Find A Quartic Function With The Given X -values As Its Only Real Zeros. X=4 And X=2 .

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Understanding how to construct polynomial functions based on their zeros is a fundamental skill in algebra and calculus. When tasked with creating a quartic function—meaning a polynomial of degree four—that has specific real zeros, it’s essential to understand the relationship between zeros and factors of the polynomial. In this comprehensive guide, we will explore the process of finding a quartic function whose only real zeros are at x=4 and x=2, examining the role of multiplicities, complex zeros, and the general form of such functions.

Understanding Zeros and Factors of Polynomial Functions

What Are Zeros of a Polynomial?

Zeros of a polynomial are the values of x for which the polynomial evaluates to zero. For example, if $f(x)$ is a polynomial and $f(a) = 0$, then $x = a$ is a zero of the polynomial.

Connection Between Zeros and Factors

Each zero corresponds to a factor of the polynomial, typically expressed as $(x - a)$. For real zeros, the polynomial includes factors of the form:
- $(x - a)$ if the zero is at $x = a$,
- with multiplicity indicating how many times that zero appears.

Multiplicity of Zeros

- A zero with multiplicity 1 appears once.
- A zero with multiplicity greater than 1 appears multiple times, e.g., $(x - 4)^2$.

The multiplicity influences the graph's behavior at the zero:
- Odd multiplicity zeros cross the x-axis.
- Even multiplicity zeros touch but do not cross the x-axis.

Conditions for the Quartic Function in This Problem

Given:
- The only real zeros are at $x=4$ and $x=2$.
- Since the polynomial is quartic (degree 4), the total multiplicity of all zeros must sum to 4.

Implications:
- Both zeros are real, and no other real zeros exist.
- Zeros at $x=4$ and $x=2$ may have multiplicities summing to 4 in total.

Possible configurations:
- $x=4$ with multiplicity 3, $x=2$ with multiplicity 1.
- $x=4$ with multiplicity 2, $x=2$ with multiplicity 2.
- $x=4$ with multiplicity 1, $x=2$ with multiplicity 3.

The choice among these depends on the specific shape or behavior desired for the function, but in general, all these are valid options.

Constructing the General Form of the Quartic Function

Based on the zeros and their multiplicities, the general form of the polynomial is:

$$
f(x) = k \cdot (x - 4)^{m} \cdot (x - 2)^{n}
$$

Where:
- k is a non-zero constant (determines the vertical stretch or compression),
- $m + n = 4$,
- $m, n \in \mathbb{N}$.

Possible specific forms:
- $f(x) = k \cdot (x - 4)^3 (x - 2)$,
- $f(x) = k \cdot (x - 4)^2 (x - 2)^2$,
- $f(x) = k \cdot (x - 4) (x - 2)^3$.

Choosing the Constant k

- The constant k influences the overall shape and size of the graph.
- Typically, for simplicity, $k = 1$ is used unless specific conditions (like a point the graph passes through) are given.

Explicit Examples of Quartic Functions with the Given Zeros

Let's construct specific functions based on different zero multiplicity configurations, assuming $k=1$:

Example 1: Zeros at $x=4$ (multiplicity 3) and $x=2$ (multiplicity 1)

$$
f(x) = (x - 4)^3 (x - 2)
$$

Example 2: Zeros at $x=4$ (multiplicity 2) and $x=2$ (multiplicity 2)

$$
f(x) = (x - 4)^2 (x - 2)^2
$$

Example 3: Zeros at $x=4$ (multiplicity 1) and $x=2$ (multiplicity 3)

$$
f(x) = (x - 4) (x - 2)^3
$$

Expanding the Functions for Better Understanding

Expanding these functions allows us to analyze their shape and behavior.

