Find The Equation For The Circle With A Diameter Whose Endpoints Are (-4,4) And (5,3).

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Understanding how to find the equation of a circle given specific points is a fundamental concept in coordinate geometry. When provided with the endpoints of a diameter, the task involves determining the circle's center and radius before formulating its standard equation. In this comprehensive guide, we will walk through each step meticulously, ensuring clarity and depth for learners and enthusiasts alike. Whether you're preparing for exams or seeking to strengthen your geometry skills, this article will serve as an invaluable resource.

Understanding the Fundamentals of a Circle in Coordinate Geometry

Before diving into the specific problem, it's crucial to understand the basic properties and equations related to circles in the coordinate plane.

Standard Equation of a Circle

A circle in the coordinate plane can be expressed using the standard form:

$$(x - h)^2 + (y - k)^2 = r^2$$

where:

- (h, k) is the center of the circle,
- r is the radius.

Knowing the center and radius allows us to write the equation directly.

Role of Diameter in a Circle

The diameter of a circle is a straight line passing through its center, with both endpoints on the circle. Key properties include:

- The diameter's endpoints lie on the circle.
- The length of the diameter is twice the radius: $d = 2r$.
- The midpoint of the diameter's endpoints is the circle's center.

Step-By-Step Solution to Find the Equation

Given the endpoints of the diameter $(-4, 4)$ and $(5, 3)$, our goal is to find the explicit equation of the circle.

Step 1: Find the Midpoint of the Diameter

Since the endpoints are on the circle's diameter, the midpoint of these points is the circle’s center.

Calculating the midpoint:

$$
h = \frac{x_1 + x_2}{2} = \frac{-4 + 5}{2} = \frac{1}{2} = 0.5
$$

$$
k = \frac{y_1 + y_2}{2} = \frac{4 + 3}{2} = \frac{7}{2} = 3.5
$$

Result:

$$
\boxed{
\text{Center} (h, k) = \left(\frac{1}{2}, \frac{7}{2} \right)
}
$$

This point will be used as the center in our circle's equation.

Step 2: Calculate the Length of the Diameter

The distance between the endpoints gives the length of the diameter. Use the distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Plugging in the values:

\[
d = \sqrt{(5 - (-4))^2 + (3 - 4)^2} = \sqrt{(9)^2 + (-1)^2} = \sqrt{81 + 1} = \sqrt{82}
\]

Result:

\[
d = \sqrt{82}
\]

Step 3: Find the Radius of the Circle

Since the diameter \(d = 2r\):

\[
r = \frac{d}{2} = \frac{\sqrt{82}}{2}
\]

This simplifies to:

\[
r = \frac{\sqrt{82}}{2}
\]

Step 4: Write the Equation of the Circle

Now that we have the center \(\left(\frac{1}{2}, \frac{7}{2} \right)\) and the radius \(r = \frac{\sqrt{82}}{2}\), substitute these into the standard form:

\[
(x - h)^2 + (y - k)^2 = r^2
\]

which becomes:

\[
\left(x - \frac{1}{2} \right)^2 + \left(y - \frac{7}{2} \right)^2 = \left(\frac{\sqrt{82}}{2} \right)^2
\]

Calculating \(r^2\):

$$
r^2 = \left(\frac{\sqrt{82}}{2} \right)^2 = \frac{82}{4} = \frac{41}{2}
$$

Final equation:

$$
\boxed{
\left(x - \frac{1}{2} \right)^2 + \left(y - \frac{7}{2} \right)^2 = \frac{41}{2}
}
$$

Alternative Forms of the Equation

Depending on the context or preference, the equation of the circle can be expressed in different formats.

1. Expanded Standard Form

Expand the binomials:

$$
\left(x - \frac{1}{2} \right)^2 = x^2 - x + \frac{1}{4}
$$

$$
\left(y - \frac{7}{2} \right)^2 = y^2 - 7y + \frac{49}{4}
$$

Adding these:

$$
x^2 - x + \frac{1}{4} + y^2 - 7y + \frac{49}{4} = \frac{41}{2}
$$

Combine constants:

$$
\frac{1}{4} + \frac{49}{4} = \frac{50}{4} = \frac{25}{2}
$$

So the equation becomes:

$$
x^2 - x + y^2 - 7y + \frac{25}{2} = \frac{41}{2}

\]

Subtract $\frac{41}{2}$ from both sides:

$$
x^2 - x + y^2 - 7y + \frac{25}{2} - \frac{41}{2} = 0
$$

Simplify:

$$
x^2 - x + y^2 - 7y - \frac{16}{2} = 0
$$

$$
x^2 - x + y^2 - 7y - 8 = 0
$$

This is the expanded form of the circle's equation.

