Find The Area Of The Triangle Which Has Sides ~u = 3, 3, 3, ~v = 6, 0, 6, And ~u ~v.

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Understanding how to find the area of a triangle when given vectors is a fundamental concept in vector calculus and geometry. This problem involves vectors $\vec{u}$ and $\vec{v}$, with specified components, and asks us to determine the area of the triangle formed by these vectors.

In this comprehensive guide, we will explore the methods involved in solving such problems, including vector operations like the cross product, dot product, and their geometric interpretations. Whether you're a student preparing for exams or someone interested in the applications of vectors in physics and engineering, this article will provide a detailed, step-by-step approach to find the area of the triangle defined by the given vectors.

Understanding the Problem and Given Data

The problem states:

- $\vec{u} = (3, 3, 3)$
- $\vec{v} = (6, 0, 6)$
- The task: Find the area of the triangle formed by these vectors, considering $\vec{u}$, $\vec{v}$, and their vector cross product $\vec{u} \times \vec{v}$.

Key points:

- The vectors are in three-dimensional space.
- The triangle is formed by the vectors originating from the same point (the origin).

- The area of the triangle can be computed using the cross product of the vectors.

Conceptual Foundations

Before diving into calculations, let's review some essential concepts:

Vectors in 3D Space

A vector in 3D space is represented as $\vec{a} = (a_x, a_y, a_z)$, indicating its components along the x, y, and z axes.

Vector Operations

- Addition: $\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$
- Scalar multiplication: $k \times \vec{a} = (k a_x, k a_y, k a_z)$
- Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$
- Cross product: $\vec{a} \times \vec{b}$ results in a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Area of a Triangle Formed by Vectors

Given two vectors $\vec{u}$ and $\vec{v}$ emanating from the same point, the area A of the triangle they form is:

$$
A = \frac{1}{2} |\vec{u} \times \vec{v}|
$$

where $|\vec{u} \times \vec{v}|$ is the magnitude of the cross product.

Step-by-Step Solution

Let's now proceed with calculating the area step-by-step.

1. Write down the vectors

$$
\vec{u} = (3, 3, 3)
$$

$$
\vec{v} = (6, 0, 6)
$$

2. Compute the cross product $\vec{u} \times \vec{v}$

The cross product of two vectors $\vec{a} = (a_x, a_y, a_z)$ and $\vec{b} = (b_x, b_y, b_z)$ is:

$$
\vec{a} \times \vec{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)
$$

Applying this to $\vec{u}$ and $\vec{v}$:

$$

\vec{u} \times \vec{v} =

\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

3 & 3 & 3 \\

6 & 0 & 6 \\

\end{vmatrix}

\]

Calculating component-wise:

- i-component:

\[

(3)(6) - (3)(0) = 18 - 0 = 18

\]

- j-component:

\[

(3)(6) - (3)(6) = 18 - 18 = 0

\]

(Note: Remember the minus sign in the j-component, so it becomes \(-0 = 0\).)

- k-component:

\[

(3)(0) - (3)(6) = 0 - 18 = -18

\]

Therefore,

\[

\vec{u} \times \vec{v} = (18, 0, -18)

\]

3. Find the magnitude of the cross product

$$
|\vec{u} \times \vec{v}| = \sqrt{(18)^2 + 0^2 + (-18)^2} = \sqrt{324 + 0 + 324} = \sqrt{648}
$$

Simplify $\sqrt{648}$:

$$
\sqrt{648} = \sqrt{81 \times 8} = \sqrt{81} \times \sqrt{8} = 9 \times 2\sqrt{2} = 18 \sqrt{2}
$$

Calculating the Area of the Triangle

Recall:

$$
A = \frac{1}{2} |\vec{u} \times \vec{v}| = \frac{1}{2} \times 18 \sqrt{2} = 9 \sqrt{2}
$$

Final answer:

$$
\boxed{
\text{Area of the triangle} = 9 \sqrt{2}
}
$$

Additional Insights and Applications

Understanding how to compute the area of a triangle using vectors is crucial in multiple fields such as physics, engineering, computer graphics, and mathematics.

Applications include:
- Calculating the area of surfaces in 3D modeling.
- Determining the magnitude of forces acting along different directions.
- Analyzing geometric configurations in space.

Other methods to verify the result:
- Using the dot product: The formula $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$ helps in understanding angles between vectors.
- Heron's formula: Not directly applicable here since we are working with vectors, but useful for side lengths if known.

