Expand And Simplify (4 + Root2) (6 - Root2)Give Your Answer In The Form B + C Root2

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Introduction to the Problem

In algebra, expanding and simplifying expressions involving conjugates and radicals is a fundamental skill. The problem at hand—expanding and simplifying the expression $(4 + \sqrt{2})(6 - \sqrt{2})$ and presenting the answer in the form $B + C\sqrt{2}$—serves as an excellent example of these techniques. Understanding how to manipulate such expressions not only enhances your algebraic proficiency but also prepares you for more advanced topics like rationalizing denominators and solving radical equations.

This article will guide you through the step-by-step process of expanding the given expression, simplifying it properly, and expressing the result in the form $B + C\sqrt{2}$. We will also explore related concepts, tips for dealing with radical expressions, and common mistakes to avoid.

Understanding the Components of the Expression

Before diving into the expansion, it's essential to understand the components involved:

- Radicals (Square Roots): The $\sqrt{2}$ term is an irrational number that cannot be simplified further since $\sqrt{2}$ is already in its simplest radical form.

- Binomials: Both $(4 + \sqrt{2})$ and $(6 - \sqrt{2})$ are binomials, expressions consisting of two terms.

- Conjugates: The terms $(a + \sqrt{b})$ and $(a - \sqrt{b})$ are conjugates. When multiplied, their product results in a difference of squares, which simplifies the process.

Step-by-Step Guide to Expand (4 + √2)(6 - √2)

Expanding the product involves applying the distributive property, often called the FOIL method (First, Outer, Inner, Last). Let's proceed systematically.

Step 1: Apply the FOIL Method

- First: Multiply the first terms of each binomial:

$4 \times 6 = 24$

- Outer: Multiply the outer terms:

$4 \times (-\sqrt{2}) = -4\sqrt{2}$

- Inner: Multiply the inner terms:

$\sqrt{2} \times 6 = 6\sqrt{2}$

- Last: Multiply the last terms:

$\sqrt{2} \times (-\sqrt{2}) = -(\sqrt{2} \times \sqrt{2}) = -2$

Step 2: Combine Like Terms

Now, combine all the products:

$$
24 - 4\sqrt{2} + 6\sqrt{2} - 2
$$

Group the like terms:

- Constant terms: $24 - 2 = 22$
- Radical terms: $-4\sqrt{2} + 6\sqrt{2} = 2\sqrt{2}$

Step 3: Write the Final Expression

Putting it all together:

$$
22 + 2\sqrt{2}
$$

This is the expanded form of the original expression.

Expressing the Result in the Form B + C√2

The goal is to write the simplified expression as $B + C\sqrt{2}$. From the previous step, we have:

$$
22 + 2\sqrt{2}
$$

Hence,

- $B = 22$
- $C = 2$

Final answer:

$$
\boxed{22 + 2\sqrt{2}}
$$

Additional Tips for Simplifying Radical Expressions

To efficiently work with radicals and binomials, keep these tips in mind:

- Remember the difference of squares: When multiplying conjugates $(a + \sqrt{b})(a - \sqrt{b})$, the result is $a^2 - b$. This shortcut saves time and effort.

- Simplify radicals when possible: Always check if radicals can be simplified further. For √2, it’s already in simplest form.

- Combine like terms carefully: When adding or subtracting radical terms, only combine those with the same radical part.

- Be mindful of signs: Pay attention to negative signs, especially when multiplying radicals.

Common Mistakes to Avoid

While expanding and simplifying expressions involving radicals, students often encounter these pitfalls:

1. Forgetting to multiply the radicals correctly: Remember that $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.

2. Incorrectly applying the difference of squares: When multiplying conjugates, ensure you recognize the pattern to simplify quickly.

3. Mismanaging signs: Keep track of positive and negative signs during multiplication to avoid errors.

4. Not simplifying radicals fully: Always check if radicals can be simplified further before multiplying.

Practical Applications of Expanding Binomials with Radicals

Understanding how to expand and simplify expressions like $(4 + \sqrt{2})(6 - \sqrt{2})$ has practical applications in various fields:

- Algebraic Manipulations: Fundamental for solving equations involving radicals.