2. General Form

Alternatively, the general form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

can be derived by expanding and rearranging, but the standard form is usually preferred for clarity.

Practical Applications and Problem-Solving Tips

Understanding how to derive the equation of a circle from endpoints of a diameter is essential in various fields such as engineering, architecture, and computer graphics. Here are some practical tips:

- Always verify the coordinates of the endpoints before calculations.
- Use the midpoint formula to find the center efficiently.
- Remember that the diameter's length relates directly to the radius.
- When expanding binomials, carefully combine constants to avoid errors.
- Practice with different coordinate points to strengthen your skills.

Additional Examples for Practice

To reinforce learning, here are some similar problems:

1. Find the equation of a circle with diameter endpoints at $(2, -1)$ and $(-3, 4)$.
2. Given the endpoints $(0, 0)$ and $(6, 8)$, determine the circle's equation.
3. How does the equation change if the diameter endpoints are colinear with the origin?

Attempt these problems to master the concepts discussed.

Summary

In this article, we've comprehensively covered the steps to find the equation of a circle given the endpoints of its diameter. The process involves:

- Calculating the midpoint to find the center.
- Computing the distance between endpoints to determine the diameter.
- Deriving the radius from the diameter.
- Writing the circle's equation in standard form.

Applying these steps meticulously ensures accurate results and enhances your understanding of coordinate geometry.

Conclusion

Mastering the process of deriving the equation of a circle from diameter endpoints is a vital skill in mathematics. By understanding the underlying principles—midpoint as the center, distance as the diameter, and the standard form of a circle—you can solve a wide range of geometric problems efficiently. Practice regularly, and you'll develop confidence in handling similar tasks with ease. Whether for academic purposes or practical applications, these skills form a cornerstone of analytical geometry.

Frequently Asked Questions

How do you find the center of a circle given its diameter endpoints?

The center of the circle is the midpoint of the diameter's endpoints, calculated by averaging the x-coordinates and y-coordinates of the endpoints.

What is the formula for the midpoint of two points?

The midpoint M between points (x1, y1) and (x2, y2) is given by M = ((x1 + x2)/2, (y1 + y2)/2).

What are the endpoints of the diameter given in the problem?

The endpoints are (-4, 4) and (5, 3).

How do you calculate the radius of the circle from the diameter endpoints?

The radius is half the length of the diameter, which is the distance between the endpoints divided by 2.

How do you find the length of the diameter between (-4, 4) and (5, 3)?

Use the distance formula:
$\sqrt{[(x2 - x1)^2 + (y2 - y1)^2]} = \sqrt{[(5 - (-4))^2 + (3 - 4)^2]} = \sqrt{[(9)^2 + (-1)^2]} = \sqrt{[81 + 1]} = \sqrt{82}$.

What is the equation of the circle in standard form?

The standard form of a circle's equation is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

What is the center of the circle with endpoints (-4, 4) and (5, 3)?

The center is the midpoint: ((-4 + 5)/2, (4 + 3)/2) = (0.5, 3.5).

What is the length of the diameter between the endpoints?

The length of the diameter is $\sqrt{82}$, approximately 9.055 units.

What is the radius of the circle?

The radius is half the diameter length, so $r = \sqrt{82} / 2$, approximately 4.527 units.

What is the final equation of the circle with the given diameter?

The equation is $(x - 0.5)^2 + (y - 3.5)^2 = (\sqrt{82} / 2)^2$, which simplifies to $(x - 0.5)^2 + (y$

- 3.5)^2 = 82 / 4 = 20.5.

Additional Resources

Find The Equation For The Circle With A Diameter Whose Endpoints Are (-4,4) And (5,3)

Introduction

Mathematics often reveals its elegance through the geometric relationships that underpin the shapes and figures we observe in the world. Among these, the circle stands out due to its perfect symmetry and the simplicity of its defining properties. One of the fundamental problems encountered in analytic geometry involves determining the equation of a circle given specific features—most notably, its diameter.

This article aims to thoroughly investigate the problem: Find the equation for the circle with a diameter whose endpoints are (-4, 4) and (5, 3). This problem is not only a classic exercise in coordinate geometry but also serves as a gateway to understanding how algebraic formulas translate into geometric constructs.

Through detailed exploration, we will:
- Derive the circle's center and radius based on the endpoints of the diameter.
- Present the general equation of a circle.
- Calculate the specific parameters for the given endpoints.
- Write the explicit equation of the circle.
- Discuss related concepts and potential extensions.

Fundamental Concepts in Circle Geometry

Before delving into calculations, it’s essential to review key concepts.

The Equation of a Circle

A circle in the Cartesian plane is defined as the set of all points equidistant from a fixed point called the center. The standard form of a circle's equation with center (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Diameter, Center, and Radius

- The diameter of a circle is the longest chord passing through the center, with endpoints on the circle.
- The center of the circle is the midpoint of its diameter.
- The radius is half the length of the diameter.