Expansion of $f(x) = (x - 4)^2 (x - 2)^2$

$$
(x - 4)^2 = x^2 - 8x + 16
$$

$$
(x - 2)^2 = x^2 - 4x + 4
$$

Multiply:

$$
f(x) = (x^2 - 8x + 16)(x^2 - 4x + 4)
$$

Using distributive property:

$$
f(x) = x^2 \cdot x^2 - 4x \cdot x^2 + 4 \cdot x^2 - 8x \cdot x^2 + 32x^2 - 8x \cdot 4x + 16 \cdot x^2 - 8x \cdot 4 + 16 \cdot 4
$$

But it's easier to expand systematically:

$$
f(x) = (x^2 - 8x + 16)(x^2 - 4x + 4)
$$

$$
= x^2 \cdot x^2 + x^2 \cdot (-4x) + x^2 \cdot 4 - 8x \cdot x^2 - 8x \cdot (-4x) - 8x \cdot 4 + 16 \cdot x^2 + 16 \cdot (-4x) + 16 \cdot 4
$$

Calculations:
- $x^2 \cdot x^2 = x^4$
- $x^2 \cdot (-4x) = -4x^3$
- $x^2 \cdot 4 = 4x^2$
- $-8x \cdot x^2 = -8x^3$
- $-8x \cdot (-4x) = 32x^2$
- $-8x \cdot 4 = -32x$
- $16 \cdot x^2 = 16x^2$
- $16 \cdot (-4x) = -64x$
- $16 \cdot 4 = 64$

Combine like terms:

$$
f(x) = x^4 + (-4x^3 - 8x^3) + (4x^2 + 32x^2 + 16x^2) + (-32x - 64x) + 64
$$

$$
= x^4 - 12x^3 + 52x^2 - 96x + 64
$$

Thus:

$$
f(x) = x^4 - 12x^3 + 52x^2 - 96x + 64
$$

\]

This explicit form helps in analyzing the function’s behavior, critical points, and graph.

Graphical Behavior and Zero Multiplicities

Understanding how multiplicities affect the graph:
- Zeros at \(x=4\) and \(x=2\) are the only real zeros.
- The multiplicity determines whether the graph crosses or touches the x-axis at these zeros:
- Odd multiplicity (1, 3): the graph crosses the x-axis.
- Even multiplicity (2, 4): the graph touches the x-axis and turns back.

For example:
- In the case \(f(x) = (x - 4)^2 (x - 2)^2\), the graph touches the x-axis at \(x=4\) and \(x=2\) but does not cross it.

Ensuring the Function Has No Other Real Zeros

Since the problem specifies only \(x=4\) and \(x=2\) as real zeros, ensure:
- No other real zeros exist in the factorization.
- The polynomial is constructed solely from these zeros with the appropriate multiplicities.

Complex zeros come in conjugate pairs and do not affect the real zeros, so they are not relevant unless specified.

Summary and Final Steps

To find a quartic function with the given zeros:
1. Identify the zeros: \(x=

Frequently Asked Questions

How do I find a quartic function with only real zeros at

x=4 and x=2?

Start by expressing the polynomial as a product of its factors, such as $(x - 4)^m$ and $(x - 2)^n$, where m and n are their multiplicities. Since the polynomial is quartic, $m + n = 4$. Choose multiplicities accordingly and expand to find the explicit form.

What multiplicities should I assign to the zeros at x=4 and x=2 to get a quartic function?

Since the total degree is 4, and the zeros are at x=4 and x=2, possible multiplicity combinations include (3,1), (2,2), or (1,3). For simplicity, choosing (2,2) gives a symmetric quartic like $(x - 4)^2 (x - 2)^2$.

Can a quartic function with only the zeros x=4 and x=2 have complex zeros?

No. If the zeros are only at x=4 and x=2 and these are the only real zeros, then all zeros are real, and the quartic can be written using real factors. Complex zeros would come in conjugate pairs, which would add additional zeros beyond these two points.

How do I write the general form of the quartic polynomial with zeros at x=4 and x=2?

Assuming multiplicities of 2 for each zero, the polynomial is $f(x) = a(x - 4)^2(x - 2)^2$, where a is a non-zero constant determining the leading coefficient.

