perhaps better to express X^{k} in terms of U:

$$
U = X^{27} \Rightarrow X = U^{1/27}
$$

Thus:

$$
X^2 = (U^{1/27})^2 = U^{2/27}
$$
and

$$
X^{326} = (U^{1/27})^{326} = U^{326/27}
$$

Step 3: Express dX in terms of dU

Since:

$$
dU = 27 X^{26} \, dX
$$

then:

$$
dX = \frac{dU}{27 X^{26}} = \frac{dU}{27 (U^{1/27})^{26}} = \frac{dU}{27 U^{26/27}}
$$

Step 4: Rewrite the integral in terms of U

Putting all together:

$$
I = \int X^2 \times (X^{27})^{12} \, dX = \int U^{2/27} \times U^{326/27} \times \frac{dU}{27 U^{26/27}}
$$

Simplify numerator:

$$
U^{2/27} \times U^{326/27} = U^{(2 + 326)/27} = U^{328/27}
$$

Now, divide by $27 U^{26/27}$:

$$
I = \frac{1}{27} \int U^{328/27} \times U^{-26/27} \, dU = \frac{1}{27} \int U^{(328/27) - (26/27)} \, dU
$$

Subtract exponents:

$$
\frac{328}{27} - \frac{26}{27} = \frac{328 - 26}{27} = \frac{302}{27}
$$

So:

$$
I = \frac{1}{27} \int U^{302/27} \, dU
$$

Step 5: Integrate with respect to U

Applying the power rule:

$$
\int U^{m} \, dU = \frac{U^{m+1}}{m+1} + C
$$

we get:

$$
I = \frac{1}{27} \times \frac{U^{(302/27) + 1}}{(302/27) + 1} + C
$$

Simplify denominator:

$$
(302/27) + 1 = (302/27) + (27/27) = \frac{302 + 27}{27} = \frac{329}{27}
$$

Thus:

$$
I = \frac{1}{27} \times \frac{U^{(329/27)}}{(329/27)} + C = \frac{1}{27} \times \frac{27}{329} U^{329/27} + C
$$

The 27 cancels:

$$
I = \frac{1}{329} U^{329/27} + C
$$

Step 6: Back-substitute $U = X^{27}$

Recall that:

$$
U = X^{27}
$$

then:

$$
I = \frac{1}{329} (X^{27})^{329/27} + C = \frac{1}{329} X^{27 \times (329/27)} + C = \frac{1}{329} X^{329} + C
$$

Final Result and Interpretation

The integral evaluates to:

$$
\boxed{
\int X^2 \left(X^{27} \right)^{12} \, dX = \frac{X^{329}}{329} + C
}
$$

which aligns with the straightforward power rule for polynomial integrals.

Broader Implications and Insights

The detailed process highlights several important calculus principles:

- Power Rule for Integration: When integrating X^{n}, the result is $\frac{X^{n+1}}{n+1}$ \

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