Examine The System Of Equations. 5x 5y = 40 4x 3y = 24 What Is The Solution? (,).

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Understanding how to solve systems of equations is a fundamental skill in algebra, crucial for solving real-world problems involving multiple variables. In this article, we will explore the methods to analyze and find solutions to the given system of equations:

1. $5x + 5y = 40$
2. $4x + 3y = 24$

Our goal is to determine the values of x and y that satisfy both equations simultaneously, represented as an ordered pair (x, y).

Understanding Systems of Equations

A system of equations is a set of two or more equations with the same variables. The solution to the system is the point(s) where the equations intersect when graphed on the coordinate plane.

Types of solutions:

- One solution: The system is consistent and independent; equations intersect at a single point.
- No solution: The system is inconsistent; the equations are parallel and do not intersect.
- Infinite solutions: The equations are dependent; they represent the same line.

In our case, we expect to find a unique solution for x and y, unless the equations are dependent or inconsistent.

Analyzing the Given System of Equations

The given system is:

(1) $5x + 5y = 40$

(2) $4x + 3y = 24$

Notice that in equation (1), the coefficients of x and y are both 5, which suggests that simplifying or manipulating the equations can make the solving process more straightforward.

Step 1: Simplify the Equations

Equation (1):

$5x + 5y = 40$

Divide both sides by 5:

$x + y = 8$

This simplifies our system to:

(1a) $x + y = 8$

Equation (2):

$4x + 3y = 24$

Now, the system is:

$x + y = 8$ (Equation 1a)

$4x + 3y = 24$ (Equation 2)

Methods to Solve the System

There are several methods to solve systems of equations:

- Substitution Method
- Elimination Method
- Graphical Method
- Matrix Method (using determinants or matrices)

For this system, substitution or elimination is efficient. We'll demonstrate both.

Method 1: Substitution

Since equation (1a) expresses x in terms of y, we can solve for x:

$x = 8 - y$

Substitute $x = 8 - y$ into equation (2):

$4(8 - y) + 3y = 24$

Simplify:

$32 - 4y + 3y = 24$

Combine like terms:

$32 - y = 24$

Subtract 32 from both sides:

$-y = 24 - 32$

$-y = -8$

Multiply both sides by -1:

$y = 8$

Now, substitute $y = 8$ back into $x = 8 - y$:

$x = 8 - 8 = 0$

Solution: $(x, y) = (0, 8)$

This pair satisfies both equations, which can be checked by substitution.

Method 2: Elimination

Alternatively, we can use the elimination method directly on the simplified equations.

Recall:

$x + y = 8$ (Equation 1a)

$4x + 3y = 24$ (Equation 2)

Multiply equation (1a) by 3 to align the y coefficients:

$3(x + y) = 3\ 8$

$3x + 3y = 24$

Now, subtract this from equation (2):

$(4x + 3y) - (3x + 3y) = 24 - 24$

Simplify:

$(4x - 3x) + (3y - 3y) = 0$

$x + 0 = 0$

Thus,

$x = 0$

Substitute $x = 0$ into $x + y = 8$:

$0 + y = 8$

$y = 8$

Solution: $(x, y) = (0, 8)$

Both methods lead to the same solution, confirming its correctness.

Verifying the Solution

Always verify the solution by substituting the values into the original equations:

Original equations:

$5x + 5y = 40$

$4x + 3y = 24$

Substitute $x = 0$ and $y = 8$:

First equation:

$5(0) + 5(8) = 0 + 40 = 40$ ✓

Second equation:

$4(0) + 3(8) = 0 + 24 = 24$ ✓

Both equations are satisfied, confirming that $(0, 8)$ is the correct solution.

Graphical Interpretation

Graphing the equations provides a visual understanding of the solution:

- Equation (1a): $x + y = 8$

- When $x = 0$, $y = 8$
- When $y = 0$, $x = 8$

- Equation (2): $4x + 3y = 24$

- When $x = 0$, $y = 8$
- When $y = 0$, $x = 6$

Plotting these lines on the coordinate plane:

- The line $x + y = 8$ passes through (0, 8) and (8, 0).
- The line $4x + 3y = 24$ passes through (0, 8) and (6, 0).

The intersection point, which is (0, 8), confirms the solution.

