Find The Equation Of The Line With Slope M = 5/4 That Contains The Point (-4,-2).

Find The Equation Of The Line With Slope M = 5/4 That Contains The Point (-4,-2)

Understanding how to find the equation of a line given specific information is a fundamental skill in algebra and coordinate geometry. In particular, when you know the slope of the line and a point through which it passes, you can determine its exact equation with relative ease. This article provides a detailed guide on how to find the equation of a line with a slope $M = \frac{5}{4}$ that contains the point $(-4, -2)$. We will explore the underlying concepts, step-by-step procedures, and practical examples to help you master this essential mathematical task.

Understanding the Basics: Slope-Intercept Form of a Line

Before diving into the specific problem, it’s important to familiarize yourself with the most common form of a line's equation:

The Slope-Intercept Form

The slope-intercept form of a line is written as:

$$
y = mx + b
$$

Where:
- y and x are variables representing the coordinates of any point on the line.
- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

Given the slope m, to find the complete equation, you need to determine the value of b, which depends on the point the line passes through.

Applying the Point-Slope Form

Another useful form when given a point and a slope is the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Where:
- (x_1, y_1) is a point on the line.
- m is the slope.

This form is particularly handy because it directly incorporates the known point and slope, allowing for straightforward calculation of the line's equation.

Step-by-Step Guide to Find the Equation of the Line

Let's now focus on the specific problem: find the equation of the line with slope $M = \frac{5}{4}$ passing through the point $(-4, -2)$.

Step 1: Write the point-slope form

Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Plugging in the known values:

$$
y - (-2) = \frac{5}{4}(x - (-4))
$$

Simplify:

$$
y + 2 = \frac{5}{4}(x + 4)
$$

Step 2: Simplify the equation

Distribute the slope on the right side:

$$
y + 2 = \frac{5}{4}x + \frac{5}{4} \times 4
$$

Since $\frac{5}{4} \times 4 = 5$:

$$
y + 2 = \frac{5}{4}x + 5
$$

Step 3: Solve for y to obtain slope-intercept form

Subtract 2 from both sides:

$$
y = \frac{5}{4}x + 5 - 2
$$

Simplify:

$$
y = \frac{5}{4}x + 3
$$

Final Equation:

$$
\boxed{y = \frac{5}{4}x + 3}
$$

This is the slope-intercept form of the line passing through $(-4, -2)$ with slope $\frac{5}{4}$.

Verifying the Solution

It's always good practice to verify your solution.

Check with the given point

Plug in $x = -4$:

$$
y = \frac{5}{4}(-4) + 3 = -5 + 3 = -2
$$

Since this matches the given point $(-4, -2)$, our equation is correct.

Check the slope

The coefficient of x in the equation is $\frac{5}{4}$, confirming the slope matches the given value.

Alternate Forms of the Equation

Besides the slope-intercept form, you can express the equation in other forms depending on context.

Standard Form

Starting from the slope-intercept form:

$$
y = \frac{5}{4}x + 3
$$

Multiply both sides by 4 to clear the fraction:

$$
4y = 5x + 12
$$

Rearranged:

$$
5x - 4y = -12
$$

This is the standard form of the line.

Point-Slope Form

Revisiting the point-slope form with the known point and slope:

$$
y - (-2) = \frac{5}{4}(x - (-4))
$$

$$
y + 2 = \frac{5}{4}(x + 4)
$$

This form is useful for quick calculations or graphing.

Practical Applications of Finding the Equation of a Line

Understanding how to find the equation of a line with given slope and points has numerous real-world applications, including:

- Engineering and Design: Determining the trajectory of objects or structural lines.
- Economics: Modeling relationships between economic variables.
- Navigation: Calculating routes and paths based on known directions.
- Data Analysis: Establishing trend lines in data sets.

Common Mistakes to Avoid

While solving for the line's equation, be mindful of the following pitfalls:

- Incorrect substitution: Always double-check the point coordinates and slope before plugging into formulas.
- Forgetting to simplify: Simplify fractions and expressions to avoid errors in calculations.
- Mixing forms: Be clear about which form you are deriving and ensure conversions are correct.
- Sign errors: Pay attention to signs when substituting negative values.

Summary

Finding the equation of a line when given a point and slope involves a straightforward process rooted in the point-slope and slope-intercept forms. Here is a quick recap:

1. Use the point-slope form: $y - y_1 = m(x - x_1)$.
2. Substitute the known point $(-4, -2)$ and slope $\frac{5}{4}$.
3. Simplify and solve for y to obtain the slope-intercept form.
4. Verify the equation by plugging in the point and confirming the slope.