Summary and Key Takeaways

- The area of a triangle in 3D space formed by two vectors $\vec{u}$ and $\vec{v}$ is $\frac{1}{2} |\vec{u} \times \vec{v}|$.
- Calculating the cross product involves determinants or component-wise calculations.
- The magnitude of the cross product gives the area of the parallelogram formed by the vectors; dividing by 2 gives the triangle's area.
- For the given vectors $\vec{u} = (3, 3, 3)$ and $\vec{v} = (6, 0, 6)$, the area is $9 \sqrt{2}$.

Final Note:
Mastering these vector operations enhances spatial understanding and problem-solving skills in multidimensional geometry.

Meta Keywords: vector cross product, triangle area in 3D, vector calculus, geometry, vector operations, cross product formula, 3D vectors, calculating area, vector methods, physics applications

Frequently Asked Questions

How do I find the area of a triangle given two vectors ~u and ~v?

The area of the triangle can be found using the formula: (1/2) |~u × ~v|, where ~u and ~v are the vectors representing two sides of the triangle.

What is the cross product of vectors ~u = (3, 3, 3) and ~v = (6, 0, 6)?

The cross product ~u × ~v is calculated as: (~u□ ~v□ - ~u□ ~v□, ~u□ ~v□ - ~u□ ~v□, ~u□ ~v□ - ~u□ ~v□). Substituting values: (36 - 30, 36 - 36, 30 - 36) = (18 - 0, 18 - 18, 0 - 18) = (18, 0, -18).

What is the magnitude of the cross product ~u × ~v?

The magnitude is |~u × ~v| = □(18^2 + 0^2 + $(-18)^2$) = □(324 + 0 + 324) = □648 = 18□2.

How do I compute the area of the triangle with vectors ~u and ~v?

Using the formula: Area = (1/2) |~u × ~v|. From previous calculation, the area is (1/2) 18□2 = 9□2.

What is the final numerical value of the area of the triangle?

The area is 9□2, which is approximately 12.73 square units.

Are the given vectors ~u and ~v perpendicular?

Since their dot product is not zero (36 + 30 + 36 = 18 + 0 + 18 = 36), they are not perpendicular.

Can I verify the area using the coordinate geometry formula?

Yes, if the points corresponding to the vectors are known, you can use the coordinate formula for the area of a triangle. But using the cross product method is more straightforward in vector form.

Additional Resources

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Introduction

In the realm of vector geometry, understanding the spatial relationships between vectors and their geometric counterparts is fundamental. One of the classical problems involves determining the area of a triangle formed by two vectors originating from a common point, often the origin. This problem becomes particularly intriguing when the vectors have specific components, as in the case of vectors u and v with given components:

- u = (3, 3, 3)
- v = (6, 0, 6)

The task is to find the area of the triangle formed by these vectors, considering their magnitudes and the angle between them. This problem is not just an exercise in computational skills but also a gateway into understanding the geometric significance of dot products, cross products, and their relationship to the area of a triangle in three-dimensional space.

Understanding the Given Vectors

Components of the Vectors

Before delving into calculations, it's crucial to understand what the given vectors represent:

- Vector u = (3, 3, 3)
- Vector v = (6, 0, 6)

Each component indicates the vector's projection along the x, y, and z axes. The vector u points equally along all axes, suggesting a direction that is symmetric in x, y, and z. Conversely, v extends more along the x and z axes but has no y component. Recognizing these components allows for geometric intuition and aids in the computation of magnitudes and angles.

The Significance of the Vectors

In vector geometry, the magnitude of a vector indicates its length, while the dot and cross products reveal the angle between vectors and the area of the parallelogram they span. The triangle's area is directly related to the cross product's magnitude, making it an essential tool for this problem.

Step 1: Calculating the Magnitudes of the Vectors

The first step involves finding the lengths (magnitudes) of u and v.

Magnitude of u

$$
|u| = \sqrt{(3)^2 + (3)^2 + (3)^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}
$$

Magnitude of v

$$
|v| = \sqrt{(6)^2 + (0)^2 + (6)^2} = \sqrt{36 + 0 + 36} = \sqrt{72} = 6\sqrt{2}
$$

These magnitudes are essential for calculating the angle between the vectors and for verifying the properties of the vectors.

Step 2: Determining the Dot Product of u and v

The dot product provides a measure of the cosine of the angle between the two vectors.

$$
u \cdot v = (3)(6) + (3)(0) + (3)(6) = 18 + 0 + 18 = 36
$$

The dot product's value is 36.