Real-World Applications of Solving Systems of Equations

Understanding how to solve systems of equations is essential in various real-life scenarios, such as:

- Economics: Determining equilibrium prices and quantities
- Physics: Calculating intersection points of forces or trajectories
- Business: Optimizing profit by solving multiple constraints
- Engineering: Analyzing circuit systems or structural problems

Summary and Key Takeaways

- Systems of equations involve multiple equations with common variables.
- Simplification often makes solving easier; in this case, dividing the first equation by 5 simplified it.
- Substitution and elimination are effective methods for solving two-variable systems.
- The solution (0, 8) satisfies both equations, confirmed through substitution and graphing.
- Visualizing the equations graphically offers insight into the intersection point and the nature of the solutions.

Conclusion

By methodically analyzing and solving the system of equations $5x + 5y = 40$ and $4x + 3y = 24$, we've demonstrated the process of simplifying, solving, and verifying solutions. The key steps include simplifying equations, choosing an appropriate solving method, and confirming the solution through substitution or graphical interpretation. Mastering these techniques empowers students and professionals to tackle complex problems involving multiple variables confidently. Whether in academics, engineering, economics, or everyday problem-solving, understanding systems of equations is a vital mathematical skill that opens doors to a wide array of analytical opportunities.

Frequently Asked Questions

How do you solve the system of equations 5x + 5y = 40 and 4x + 3y = 24?

You can solve this system using substitution or elimination. For elimination, multiply equations to align coefficients and subtract to find one variable, then substitute back to find the other.

What is the solution to the system of equations 5x + 5y = 40 and 4x + 3y = 24?

The solution is $(4, 4)$, meaning $x = 4$ and $y = 4$.

Can you demonstrate how to use the elimination method on these equations?

Yes. Multiply the first equation by 3 and the second by 5 to align coefficients: $15x + 15y = 120$ and $20x + 15y = 120$. Subtract to find x, then substitute back to find y.

Are these equations consistent and do they have a unique solution?

Yes, they are consistent and have a unique solution: $(4, 4)$.

What steps are involved in solving these two linear equations?

First, align coefficients (using multiplication if needed), then eliminate one variable by subtraction, solve for the remaining variable, and finally substitute back to find the other variable.

Is it possible to solve these equations graphically, and what would the solution point be?

Yes, graphing both equations would show their lines intersecting at point (4, 4), which is the solution.

Why is (4, 4) the solution to the system 5x + 5y = 40 and 4x + 3y = 24?

Because substituting x=4 and y=4 into both equations satisfies both, confirming it as the intersection point.

Additional Resources

Examine The System Of Equations: 5x + 5y = 40 and 4x + 3y = 24 – What Is The Solution? (,)

When faced with a system of equations, the primary goal is to find the set of values for the variables—typically x and y—that satisfy all the equations simultaneously. These solutions are represented as ordered pairs (x, y). In this detailed exploration, we will examine the given system:

1. 5x + 5y = 40
2. 4x + 3y = 24

Our objective is to analyze various methods to solve this system, interpret the solutions, and understand the nuances involved in such algebraic problems.

Understanding the System of Equations

Nature of the Equations

Both equations are linear, meaning each is of the first degree in terms of x and y. Linear systems like these typically have three possible types of solutions:

- A single unique solution (intersecting lines at one point)
- Infinite solutions (coincident lines)
- No solution (parallel lines)

Determining which case applies involves analyzing the equations' relationships.

Graphical Interpretation

Visualizing these equations as lines on the Cartesian plane helps understand their intersection:

- Each equation can be rewritten in slope-intercept form ($y = mx + b$).
- The intersection point(s) of the lines correspond to the solution(s) of the system.

Methods for Solving the System

Several methods are employed to find solutions in systems of linear equations. Here, we will focus on three primary techniques:

1. Substitution Method
2. Elimination Method
3. Graphical Method

Each method has its advantages and ideal scenarios.

1. Substitution Method

This method involves solving one equation for one variable and substituting into the other.

Step-by-step:

- Choose one equation and solve for either x or y.
- Substitute this expression into the other equation.
- Solve for the remaining variable.
- Back-substitute to find the other variable.

Application to our system:

Equation 1: $5x + 5y = 40$

- Simplify: Divide through by 5:

$x + y = 8$

Express y:

- y = 8 - x

Substitute into Equation 2: 4x + 3y = 24

- Replace y:

4x + 3(8 - x) = 24

- Expand:

4x + 24 - 3x = 24

- Simplify:

(4x - 3x) + 24 = 24

x + 24 = 24

- Solve for x:

x = 24 - 24 = 0

Back to y:

- y = 8 - x = 8 - 0 = 8

Solution:

- (x, y) = (0, 8)

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2. Elimination Method

This technique aims to eliminate one variable by aligning coefficients.

