

Find The Distance Between The Two Points In Simplest Radical Form (-7,-3) And (-3,-5)

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Understanding how to determine the distance between two points on a coordinate plane is a fundamental skill in geometry and algebra. When given two points, such as (-7, -3) and (-3, -5), finding the precise distance involves applying the distance formula derived from the Pythagorean theorem. The goal is to compute this distance and express it in its simplest radical form, which often provides a more exact and elegant answer compared to decimal approximations. This guide will walk you through the entire process, from understanding the foundational concepts to performing the calculations step-by-step, ensuring clarity and confidence in solving similar problems.

Understanding the Distance Formula

What Is the Distance Formula?

The distance formula calculates the straight-line distance between two points in a coordinate plane. Given any two points, (x_1, y_1) and (x_2, y_2) , the distance (d) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is derived from the Pythagorean theorem, which relates the sides of a right triangle. Visualize the two points as vertices of a right triangle, with the differences in their x and y coordinates forming the legs, and the distance being the hypotenuse.

Why Use the Distance Formula?

- To find the shortest path between two points in a plane.
- To solve geometric problems involving length measurements.
- To facilitate further calculations in coordinate geometry, such as finding the midpoint or verifying the length of segments.

Step-by-Step Calculation of the Distance

Let's now apply the distance formula to the specific points: (-7, -3) and (-3, -5).

Step 1: Identify Coordinates

- First point: $((x_1, y_1) = (-7, -3))$
- Second point: $((x_2, y_2) = (-3, -5))$

Step 2: Compute the Differences in Coordinates

Calculate the differences:

$$\Delta x = x_2 - x_1 = -3 - (-7) = -3 + 7 = 4$$

$$\Delta y = y_2 - y_1 = -5 - (-3) = -5 + 3 = -2$$

Step 3: Square the Differences

Square both differences:

$$(\Delta x)^2 = 4^2 = 16$$

$$(\Delta y)^2 = (-2)^2 = 4$$

Step 4: Sum the Squares

Add the squared differences:

$$16 + 4 = 20$$

Step 5: Take the Square Root

Apply the square root to find the distance:

$$d = \sqrt{20}$$

Expressing the Distance in Simplest Radical Form

Simplify $\sqrt{20}$

The radical $\sqrt{20}$ can be simplified by factoring out perfect squares:

$$\begin{aligned} 20 &= 4 \times 5 \end{aligned}$$

Since 4 is a perfect square, rewrite the radical as:

$$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

Final Answer:

$$\boxed{\text{The distance between the points } (-7, -3) \text{ and } (-3, -5) \text{ in simplest radical form is } 2\sqrt{5}}$$

Additional Insights and Related Concepts

Understanding Radical Simplification

Simplifying radicals involves factoring the radicand (the number inside the radical) into its prime factors and extracting any perfect squares. This process ensures the radical is expressed in the most reduced form, making it easier to interpret and use in further calculations.

Practical Applications of Distance Calculation

- Navigation and Mapping: Determining the shortest path between two locations.
- Engineering: Calculating lengths and distances in design plans.
- Computer Graphics: Measuring distances between points on the screen.
- Physics: Finding the displacement between two positions.

Common Mistakes to Avoid

- Forgetting to square the differences before summing.
- Mixing signs when calculating differences (remember, squaring removes the sign).
- Forgetting to simplify the radical to its simplest form.
- Mislabeling coordinates, leading to incorrect differences.

Practice Problems for Mastery

To reinforce your understanding, try solving these problems:

1. Find the distance between points (3, 4) and (7, 1) in simplest radical form.
2. Calculate the distance between points (-2, -8) and (4, 2), simplifying your answer.
3. Determine the length of the segment connecting (0, 0) and (5, -12) in simplest radical form.

Summary

Calculating the distance between two points in the coordinate plane involves a straightforward application of the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

By carefully computing the differences, squaring, summing, and simplifying the radical, you can find an exact measure of the distance. For the points (-7, -3) and (-3, -5), the distance is $(2\sqrt{5})$, a neat and simplified radical expression. Mastering this process enhances your problem-solving skills in geometry and prepares you for more advanced topics involving distances and coordinate geometry.