- Rationalizing Denominators: When radicals appear in denominators, expansion techniques help in rationalizing them.

- Geometry and Trigonometry: Calculations involving lengths, areas, or angles often involve radicals and binomials.

- Physics and Engineering: Many formulas involve radicals, and simplifying expressions is crucial for accurate computation.

Further Practice Problems

To reinforce your understanding, try expanding and simplifying the following expressions:

1. $(3 + \sqrt{5})(7 - \sqrt{5})$

2. $(2 + \sqrt{3})(5 - \sqrt{3})$

3. $(1 + \sqrt{7})(4 - \sqrt{7})$

Solutions:

- Use the same step-by-step approach: FOIL, combine like terms, express in $B + C\sqrt{2}$ form.

Conclusion

Expanding and simplifying algebraic expressions involving radicals is a vital skill in mathematics. The process of multiplying binomials, especially conjugates, reveals the elegance of algebraic identities like the difference of squares. In our example, we successfully expanded $(4 + \sqrt{2})(6 - \sqrt{2})$, simplified the expression to $22 + 2\sqrt{2}$, and expressed it in the desired form $B + C\sqrt{2}$.

Mastering these techniques enhances your problem-solving toolkit, making complex radical expressions manageable and paving the way for tackling more advanced mathematical concepts. Remember to practice regularly, watch for sign errors, and always look for opportunities to apply algebraic identities to simplify your work.

Keywords: expand and simplify, radicals, conjugates, difference of squares, algebra, binomials, radical expressions, mathematical skills, simplifying radicals

Frequently Asked Questions

How do I expand the expression $(4 + \sqrt{2})(6 - \sqrt{2})$?

To expand, apply the distributive property: $(4)(6) + (4)(-\sqrt{2}) + (\sqrt{2})(6) + (\sqrt{2})(-\sqrt{2})$. Simplify to get $24 - 4\sqrt{2} + 6\sqrt{2} - 2$.

What is the simplified form of $(4 + \sqrt{2})(6 - \sqrt{2})$ in the form $B + C\sqrt{2}$?

Simplify the expression to get $22 + 2\sqrt{2}$, so the answer is $22 + 2\sqrt{2}$.

How do I convert 22 + 2√2 into the form B + C√2?

It's already in the form B + C√2 with B = 22 and C = 2.

What is the step-by-step process to expand and simplify (4 + √2)(6 - √2)?

First, multiply each term: 4×6 = 24, 4×(-√2) = -4√2, √2×6 = 6√2, √2×(-√2) = -2. Then, combine like terms: 24 + (-4√2 + 6√2) - 2 = 24 + 2√2 - 2 = 22 + 2√2.

Why does (√2)(-√2) equal -2 in the expansion?

Because √2 × √2 = 2, and with a negative sign, it becomes -2.

Can you verify the expansion of (4 + √2)(6 - √2) using the difference of squares?

Yes, it can be viewed as (4 + √2)(6 - √2) = (4)(6) + (4)(-√2) + (√2)(6) - (√2)(√2). Simplifying as before confirms the result is 22 + 2√2.

Additional Resources

Expand And Simplify (4 + √2)(6 - √2) Give Your Answer In The Form B + C√2

When tackling algebraic expressions involving binomials with radicals, such as Expand And Simplify (4 + √2)(6 - √2), it's essential to approach the problem systematically. The goal here is to expand the product fully, simplify the resulting expression, and express the answer in the form B + C√2, where B and C are rational numbers. This process not only reinforces fundamental algebraic skills but also deepens understanding of how radicals interact within algebraic expressions.

Understanding the Problem

Before diving into the calculations, let's clarify the key objectives:

- Expand: Apply the distributive property (FOIL or binomial expansion) to multiply the two binomials.
- Simplify: Combine like terms and simplify radical expressions where possible.
- Express in the form B + C√2: Rearrange the simplified expression to clearly identify the rational part B and the radical part C√2.