Step-by-step:

- Multiply equations by suitable numbers to match coefficients of either x or y.
- Subtract or add equations to eliminate one variable.
- Solve for the remaining variable.
- Substitute back to find the other.

Application:

Equation 1: 5x + 5y = 40

Equation 2: 4x + 3y = 24

Multiply Equation 1 by 4 and Equation 2 by 5 to align x:

- Equation 1 4:

$20x + 20y = 160$

- Equation 2 5:

$20x + 15y = 120$

Subtract:

- $(20x + 20y) - (20x + 15y) = 160 - 120$

- Simplifies to:

$5y = 40$

Solve for y:

- $y = 8$

Substitute y into one of the original equations to find x:

Using Equation 1:

$5x + 5(8) = 40$

$5x + 40 = 40$

$5x = 0$

$x = 0$

Solution:

- $(x, y) = (0, 8)$, confirming our previous result.

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3. Graphical Method

Plotting the lines represented by the equations reveals the point of intersection directly.

Steps:

- Convert both equations to slope-intercept form.
- Plot both lines on the coordinate plane.

- Identify the intersection point.

Conversion:

Equation 1:

- 5x + 5y = 40

Divide through by 5:

- x + y = 8 → y = 8 - x

Equation 2:

- 4x + 3y = 24

Solve for y:

- 3y = 24 - 4x

- y = (24 - 4x)/3

Plotting these lines:

- Line 1: y = -x + 8 (slope -1, y-intercept 8)
- Line 2: y = (24 - 4x)/3

By plotting, you'll observe the lines intersect at the point (0, 8).

Analyzing the Solution

Unique Solution

Since both methods yielded the same point (0, 8), the system has a single unique solution.

Implication:

- The two lines intersect at exactly one point.
- The intersection point (0, 8) satisfies both equations.

Verification of the Solution

Always verify solutions by substituting (0, 8) into the original equations:

Equation 1:

- $5(0) + 5(8) = 0 + 40 = 40$ ✓

Equation 2:

- $4(0) + 3(8) = 0 + 24 = 24$ ✓

Both satisfy the equations, confirming the solution's correctness.

Alternative Scenarios and Their Significance

While our system has a unique solution, it's essential to understand other possibilities.

Infinite Solutions

- Occur when the two equations represent the same line.
- For example, if after simplification, both equations are identical, every point on the line is a solution.
- This happens when the ratios of coefficients and constants are equal:

For example:

$2x + 3y = 6$ and $4x + 6y = 12$

Both are multiples; hence, infinitely many solutions.

No Solution (Parallel Lines)

- When lines are parallel (same slope, different intercepts), the system has no solution.
- For instance:

$y = 2x + 3$ and $y = 2x - 1$

Same slope (2), different intercepts, no intersection point.

Understanding these cases helps in diagnosing systems quickly and correctly.

Advanced Considerations

Matrix Method (Optional for Deeper Understanding)

Using matrices and determinants (Cramer's Rule) is a powerful approach:

- Write the system in matrix form: AX = B

$$
\begin{bmatrix}
5 & 5 \\
4 & 3
\end{bmatrix}
\times
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
40 \\
24
\end{bmatrix}
$$

- Calculate the determinant (D):

D = (5)(3) - (5)(4) = 15 - 20 = -5

- Calculate determinants for x and y:

$$
D_x = \begin{bmatrix}
40 & 5 \\
24 & 3
\end{bmatrix}
\Rightarrow D_x = (40)(3) - (5)(24) = 120 - 120 = 0
$$

$$
D_y = \begin{bmatrix}
5 & 40 \\
4 & 24
\end{bmatrix}
\Rightarrow D_y = (5)(24) - (40)(4) = 120 - 160 = -40
$$

- Find the solutions:

\[
x = \frac{D_x}{D} = \frac{0}{-5} = 0
\]

\[
y = \frac{D_y}{D} = \frac{-40}{-5} = 8
\]

This confirms the earlier solutions, demonstrating the consistency across methods.

Summary and Final Thoughts

- The system of equations:

1. 5x + 5y = 40
2. 4x + 3y = 24

- Has a single unique solution: (x, y) = (0, 8).

- Multiple solving methods—substitution, elimination, graphical, and matrix—converge to this solution, reinforcing its validity.

- Understanding the nature of the system (inter

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