

FIND THE EQUATION $Y = Ax^2 + Bx + C$ FOR THE FUNCTION: (3,10), (5,36), AND (-2,15)

FIND THE EQUATION $Y = Ax^2 + Bx + C$ FOR THE FUNCTION: (3,10), (5,36), AND (-2,15)

ARE YOU STRUGGLING TO FIND THE QUADRATIC EQUATION THAT PASSES THROUGH THREE SPECIFIC POINTS? WHETHER YOU'RE A STUDENT WORKING ON ALGEBRA HOMEWORK OR A PROFESSIONAL NEEDING TO MODEL DATA, UNDERSTANDING HOW TO DERIVE THE QUADRATIC FUNCTION $Y = Ax^2 + Bx + C$ FROM GIVEN POINTS IS A VITAL SKILL. IN THIS COMPREHENSIVE GUIDE, WE'LL WALK THROUGH THE STEP-BY-STEP PROCESS OF DETERMINING THE COEFFICIENTS A, B, AND C FOR THE QUADRATIC FUNCTION THAT PASSES THROUGH THE POINTS (3, 10), (5, 36), AND (-2, 15). BY THE END OF THIS ARTICLE, YOU'LL BE EQUIPPED WITH THE KNOWLEDGE TO SOLVE SIMILAR PROBLEMS CONFIDENTLY.

UNDERSTANDING THE QUADRATIC FUNCTION AND ITS COMPONENTS

WHAT IS A QUADRATIC FUNCTION?

A QUADRATIC FUNCTION IS A POLYNOMIAL FUNCTION OF DEGREE 2, EXPRESSED IN THE FORM:

$$Y = Ax^2 + Bx + C$$

WHERE:

- A DETERMINES THE PARABOLA'S OPENING DIRECTION AND CURVATURE
- B SHIFTS THE PARABOLA HORIZONTALLY
- C SHIFTS THE PARABOLA VERTICALLY

GIVEN THREE POINTS, THE GOAL IS TO FIND THE SPECIFIC VALUES OF A, B, AND C THAT DEFINE THE PARABOLA PASSING THROUGH THESE POINTS.

WHY THREE POINTS ARE ENOUGH

A QUADRATIC FUNCTION HAS THREE COEFFICIENTS, SO YOU NEED AT LEAST THREE POINTS WITH DISTINCT X-VALUES TO UNIQUELY DETERMINE THE FUNCTION. EACH POINT PROVIDES AN EQUATION WHEN SUBSTITUTED INTO THE QUADRATIC FORMULA, LEADING TO A SYSTEM OF THREE EQUATIONS WITH THREE UNKNOWNNS.

STEP-BY-STEP PROCESS TO FIND THE COEFFICIENTS

1. SET UP THE SYSTEM OF EQUATIONS

GIVEN POINTS: (3, 10), (5, 36), AND (-2, 15).

FOR EACH POINT, SUBSTITUTE X AND Y INTO THE QUADRATIC EQUATION:

Point (3, 10):

$$10 = A(3)^2 + B(3) + C$$

$$10 = 9A + 3B + C$$

Point (5, 36):

$$36 = A(5)^2 + B(5) + C$$

$$36 = 25A + 5B + C$$

Point (-2, 15):

$$15 = A(-2)^2 + B(-2) + C$$

$$15 = 4A - 2B + C$$

THIS YIELDS THE FOLLOWING SYSTEM OF EQUATIONS:

$$\text{Equation 1: } 9A + 3B + C = 10$$

$$\text{Equation 2: } 25A + 5B + C = 36$$

$$\text{Equation 3: } 4A - 2B + C = 15$$

2. SOLVE THE SYSTEM OF EQUATIONS

TO FIND A, B, AND C, WE'LL USE SUBSTITUTION OR ELIMINATION METHODS.

STEP 1: SUBTRACT EQUATION 1 FROM EQUATION 2:

$$(25A - 9A) + (5B - 3B) + (C - C) = 36 - 10$$

$$16A + 2B = 26$$

SIMPLIFY:

$$16A + 2B = 26$$

DIVIDE BOTH SIDES BY 2:

$$8A + B = 13$$

THIS IS OUR FIRST SIMPLIFIED EQUATION.

STEP 2: SUBTRACT EQUATION 3 FROM EQUATION 1:

$$(9A - 4A) + (3B - (-2B)) + (C - C) = 10 - 15$$

$$5A + 5B = -5$$

DIVIDE BOTH SIDES BY 5:

$$A + B = -1$$

NOW, WE HAVE TWO SIMPLER EQUATIONS:

$$- 8A + B = 13$$

$$- A + B = -1$$

STEP 3: SUBTRACT THE SECOND FROM THE FIRST:

$$(8A + B) - (A + B) = 13 - (-1)$$

$$7A = 14$$

SOLVE FOR A:

$$A = 2$$

STEP 4: FIND B USING $A + B = -1$:

$$2 + B = -1$$

$$B = -3$$

STEP 5: FIND C USING EQUATION 1:

$$9A + 3B + C = 10$$

$$9(2) + 3(-3) + C = 10$$

$$18 - 9 + C = 10$$

$$9 + C = 10$$

$$C = 1$$

FINAL EQUATION OF THE QUADRATIC FUNCTION

AFTER SOLVING THE SYSTEM, THE COEFFICIENTS ARE:

