Find A Quadratic Equation Whose Sum And Product Of The Roots Are 7 And 5 Respectively.

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Understanding how to find a quadratic equation given specific conditions about its roots is a fundamental concept in algebra. In particular, when the sum and product of the roots are provided, it becomes straightforward to construct the quadratic equation using Vieta’s formulas. This article provides a comprehensive guide to find a quadratic equation whose roots have a sum of 7 and a product of 5, including detailed explanations, step-by-step procedures, and practical examples. Whether you're a student preparing for exams or someone interested in algebraic problem-solving, this guide will enhance your understanding and skills.

Understanding Quadratic Equations and Roots

What is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$$ax^2 + bx + c = 0$$

where:
- $a \neq 0$,
- b and c are real numbers,
- x represents the variable.

Quadratic equations are fundamental in algebra because they model many real-world phenomena such as projectile motion, area calculations, and optimization problems.

The Roots of a Quadratic Equation

Roots (or solutions) of a quadratic equation are the values of x that satisfy the equation, making it true. These roots can be real or complex numbers depending on the discriminant $\Delta = b^2 - 4ac$.

The roots are often denoted as r_1 and r_2. The relationship between the roots and the coefficients of the quadratic equation is described by Vieta’s formulas:

- Sum of roots: $r_1 + r_2 = -\frac{b}{a}$
- Product of roots: $r_1 \times r_2 = \frac{c}{a}$

These formulas are central to solving problems where roots are known or given, and the goal is to find the corresponding quadratic equation.

Using Vieta’s Formulas to Find the Quadratic Equation

When given the sum and product of roots, you can directly construct the quadratic equation by reversing Vieta’s relationships.

Step-by-Step Approach

1. Identify the sum and product of the roots:
- Sum $r_1 + r_2 = S$
- Product $r_1 \times r_2 = P$

2. Translate the sum and product into the quadratic form:
- The quadratic equation with roots r_1 and r_2 can be written as:

$$x^2 - (r_1 + r_2) x + r_1 r_2 = 0$$

- Using the given values:

$$x^2 - S x + P = 0$$

3. Substitute the known values:
- For our specific problem, $S = 7$ and $P = 5$:

$$x^2 - 7x + 5 = 0$$

4. Verify the roots (optional):
- Solve the quadratic to find the roots and verify their sum and product.

Constructed Quadratic Equation

Based on the above steps, the quadratic equation whose roots have a sum of 7 and a product of 5 is:

$$\boxed{x^2 - 7x + 5 = 0}$$

This is the most straightforward way to find the quadratic equation from the roots' sum and product.

Example Problems and Solutions

Example 1: Find the quadratic equation with roots summing to

7 and product 5

Solution:
- Use the formula:

$$ x^2 - (r_1 + r_2) x + r_1 r_2 = 0 $$

- Substitute $r_1 + r_2 = 7$ and $r_1 r_2 = 5$:

$$ x^2 - 7x + 5 = 0 $$

- Answer: The quadratic equation is $x^2 - 7x + 5 = 0$.

Example 2: Find roots of the quadratic and verify their sum and product

Solution:
- Given quadratic: $x^2 - 7x + 5 = 0$.
- Use the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

Here, $a=1$, $b=-7$, $c=5$.

- Calculate discriminant:

$$ \Delta = (-7)^2 - 4 \times 1 \times 5 = 49 - 20 = 29 $$

- Roots:

$$ x = \frac{7 \pm \sqrt{29}}{2} $$

- Sum of roots:

$$ \frac{7 + \sqrt{29}}{2} + \frac{7 - \sqrt{29}}{2} = 7 $$

- Product of roots:

$$ \left(\frac{7 + \sqrt{29}}{2}\right) \times \left(\frac{7 - \sqrt{29}}{2}\right) = \frac{(7)^2 - (\sqrt{29})^2}{4} = \frac{49 - 29}{4} = \frac{20}{4} = 5 $$

- Verification confirms the roots satisfy the original conditions.

Additional Methods to Find the Quadratic Equation

While the primary method involves directly using the sum and product, there are alternative approaches and considerations.

Method 1: Using Factoring

If roots are rational and factors are known or easily guessed, you can factor the quadratic directly:

- Search for two numbers that multiply to $P=5$ and add to $S=7$.
- Factors of 5 are 1 and 5.
- Sum: 1 + 5 = 6, which is not 7.
- No other integer factors; hence, roots are irrational, and factoring over integers isn't straightforward.