How do I expand (x - 4)^2(x - 2)^2 to get the explicit quartic form?

First expand each quadratic: $(x - 4)^2 = x^2 - 8x + 16$ and $(x - 2)^2 = x^2 - 4x + 4$. Then multiply these two quadratics to get the quartic: $(x^2 - 8x + 16)(x^2 - 4x + 4)$.

What is the expanded form of (x^2 - 8x + 16)(x^2 - 4x + 4)?

Multiplying term-by-term: $x^2 x^2 = x^4$, $x^2 (-4x) = -4x^3$, $x^2 4 = 4x^2$, $-8x x^2 = -8x^3$, $-8x (-4x) = 32x^2$, $-8x 4 = -32x$, $16 x^2 = 16x^2$, $16 (-4x) = -64x$, $16 4 = 64$. Summing these gives: $x^4 - 12x^3 + 52x^2 - 96x + 64$.

What is the final quartic function with zeros at x=4 and x=2?

Assuming a leading coefficient of 1, the quartic function is $f(x) = (x - 4)^2 (x - 2)^2 = x^4 - 12x^3 + 52x^2 - 96x + 64$.

Additional Resources

Finding a Quartic Function With Given X-Values as Its Only Real Zeros: X=4 and X=2

When approaching polynomial functions, especially quartic (degree four) functions, one fundamental task often involves constructing a polynomial with specific zeros. In this case, the goal is to find a quartic function whose only real zeros are at X=4 and X=2. This problem involves understanding polynomial roots, their multiplicities, and how these influence the shape and properties of the graph. In this detailed exploration, we will dissect the problem step-by-step, analyze the underlying concepts, and provide a comprehensive methodology for constructing such a quartic polynomial.

Understanding the Basics of Polynomial Zeros and Their Multiplicities

Before diving into the specifics of constructing the quartic function, it’s crucial to revisit some core concepts:

What Are Polynomial Zeros?

- Zeros (or roots) of a polynomial are the values of x for which the polynomial evaluates to zero.
- For a polynomial $f(x)$, if $f(a) = 0$, then $x = a$ is a zero of the polynomial.

Multiplicity of Zeros

- The multiplicity of a zero refers to how many times that zero appears as a factor in the polynomial's factored form.
- For example, if $f(x) = (x - 2)^3 (x - 4)^2$, then:
- Zero at $x=2$ has multiplicity 3.
- Zero at $x=4$ has multiplicity 2.

Impact of Multiplicity on Graph Behavior

- Zeros with odd multiplicity (like 1, 3, 5, ...) cause the graph to cross the x-axis at that zero.
- Zeros with even multiplicity (2, 4, 6, ...) cause the graph to touch the x-axis and turn around at that zero, without crossing.
- The multiplicity also influences the shape near the zero: higher multiplicities cause the graph to flatten or "stick" to the x-axis more prominently.

Identifying the Constraints for the Quartic Function

Given the problem:

- The only real zeros are at $x=2$ and $x=4$.
- The polynomial is quartic, i.e., degree four.

This implies:

- The zeros are limited to these two points.
- The zeros' multiplicities must sum to 4 (since degree four).

Key implications:

- The polynomial can be expressed in factored form as:

$$
f(x) = k \cdot (x - 2)^{m} \cdot (x - 4)^{n}
$$

where:
- k is a non-zero constant (leading coefficient).
- $m, n \geq 1$, and $m + n = 4$.

- Because these are the only real zeros, and we want the zeros at $x=2$ and $x=4$ to be the only real solutions, the polynomial should have no other real roots.

Possible Combinations of Zero Multiplicities

Since $m + n = 4$, and both $m, n \geq 1$, possible pairs are:

1. $m=1, n=3$
2. $m=3, n=1$
3. $m=2, n=2$

Each combination results in different behaviors and properties:

Case 1: $m=1, n=3$
- Polynomial form: $f(x) = k \cdot (x - 2)^1 \cdot (x - 4)^3$
- Zero at $x=2$ has multiplicity 1: the graph crosses the x-axis at $x=2$.