The resulting line equation:

$$
\boxed{y = \frac{5}{4}x + 3}
$$

accurately describes the line with slope $\frac{5}{4}$ passing through $(-4, -2)$.

Additional Tips for Mastering Line Equations

- Practice with different points and slopes to become comfortable with various scenarios.
- Convert between different forms of line equations to enhance flexibility.
- Use graphing tools or graph paper to visualize the lines and verify your equations.
- Remember that understanding the underlying concepts is more important than memorizing formulas.

Conclusion

Mastering how to find the equation of a line given a slope and a point is a key skill that forms the foundation for more advanced topics in mathematics, physics, and engineering. By following a systematic approach—using point-slope form, simplifying, and verifying—you can confidently determine the precise equation of any line under similar conditions. With practice, this process becomes second nature, empowering you to solve a wide range of real-world problems involving linear relationships.

Frequently Asked Questions

How do you find the equation of a line with a given slope and a point on the line?

Use the point-slope form: $y - y_1 = m(x - x_1)$, substituting the given slope and point coordinates.

What is the equation of the line with slope 5/4 passing through the point (-4, -2)?

Using point-slope form: $y - (-2) = (5/4)(x - (-4))$, which simplifies to $y + 2 = (5/4)(x + 4)$.

How do you convert the point-slope form to slope-intercept form for this line?

Distribute the slope: $y + 2 = (5/4)x + 5$, then subtract 2 from both sides to get $y = (5/4)x + 3$.

What is the final equation of the line with slope 5/4 passing through (-4, -2)?

The slope-intercept form: y = (5/4)x + 3.

How can I verify that the point (-4, -2) lies on the line y = (5/4)x + 3?

Substitute x = -4 into the equation: y = (5/4)(-4) + 3 = -5 + 3 = -2, confirming the point is on the line.

What is the importance of the slope in determining the line's equation?

The slope indicates the steepness and direction of the line; it’s essential for formulating the line’s equation accurately.

Additional Resources

Find the Equation of the Line With Slope M = 5/4 That Contains the Point (-4, -2)

When studying the fundamentals of coordinate geometry, one of the most essential skills is understanding how to determine the equation of a line given certain parameters. In particular, knowing how to find the equation of a line when its slope and a point on the line are provided is a foundational concept that paves the way for more advanced topics such as linear transformations, analytic geometry, and algebraic problem solving.

In this comprehensive guide, we will explore the process of deriving the equation of a line with a specific slope $M = \frac{5}{4}$ passing through the point $(-4, -2)$. We will delve into the core concepts involved, step-by-step procedures, formula derivations, and practical tips to master this skill.

Understanding the Basics of Line Equations

Before diving into the specific problem, it’s important to establish a clear understanding of the fundamental forms of line equations and the key parameters involved.

Standard Forms of Line Equations

- Slope-Intercept Form:
$$ y = mx + b $$

where m is the slope and b is the y-intercept.

- Point-Slope Form:
$$y - y_1 = m(x - x_1)$$
where m is the slope, and (x_1, y_1) is a point on the line.

- Two-Point Form:
Used when two points are known to determine the line.

- General Form:
$$Ax + By + C = 0$$

In our case, since the slope and a point are given, the point-slope form is the most straightforward starting point.

The Significance of the Slope $M = \frac{5}{4}$

The slope m indicates the rate at which y increases or decreases relative to x. A slope of $\frac{5}{4}$ implies that for every 4 units increase in x, y increases by 5 units. This positive slope suggests the line rises as it moves from left to right.

Step-by-Step Derivation of the Line Equation

Let's now systematically derive the equation of the line that has slope $\frac{5}{4}$ and passes through the point $(-4, -2)$.

Step 1: Write the Point-Slope Equation

The point-slope form of a line’s equation is:

$$
y - y_1 = m(x - x_1)
$$

Where:

- $(x_1, y_1) = (-4, -2)$
- $m = \frac{5}{4}$

Substituting these values:

$$
y - (-2) = \frac{5}{4}(x - (-4))

\]

Simplify:

\[
y + 2 = \frac{5}{4}(x + 4)
\]

This form is convenient because it directly incorporates the known point and slope.

Step 2: Simplify the Equation

Distribute the slope:

\[
y + 2 = \frac{5}{4}x + \frac{5}{4} \times 4
\]

Calculate:

\[
\frac{5}{4} \times 4 = 5
\]

So, the equation becomes:

\[
y + 2 = \frac{5}{4}x + 5
\]

Next, isolate \(y \):

\[
y = \frac{5}{4}x + 5 - 2
\]

\[
y = \frac{5}{4}x + 3
\]

This is the slope-intercept form of the line.