Method 2: Completing the Square or Graphical Approach

These are less direct but can provide insights or approximate roots, especially for more complex scenarios.

Practical Applications of Quadratic Equations with Given Roots

Understanding how to construct quadratic equations based on roots' sums and products has diverse applications:

- Physics: Modeling projectile trajectories where initial velocity and time are related.
- Engineering: Designing systems with specific resonance frequencies.
- Economics: Calculating profit or cost functions with certain constraints.
- Mathematics Education: Enhancing problem-solving skills and understanding algebraic relationships.

Key Takeaways and Tips

- Always remember Vieta’s formulas link roots and coefficients directly.
- When given roots' sum and product, the quadratic equation can be constructed easily using the standard form $x^2 - S x + P = 0$.
- Verify solutions by solving the quadratic and checking the sum and product of roots.
- Practice with various values to strengthen understanding.

Conclusion

Finding a quadratic equation whose roots have specific sum and product is a fundamental skill in algebra, rooted in Vieta’s formulas. For the case where the sum is 7 and the product is 5, the quadratic equation is simply $x^2 - 7x + 5 = 0$. This method can be applied broadly to similar problems, making it an essential technique for students and professionals alike. Mastering this process enhances problem-solving efficiency and deepens understanding of quadratic functions and their properties.

Remember: The key is to start with the known sum and product, then use the standard quadratic form derived from these values. Practice with different sets of roots to become proficient in constructing quadratic equations from root conditions.

Frequently Asked Questions

How do you find a quadratic equation when the sum and product of its roots are given?

Use the relationships x + y = sum of roots and xy = product of roots, then form the quadratic as x^2 - (sum)x + (product) = 0.

What is the quadratic equation when the sum of roots is 7 and the product is 5?

The quadratic equation is x^2 - 7x + 5 = 0.

How can I verify if a quadratic equation has roots with sum 7 and product 5?

Calculate the roots of the quadratic and check if their sum is 7 and their product is 5, or use the relationships directly in the quadratic formula.

Are there multiple quadratic equations with the same sum and product of roots?

No, for given sum and product, the quadratic equation x^2 - (sum)x + (product) = 0 is unique.

What is the discriminant of the quadratic equation with roots summing to 7 and multiplying to 5?

The discriminant is (sum)^2 - 4(product) = 7^2 - 45 = 49 - 20 = 29.

Can I find the roots of the quadratic equation x^2 - 7x + 5 = 0?

Yes, using the quadratic formula: x = [7 ± sqrt(29)] / 2.

What are the approximate roots of the quadratic x^2 - 7x + 5 = 0?

The roots are approximately (7 + sqrt(29))/2 ≈ 6.19 and (7 - sqrt(29))/2 ≈ 0.81.

How does the sum and product of roots relate to the coefficients of a quadratic?

In a quadratic ax^2 + bx + c = 0, the sum of roots is -b/a and the product is c/a.

Can this method be used for quadratic equations with different sum and product values?

Yes, by substituting the given sum and product into the quadratic formula, you can find the specific equation for any roots.

Additional Resources

Find A Quadratic Equation Whose Sum And Product Of The Roots Are 7 And 5 Respectively

When tackling quadratic equations, one of the fundamental concepts involves understanding the relationship between the roots of the equation and its coefficients. Specifically, given the sum and product of the roots, you can determine the quadratic equation itself. In this comprehensive guide, we will explore find a quadratic equation whose sum and product of the roots are 7 and 5 respectively, providing you with a step-by-step methodology, examples, and insights to master this essential algebraic skill.

Understanding the Basics: Roots, Coefficients, and Vieta's Formulas

Before diving into the problem, it's crucial to understand the foundational principles that connect roots and coefficients in quadratic equations.

What is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$ax^2 + bx + c = 0$

where:
- $a \neq 0$
- b and c are real numbers

Roots of a Quadratic Equation

The roots (also called solutions or zeros) of the quadratic are the values of x that satisfy the equation. They can be real or complex, and their properties are linked to the coefficients.

Vieta's Formulas

Vieta's formulas provide a direct relationship between the roots and coefficients of a polynomial. For a quadratic equation with roots α and β:

- Sum of roots: $\alpha + \beta = -b / a$
- Product of roots: $\alpha \beta = c / a$

This means once you know the sum and product of roots, you can reconstruct the quadratic equation, especially if you choose a convenient value for a.

Step-by-Step Approach to Find the Quadratic Equation

Let's now focus on the problem: Find a quadratic equation whose sum of roots is 7 and product of roots is 5.