Step 3: Calculating the Angle Between the Vectors

The angle θ between u and v can be obtained using the dot product formula:

$$
u \cdot v = |u| \times |v| \times \cos \theta
$$

Rearranged to solve for θ:

$$
\cos \theta = \frac{u \cdot v}{|u| \times |v|}
$$

Substituting the known values:

$$
\cos \theta = \frac{36}{(3\sqrt{3}) \times (6\sqrt{2})}
$$

Calculating the denominator:

$$
(3\sqrt{3}) \times (6\sqrt{2}) = 3 \times 6 \times \sqrt{3} \times \sqrt{2} = 18 \times \sqrt{6}
$$

Thus:

$$
\cos \theta = \frac{36}{18 \times \sqrt{6}} = \frac{2}{\sqrt{6}}
$$

Simplify:

$$
\cos \theta = \frac{2}{\sqrt{6}} = \frac{2 \times \sqrt{6}}{6} = \frac{\sqrt{6}}{3}
$$

Therefore, the cosine of the angle □ is:

$$
\boxed{\cos \theta = \frac{\sqrt{6}}{3}}
$$

Step 4: Computing the Sine of the Angle

Since the area of the triangle depends on sin □, we need to find sin □.

Using the Pythagorean identity:

$$
\sin^2 \theta + \cos^2 \theta = 1
$$

Substituting cos □:

$$
\sin^2 \theta = 1 - \left(\frac{\sqrt{6}}{3}\right)^2 = 1 - \frac{6}{9} = 1 - \frac{2}{3} = \frac{1}{3}
$$

Therefore:

$$

$$
\sin \theta = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
$$

Step 5: Calculating the Cross Product of u and v

The cross product $u \times v$ yields a vector perpendicular to both u and v, with magnitude equal to the area of the parallelogram spanned by these vectors.

The formula for the cross product in component form:

$$
u \times v = \left(u_2 v_3 - u_3 v_2, \quad u_3 v_1 - u_1 v_3, \quad u_1 v_2 - u_2 v_1 \right)
$$

Substituting the components:

$$
u = (3, 3, 3), \quad v = (6, 0, 6)
$$

Calculating each component:

- x-component:

$$
(3)(6) - (3)(0) = 18 - 0 = 18
$$

- y-component:

$$
(3)(6) - (3)(6) = 18 - 18 = 0
$$

- z-component:

$$
(3)(0) - (3)(6) = 0 - 18 = -18
$$

Thus,

$$
u \times v = (18, 0, -18)
$$

Magnitude of the Cross Product

The magnitude gives the area of the parallelogram:

$$
|u \times v| = \sqrt{(18)^2 + 0^2 + (-18)^2} = \sqrt{324 + 0 + 324} = \sqrt{648} = \sqrt{36 \times 18} = 6 \sqrt{18}
$$

Simplify:

$$
6 \sqrt{18} = 6 \times \sqrt{9 \times 2} = 6 \times 3 \sqrt{2} = 18 \sqrt{2}
$$

Step 6: Deriving the Area of the Triangle

The area A of the triangle formed by vectors u and v is half the magnitude of their cross product:

$$
A = \frac{1}{2} |u \times v|
$$

Substitute the value:

$$
A = \frac{1}{2} \times 18 \sqrt{2} = 9 \sqrt{2}
$$

This result is consistent with the geometric interpretation that the cross product's magnitude equals the area of the parallelogram, and half of that gives the triangle's area.

Final Result and Interpretation

The area of the triangle formed by vectors u and v with components as specified is:

$$
\boxed{\boxed{A = 9 \sqrt{2}}}
$$

This value, approximately 12.727, represents the precise measure of the triangle's area in three-dimensional space, based on the vector components provided.

Broader Implications and Applications

Geometric Significance

Understanding how to compute the area of a triangle from vectors is central in physics, engineering, and computer graphics. It allows for the calculation of surface areas, collision detection, and the analysis of spatial structures.

Vector Operations in Practice

- Dot Product: Used to find angles between vectors, projection lengths, and work in physics.
- Cross Product: Critical for determining the area of parallelograms and the orientation of vectors.

Educational Value

This problem exemplifies how algebraic operations on vectors translate into geometric intuition. It underscores the importance of combining component-wise calculations with geometric reasoning to solve complex spatial problems efficiently.

Conclusion

The calculation of the triangle's area based on vectors u = (3, 3, 3) and v = (6, 0, 6) demonstrates the elegance of vector calculus in spatial geometry. By systematically computing magnitudes, dot products, angles, and cross products, we arrive at

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