- Zero at $x=4$ has multiplicity 3: the graph crosses the x-axis at $x=4$, but with a different "shape" (more flattened near the zero).

Case 2: $m=3, n=1$

- Polynomial form: $f(x) = k \cdot (x - 2)^3 \cdot (x - 4)^1$
- Zero at $x=2$ has multiplicity 3: graph crosses with a flattened "cusp."
- Zero at $x=4$ has multiplicity 1: graph crosses normally at $x=4$.

Case 3: $m=2, n=2$

- Polynomial form: $f(x) = k \cdot (x - 2)^2 \cdot (x - 4)^2$
- Both zeros have even multiplicity (2): the graph touches and turns around at both zeros; it does not cross the x-axis at these points but just 'bounces' off.

Important note: Since the problem states that the only real zeros are at $x=2$ and $x=4$, and these are the only zeros, these are the plausible multiplicity combinations.

Choosing the Appropriate Zero Multiplicities Based on Graph Behavior

The key to constructing the correct quartic function is understanding what kind of graph behavior is desired and ensuring the zeros' multiplicities match that behavior.

- If the goal is for the graph to cross the x-axis at both zeros, then the zeros should have odd multiplicities.
- If the goal is for the graph to touch the x-axis and turn around at the zeros, then the zeros should have even multiplicities.

Given the zeros are the only real zeros:

- If the zeros are at $x=2$ and $x=4$ and both have odd multiplicities (e.g., 1 and 3), the graph will cross the x-axis at both points.
- If both are even (e.g., 2 and 2), the graph will touch and turn around at those points.
- Mixed cases are also possible but depend on the specific graph behavior desired.

Constructing Explicit Examples of the Quartic Polynomial

Let’s now formulate explicit polynomial functions for each case, choosing a simple scale factor $k=1$ for simplicity, which can be adjusted later.

Example 1: Zero multiplicities $m=1, n=3$

$$
f(x) = (x - 2)^1 \cdot (x - 4)^3
$$

- Expanded form:

$$
f(x) = (x - 2) \cdot (x - 4)^3
$$

Calculate $(x - 4)^3$:

$$
(x - 4)^3 = x^3 - 12x^2 + 48x - 64
$$

Multiply:

$$
f(x) = (x - 2)(x^3 - 12x^2 + 48x - 64)
$$

Perform the multiplication:

$$
f(x) = x \cdot (x^3 - 12x^2 + 48x - 64) - 2 \cdot (x^3 - 12x^2 + 48x - 64)
$$

$$
= x^4 - 12x^3 + 48x^2 - 64x - 2x^3 + 24x^2 - 96x + 128
$$

Combine like terms:

$$
f(x) = x^4 - (12x^3 + 2x^3) + (48x^2 + 24x^2) + (-64x - 96x) + 128
$$

$$
f(x) = x^4 - 14x^3 + 72x^2 - 160x + 128
$$

Graph behavior:
- Zero at $x=2$ crosses the x-axis.
- Zero at $x=4$ crosses the x-axis, with a cubic multiplicity, resulting in a smoother crossing.

Example 2: Zero multiplicities $m=3, n=1$

$$
f(x) = (x - 2)^3 \cdot (x - 4)
$$

- Expand:

$$
(x - 2)^3 = x^3 - 6x^2 + 12x - 8
$$

- Multiply by $(x - 4)$:

$$
f(x) = (x^3 - 6x^2 + 12x - 8)(x - 4)
$$

- Expand:

$$
x^3 \cdot x = x^4
$$
$$
x^3 \cdot (-4) = -4x^3
$$
$$
-6x^2 \cdot x = -6x^3
$$
$$
-6x^2 \cdot (-4) = 24x^2
$$
$$
12x \cdot
$$

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