Step 1: Recall Vieta's Formulas

Given:
- Sum of roots, $\alpha + \beta = S = 7$
- Product of roots, $\alpha \beta = P = 5$

From Vieta's formulas for the quadratic $ax^2 + bx + c = 0$:

- $\alpha + \beta = -b / a$
- $\alpha \beta = c / a$

Step 2: Choose a convenient value for a

Typically, to simplify calculations, choose a = 1. This makes the quadratic monic (leading coefficient 1), simplifying the formulae:

- $\alpha + \beta = -b / 1 = -b$
- $\alpha \beta = c / 1 = c$

Thus:

- $-b = 7 \rightarrow b = -7$
- $c = 5$

Step 3: Write the quadratic equation

Using these coefficients:

$x^2 + bx + c = 0$

becomes:

$x^2 - 7x + 5 = 0$

This is the quadratic equation whose roots have a sum of 7 and a product of 5.

Variations and Additional Considerations

While setting $a = 1$ is the simplest approach, you can choose any non-zero coefficient for a to generate different but equivalent quadratic equations.

General form with arbitrary leading coefficient

Suppose a is any non-zero real number, then:

- $b = -a$ (sum of roots) $= -a\ 7$
- $c = a$ (product of roots) $= a\ 5$

The general quadratic equation is:

$a x^2 + (-a\ 7) x + (a\ 5) = 0$

or equivalently,

$a x^2 - 7a x + 5a = 0$

You can choose any value for a to generate an infinite family of quadratic equations satisfying the given roots' sum and product.

Practical Examples

Let's explore some explicit examples with different values of a:

Example 1: $a = 2$

Equation:

$2x^2 - 14x + 10 = 0$

Example 2: $a = 3$

Equation:

$3x^2 - 21x + 15 = 0$

Example 3: $a = 0.5$

Equation:

$0.5x^2 - 3.5x + 2.5 = 0$

All these equations have roots with sum 7 and product 5, just scaled differently.

Verifying the Roots

It's good practice to verify that the roots of the derived quadratic satisfy the given sum and product.

Step 1: Find the roots using the quadratic formula

Given quadratic:

$ax^2 + bx + c = 0$

Roots are:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Step 2: Apply to our example

Using the monic quadratic:

$x^2 - 7x + 5 = 0$

Discriminant:

$\Delta = (-7)^2 - 4 1 5 = 49 - 20 = 29$

Roots:

$x = [7 \pm \sqrt{29}] / 2$

Sum:

$x_1 + x_2 = (7 + \sqrt{29})/2 + (7 - \sqrt{29})/2 = 7$

Product:

$x_1 x_2 = [(7 + \sqrt{29})/2] [(7 - \sqrt{29})/2] = [(7)^2 - (\sqrt{29})^2] / (22) = (49 - 29) / 4 = 20/4 = 5$

This confirms the roots have the desired sum and product.

Additional Tips and Common Mistakes

- Always verify your roots: Calculating roots explicitly can help confirm the correctness of your quadratic.
- Choosing the coefficient a: While setting a=1 simplifies calculations, remember that other choices of a produce equivalent quadratics scaled accordingly.
- Remember the signs: Pay attention to the signs in Vieta’s formulas; the sum of roots relates to -b/a, not b/a.
- Complex roots: If the discriminant is negative, roots are complex conjugates, but the sum and

product formulas still hold.

Summary

To find a quadratic equation whose roots have a specific sum and product:

1. Use Vieta’s formulas to relate roots to coefficients.
2. Decide on a convenient value for the leading coefficient a (often 1 for simplicity).
3. Calculate b and c accordingly:
- b = -a (sum of roots)
- c = a (product of roots)
4. Write the quadratic in the form:

$a x^2 + b x + c = 0$

5. Verify the roots using the quadratic formula if desired.

Applying this to the problem where the sum of roots is 7 and the product is 5, the quadratic equation with a=1 is:

$x^2 - 7x + 5 = 0$

Final Thoughts

Understanding how to find a quadratic equation given the sum and product of its roots is a powerful algebraic skill. It demonstrates the elegance of polynomial relationships and the utility of Vieta’s formulas. Whether you're solving textbook problems or preparing for exams, mastering this technique enhances your problem-solving toolkit and deepens your understanding of quadratic functions and their roots.

Remember, the key is to start with the fundamental relationships and then adapt them to the specific context of your problem. With practice, you'll find it becomes a quick and intuitive process to construct quadratic equations from root information.

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