Step 3: Final Equation in Slope-Intercept Form

\[
\boxed{
\textbf{y = } \frac{5}{4}x + 3

}
\]

This equation fully describes the line with the given parameters.

Alternative Forms of the Equation

While the slope-intercept form is most common for graphing and understanding, other forms are useful in different contexts.

1. Point-Slope Form

Already derived:

$$
y + 2 = \frac{5}{4}(x + 4)
$$

This is useful when you want to verify the line passes through a given point or for quick graphing.

2. Standard Form

To write the equation in the standard form $Ax + By + C = 0$:

Start from:

$$
y = \frac{5}{4}x + 3
$$

Multiply through by 4 to clear fractions:

$$
4y = 5x + 12
$$

Bring all to one side:

$$
5x - 4y + 12 = 0
$$

This form is often used in algebraic manipulations and when analyzing line properties such as intercepts and perpendicularity.

Verifying the Derived Equation

It’s important to verify that the equation correctly represents the line passing through $(-4, -2)$ with slope $\frac{5}{4}$.

- Check the point:
Substitute $x = -4$ into the slope-intercept form:

$$
y = \frac{5}{4}(-4) + 3 = -5 + 3 = -2
$$

The point $(-4, -2)$ satisfies the equation.

- Check the slope:
The equation in slope-intercept form has a slope of $\frac{5}{4}$, consistent with the given.

Graphical Interpretation

Visualizing the line helps in understanding its behavior and relation to the given point.

- The line passes through $(-4, -2)$.
- The slope of $\frac{5}{4}$ indicates a moderate upward incline.
- For every 4 units moved horizontally to the right, the line rises 5 units vertically.
- The y-intercept 3 indicates that the line crosses the y-axis at $(0, 3)$.

Plotting these points and drawing the line confirms the accuracy of the equation.

Application and Real-World Contexts

Understanding how to find the equation of a line with a given slope and point is crucial in various fields, such as:

- Physics: Describing motion with constant velocity.
- Economics: Modeling linear relationships such as cost and revenue functions.
- Engineering: Designing slopes in infrastructure or analyzing stress lines.

- Computer Graphics: Rendering lines with specific orientations.

Common Pitfalls and Tips

While the process appears straightforward, several common pitfalls can occur:

- Incorrect substitution: Always double-check the point coordinates before substituting.
- Fractions handling: When working with slopes like $\frac{5}{4}$, be careful to distribute and simplify correctly.
- Sign errors: Pay attention to signs when substituting and simplifying.
- Choosing the right form: Use the form most suitable for the problem at hand; for graphing, slope-intercept is often easiest, while for algebraic analysis, standard form may be preferable.

Practice Problems for Reinforcement

To solidify understanding, consider practicing with similar problems:

1. Find the equation of a line with slope $-\frac{3}{2}$ passing through $(2, 5)$.
2. Determine the line passing through points $(1, 2)$ and $(3, 8)$.
3. Write the equation of a line with slope 4 that passes through $(0, -1)$.

Solving these will improve familiarity with deriving line equations and applying the concepts discussed.

Conclusion

In this detailed exploration, we've navigated through the process of finding the equation of a line when given a specific slope and point. Starting from the point-slope form, simplifying to slope-intercept form, and converting to standard form, we covered all essential steps. The key takeaways include:

- Recognizing the importance of the point-slope form as a starting point.
- Carefully substituting known values to avoid errors.
- Simplifying expressions accurately, especially when fractions are involved.
- Verifying solutions by plugging in known points and slopes.

Mastering this skill provides a solid foundation for more advanced topics in algebra, calculus, and geometry. It emphasizes analytical thinking, precision, and clarity—all vital in mathematical problem-solving. Whether you're preparing for exams, tackling real-world problems, or just enhancing your mathematical intuition, being comfortable with deriving line equations is a valuable asset.

Remember: Practice makes perfect. Regularly solving similar problems will build confidence and deepen your understanding of the beautiful interplay between algebra and geometry.

Find The Equation Of The Line With Slope M 5 4 That Contains The Point 4 2

Find The Equation Of The Line With Slope M 5 4 That Contains The Point 4 2

Related Articles

- What Region Of The Continent Did The French And British Do Most Of Their Fighting?
- 10. We Had Hoped To Go To Spain,we Ended Up In France.(otherwise, Instead, Again)
- How Many Ways Berlin Symbolized The Global Struggle Between Worlds From The Cold War

Back to Home