Remember: Practice makes perfect. Use similar problems to solidify your understanding and become proficient in calculating distances in various contexts.

Frequently Asked Questions

How do you find the distance between the points (-7, -3) and (-3, -5)?

Use the distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Plugging in the values: $\sqrt{(-3 - (-7))^2 + (-5 - (-3))^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20}$.

What is the simplified radical form of the distance between (-7, -3) and (-3, -5)?

Since $\sqrt{20}$ can be simplified to $2\sqrt{5}$, the distance is $2\sqrt{5}$.

Can you walk through the step-by-step calculation of the distance between these two points?

Certainly. First, find the differences: $\Delta x = -3 - (-7) = 4$, $\Delta y = -5 - (-3) = -2$. Then, square these: $4^2 = 16$, $(-2)^2 = 4$. Sum: $16 + 4 = 20$. Finally, take the square root: $\sqrt{20} = 2\sqrt{5}$.

Why do we simplify the radical $\sqrt{20}$ to $2\sqrt{5}$ in calculating the distance?

Because $\sqrt{20}$ factors into $\sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$, simplifying the radical to its simplest radical form.

What is the importance of expressing the distance in simplest radical form?

Expressing the distance in simplest radical form provides the most reduced and exact expression, making it easier to work with in further calculations or proofs.

Is the distance between the points (-7, -3) and (-3, -5) rational or irrational?

It is irrational because the simplified radical form, $2\sqrt{5}$, involves the radical $\sqrt{5}$, which is irrational.

If the points were (-7, -3) and (-3, -5), how would the distance change if the points were moved to (-7, -3) and (-3, -1)?

Calculating similarly: $\Delta x = 4$, $\Delta y = 2$. Distance: $\sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$. So, the distance remains the same at $2\sqrt{5}$.

Are the points (-7, -3) and (-3, -5) located in the same quadrant?

No, the point (-7, -3) is in the third quadrant, while (-3, -5) is in the third or fourth quadrant depending on the coordinate plane, but both are in the third quadrant if considering negative x and y.

values. However, since both have negative x and y, they are in the third quadrant.

Additional Resources

Finding the distance between two points in coordinate geometry is a fundamental skill that underpins various mathematical applications, from basic problem-solving to advanced engineering and scientific endeavors. In this article, we will explore the process of calculating the distance between the points $(-7, -3)$ and $(-3, -5)$, focusing on deriving the answer in its simplest radical form. This comprehensive examination will not only provide step-by-step instructions but also delve into the underlying principles, historical context, and practical significance of the distance formula in mathematics.

Understanding the Concept of Distance in Coordinate Geometry

The Coordinate Plane and Points

The Cartesian coordinate plane is a two-dimensional system where any point can be represented by an ordered pair (x, y) . The x-coordinate indicates the point's horizontal position, while the y-coordinate indicates its vertical position. In our case, the points are:

- Point A: $(-7, -3)$
- Point B: $(-3, -5)$

Understanding how these points are situated on the plane is essential to visualizing the problem and applying the correct mathematical tools to find the distance between them.

The Significance of Calculating Distance

Determining the distance between two points has numerous applications:

- Navigation: Calculating the shortest path between two locations.
- Engineering: Designing components with precise measurements.
- Computer Graphics: Rendering accurate spatial relationships.
- Physics: Analyzing motion and displacement.

The fundamental formula used in these calculations is derived from the Pythagorean theorem, which relates the lengths of the sides of a right triangle to its hypotenuse.

The Distance Formula: Derivation and Explanation

Historical Context

The distance formula's origins trace back to the Pythagorean theorem, formulated by the ancient Greeks. It provides a way to find the hypotenuse of a right triangle when the lengths of the other two sides are known. In coordinate geometry, this concept extends to calculating the straight-line distance between two points in the plane.