This problem provides an excellent opportunity to revisit binomial multiplication involving radicals and to practice presenting answers in a standardized form.

Step 1: Recognize the Binomials and Their Components

The given expression is:

$(4 + \sqrt{2})(6 - \sqrt{2})$

- First binomial: $4 + \sqrt{2}$
- Second binomial: $6 - \sqrt{2}$

Both are binomials containing rational numbers and radicals.

Step 2: Expand Using the FOIL Method

The FOIL method stands for:

- First: Multiply the first terms of each binomial
- Outer: Multiply the outer terms
- Inner: Multiply the inner terms
- Last: Multiply the last terms

Applying FOIL:

1. First: $4 \times 6 = 24$
2. Outer: $4 \times (-\sqrt{2}) = -4\sqrt{2}$
3. Inner: $\sqrt{2} \times 6 = 6\sqrt{2}$
4. Last: $\sqrt{2} \times (-\sqrt{2}) = - (\sqrt{2} \times \sqrt{2})$

Let's evaluate each:

- The first term: 24 (rational)
- The outer term: $-4\sqrt{2}$
- The inner term: $6\sqrt{2}$
- The last term: $- (\sqrt{2} \times \sqrt{2})$

Recall that $\sqrt{2} \times \sqrt{2} = 2$ (since $\sqrt{a} \times \sqrt{a} = a$). So:

- Last term: -2

Putting it all together:

$24 - 4\sqrt{2} + 6\sqrt{2} - 2$

Step 3: Combine Like Terms

Now, combine the rational parts and radical parts separately:

- Rational parts: $24 - 2 = 22$
- Radical parts: $-4\sqrt{2} + 6\sqrt{2} = (6\sqrt{2} - 4\sqrt{2}) = 2\sqrt{2}$

So, the simplified expression becomes:

$22 + 2\sqrt{2}$

Step 4: Express in the Form $B + C\sqrt{2}$

The problem requests the answer in the form $B + C\sqrt{2}$, where B and C are rational numbers:

$B = 22$

$C = 2$

Thus, the final answer is:

$22 + 2\sqrt{2}$

Additional Tips for Expansion and Simplification

When working with binomials involving radicals, keep these tips in mind:

- Always perform multiplication carefully, respecting the distributive property.
- Remember that $\sqrt{a} \times \sqrt{a} = a$ for any positive a.
- Combine like terms after expansion, especially radicals.
- Express your answer in the simplest radical form, and in the desired format.

Common Mistakes to Avoid

- Forgetting to multiply the radicals properly: For example, confusing $\sqrt{2} \times \sqrt{2}$ with $\sqrt{4}$ or missing the negative signs.
- Not combining like terms: Always double-check radicals and rational parts before finalizing your answer.
- Misplacing signs: Keep track of plus and minus signs carefully during expansion.

Summary of the Solution

Step	Description	Result
1	Recognize the binomials	$(4 + \sqrt{2})(6 - \sqrt{2})$
2	Expand using FOIL	$24 - 4\sqrt{2} + 6\sqrt{2} - 2$
3	Combine like terms	$22 + 2\sqrt{2}$
4	Write in form $B + C\sqrt{2}$	$22 + 2\sqrt{2}$

Final Answer:

The product of $(4 + \sqrt{2})$ and $(6 - \sqrt{2})$ simplified and expressed in the form $B + C\sqrt{2}$ is:

$22 + 2\sqrt{2}$

Conclusion

Mastering the process of expanding and simplifying expressions involving radicals is crucial for success in algebra. By systematically applying the FOIL method, carefully simplifying radicals, and organizing terms, you can confidently handle a wide range of similar expressions. Remember to always express your final answer clearly in the form $B + C\sqrt{2}$, as it is a standard way of presenting solutions involving radicals. Keep practicing with various binomials to strengthen your skills and become more comfortable with algebraic manipulations involving radicals.

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