Given the endpoints of a diameter, the center can be found as the midpoint of these points, and the radius as half the length of the diameter.

Determining the Center of the Circle

Step 1: Identify the Endpoints

Given endpoints:

- $A(-4, 4)$
- $B(5, 3)$

These points are the endpoints of the diameter.

Step 2: Calculate the Midpoint

The center (h, k) of the circle is the midpoint of A and B:

$$
h = \frac{x_A + x_B}{2} = \frac{-4 + 5}{2} = \frac{1}{2} = 0.5
$$

$$
k = \frac{y_A + y_B}{2} = \frac{4 + 3}{2} = \frac{7}{2} = 3.5
$$

Thus, the center is at:

$$
\boxed{(0.5, 3.5)}
$$

Calculating the Radius

Step 1: Find the Length of the Diameter

The length of the diameter d is the distance between points A and B:

$$
d = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}
$$

Substituting the values:

$$
d = \sqrt{(5 - (-4))^2 + (3 - 4)^2} = \sqrt{(9)^2 + (-1)^2} = \sqrt{81 + 1} = \sqrt{82}
$$

Step 2: Find the Radius

Since the radius r is half the diameter:

$$
r = \frac{d}{2} = \frac{\sqrt{82}}{2}
$$

Formulating the Equation of the Circle

Using the center $(h, k) = (0.5, 3.5)$ and radius $r = \frac{\sqrt{82}}{2}$, the standard form of the circle's equation becomes:

$$
(x - 0.5)^2 + (y - 3.5)^2 = \left(\frac{\sqrt{82}}{2}\right)^2
$$

Simplify the right side:

$$
\left(\frac{\sqrt{82}}{2}\right)^2 = \frac{82}{4} = \frac{41}{2}
$$

Therefore, the explicit equation is:

$$
(x - 0.5)^2 + (y - 3.5)^2 = \frac{41}{2}
$$

Expressing the Equation in Standard and General Forms

Standard Form:

$$
\boxed{
(x - 0.5)^2 + (y - 3.5)^2 = \frac{41}{2}
}
$$

This form clearly displays the center and radius.

Expanded (General) Form:

Expanding the equation:

$$
(x - 0.5)^2 + (y - 3.5)^2 = \frac{41}{2}

\]

\[
x^2 - 2 \times 0.5 \times x + (0.5)^2 + y^2 - 2 \times 3.5 \times y + (3.5)^2 = \frac{41}{2}
\]

Simplify:

\[
x^2 - x + 0.25 + y^2 - 7y + 12.25 = \frac{41}{2}
\]

Bring all to one side:

\[
x^2 - x + y^2 - 7y + 0.25 + 12.25 - \frac{41}{2} = 0
\]

Calculate the constants:

\[
0.25 + 12.25 = 12.5
\]

\[
\frac{41}{2} = 20.5
\]

So,

\[
x^2 - x + y^2 - 7y + 12.5 - 20.5 = 0
\]

\[
x^2 - x + y^2 - 7y - 8 = 0
\]

Final expanded form:

\[
x^2 - x + y^2 - 7y = 8
\]

Additional Insights and Applications

Geometric Significance

- The derived equation confirms the circle's symmetry and precise location based on the provided endpoints.
- The process underscores the importance of midpoints and distances in coordinate geometry.

Potential Extensions

- Find the equation of a circle passing through three given points.
- Determine tangents or secants to the circle.
- Calculate the area and circumference based on the radius.

Summary

In conclusion, the task of finding the equation of a circle given the endpoints of its diameter involves a systematic approach rooted in fundamental geometric principles.

Key steps include:

1. Calculating the center as the midpoint of the diameter endpoints.
2. Determining the radius as half the length of the diameter.
3. Formulating the equation using the standard form $(x - h)^2 + (y - k)^2 = r^2$.

Applying these steps to the endpoints $(-4, 4)$ and $(5, 3)$, we derive the circle's equation:

$$
\boxed{
(x - 0.5)^2 + (y - 3.5)^2 = \frac{41}{2}
}
$$

or equivalently in expanded form:

$$
x^2 - x + y^2 - 7y = 8
$$

This comprehensive analysis not only resolves the specific problem but also highlights the interconnectedness of algebra and geometry, illustrating the power of coordinate methods in understanding geometric figures.

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Closing Remarks

The process of deriving the equation of a circle from its diameter endpoints is a foundational skill in analytic geometry, blending algebraic manipulation with geometric intuition. Whether for academic pursuits or practical applications in engineering and design, mastering this process enhances spatial reasoning and problem-solving capabilities.

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