Mathematical Derivation

Given two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the distance d between them is expressed as:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is a direct application of the Pythagorean theorem, where:

- $(x_2 - x_1)$ and $(y_2 - y_1)$ are the differences in x and y coordinates, representing the legs of a right triangle.
- The distance d is the hypotenuse, representing the direct straight-line distance between the points.

Why the Formula Works

The key idea is that the difference in x-coordinates and y-coordinates form the legs of a right triangle when connecting the two points. The hypotenuse of this triangle is the shortest path between the points, which is the distance we're seeking. By applying the Pythagorean theorem to these differences, we obtain the formula to compute the distance directly.

Applying the Distance Formula to (-7, -3) and (-3, -5)

Step 1: Assign the Coordinates

Let:

- $x_1 = -7$
- $y_1 = -3$
- $x_2 = -3$
- $y_2 = -5$

Step 2: Calculate the Differences in Coordinates

Compute the horizontal and vertical differences:

- $\Delta x = x_2 - x_1 = -3 - (-7) = -3 + 7 = 4$
- $\Delta y = y_2 - y_1 = -5 - (-3) = -5 + 3 = -2$

These differences represent the legs of our right triangle:

- Horizontal leg: 4
- Vertical leg: -2

While the differences can be negative, the squared differences will always be positive, so the sign does not affect the calculation.

Step 3: Square the Differences

- $(\Delta x)^2 = 4^2 = 16$
- $(\Delta y)^2 = (-2)^2 = 4$

Step 4: Sum the Squares

Add the squared differences:

$$16 + 4 = 20$$

Step 5: Take the Square Root

The distance is the square root of this sum:

$$d = \sqrt{20}$$

Expressing the Distance in Simplest Radical Form

Why Simplify Radicals?

Simplifying radicals makes the expression more elegant and easier to interpret. It also aligns with mathematical standards for presenting exact answers, especially in academic and professional contexts.

Factorize the Radicand

To simplify $\sqrt{20}$, factor 20 into its prime factors:

$$20 = 4 \times 5$$

Since 4 is a perfect square, we can rewrite the radical as:

$$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

\]

Result:

\[

$$\boxed{d = 2\sqrt{5}}$$

\]

This is the simplest radical form of the distance between the points (-7, -3) and (-3, -5).

Interpreting the Result and Its Significance

Numerical Approximation

For a better intuitive understanding, approximate the radical:

\[

$$\sqrt{5} \approx 2.236$$

\]

Thus,

\[

$$d \approx 2 \times 2.236 = 4.472$$

\]

This approximation indicates that the two points are roughly 4.472 units apart in the coordinate plane.

Implications of the Simplest Radical Form

Using radical form preserves the exactness of the distance measurement, which is particularly important in precise calculations, proofs, or when further algebraic manipulations are required.

Additional Considerations and Applications

Generalization to Other Points

The process outlined here can be generalized to any two points in the plane. The key steps involve:

- Identifying coordinate differences
- Squaring and summing these differences
- Simplifying the radical

Distance in Three Dimensions

The same principles extend into three-dimensional space, where the distance formula incorporates an additional coordinate (z):

\[

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Real-World Applications

- Urban Planning: Calculating shortest paths between locations.
- Robotics: Determining distances for navigation.
- Astronomy: Measuring distances between celestial objects projected onto a 2D plane.
- Computer Graphics and Gaming: Collision detection and rendering.

Conclusion: The Elegance of Exact Distance Calculation

The process of calculating the distance between the points (-7, -3) and (-3, -5) illustrates the power and elegance of coordinate geometry. By systematically applying the distance formula, we arrive at the precise and simplified radical form $(2\sqrt{5})$. This not only provides an exact measure of the spatial separation but also exemplifies the importance of foundational mathematical principles like the Pythagorean theorem. Whether for educational purposes, professional applications, or theoretical exploration, mastering such calculations enhances our understanding of spatial relationships and the inherent beauty of mathematics.

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