1. $A = 2$

2. $B = -3$

3. $C = 1$

THEREFORE, THE QUADRATIC FUNCTION PASSING THROUGH THE POINTS $(3, 10)$, $(5, 36)$, AND $(-2, 15)$ IS:

$$Y = 2x^2 - 3x + 1$$

GRAPHICAL INTERPRETATION AND VALIDATION

PLOTTING THE FUNCTION

VISUALIZING THE QUADRATIC FUNCTION HELPS VERIFY ITS CORRECTNESS. PLOT THE POINTS AND THE PARABOLA DEFINED BY $Y = 2x^2 - 3x + 1$. THE POINTS SHOULD LIE EXACTLY ON THE PARABOLA.

VALIDATION BY SUBSTITUTION

CHECK THE ORIGINAL POINTS:

- For $x = 3$:

$$Y = 2(3)^2 - 3(3) + 1 = 29 - 9 + 1 = 18 - 9 + 1 = 10$$

MATCHES THE POINT (3, 10).

- For $x = 5$:

$$Y = 225 - 35 + 1 = 50 - 15 + 1 = 36$$

MATCHES THE POINT (5, 36).

- For $x = -2$:

$$Y = 24 - 3(-2) + 1 = 8 + 6 + 1 = 15$$

MATCHES THE POINT (-2, 15).

THIS CONFIRMS THE ACCURACY OF THE DERIVED QUADRATIC EQUATION.

PRACTICAL APPLICATIONS OF FINDING QUADRATIC EQUATIONS

REAL-WORLD USES

QUADRATIC FUNCTIONS ARE INSTRUMENTAL IN VARIOUS FIELDS:

- **PHYSICS:** MODELING PROJECTILE MOTION, SUCH AS THE TRAJECTORY OF A BALL.
- **ECONOMICS:** CALCULATING PROFIT MAXIMIZATION OR LOSS MINIMIZATION SCENARIOS.
- **ENGINEERING:** DESIGNING PARABOLIC REFLECTORS OR ARCHES.
- **BIOLOGY:** MODELING GROWTH RATES UNDER SPECIFIC CONDITIONS.

EDUCATIONAL IMPORTANCE

LEARNING TO DERIVE QUADRATIC EQUATIONS FROM DATA POINTS ENHANCES PROBLEM-SOLVING SKILLS AND DEEPENS UNDERSTANDING OF ALGEBRAIC CONCEPTS, ESSENTIAL FOR HIGHER-LEVEL MATHEMATICS AND STEM CAREERS.

ADDITIONAL TIPS FOR SOLVING SIMILAR PROBLEMS

1. **ALWAYS VERIFY THE POINTS:** ENSURE THE X-VALUES ARE DISTINCT TO AVOID DEGENERATE SYSTEMS.
2. **USE SUBSTITUTION OR MATRIX METHODS:** FOR LARGER SYSTEMS, MATRIX ALGEBRA OR CALCULATORS CAN SIMPLIFY THE PROCESS.
3. **CHECK YOUR WORK:** SUBSTITUTING COEFFICIENTS BACK INTO THE ORIGINAL POINTS CONFIRMS CORRECTNESS.
4. **PRACTICE WITH VARIOUS DATA SETS:** THE MORE YOU PRACTICE, THE MORE INTUITIVE SOLVING BECOMES.

CONCLUSION

FINDING THE QUADRATIC EQUATION THAT PASSES THROUGH THREE GIVEN POINTS INVOLVES SETTING UP A SYSTEM OF THREE EQUATIONS, SOLVING FOR THE COEFFICIENTS A, B, AND C, AND VERIFYING THE SOLUTION. IN THIS CASE, THE POINTS (3, 10), (5, 36), AND (-2, 15) LEAD US TO THE QUADRATIC FUNCTION:

$$Y = 2x^2 - 3x + 1$$

THIS PROCESS NOT ONLY HELPS IN ACADEMIC SETTINGS BUT ALSO EQUIPS YOU WITH PRACTICAL SKILLS APPLICABLE IN VARIOUS SCIENTIFIC AND ENGINEERING FIELDS. MASTERY OF THESE TECHNIQUES ENHANCES YOUR ANALYTICAL THINKING AND PROBLEM-SOLVING CAPABILITIES, ESSENTIAL FOR SUCCESS IN STEM DISCIPLINES.

META-DESCRIPTION: LEARN HOW TO FIND THE QUADRATIC EQUATION PASSING THROUGH POINTS (3, 10), (5, 36), AND (-2, 15). STEP-BY-STEP GUIDE WITH DETAILED EXPLANATIONS AND PRACTICAL TIPS.

FREQUENTLY ASKED QUESTIONS

HOW DO I FIND THE QUADRATIC EQUATION $Y = Ax^2 + Bx + C$ GIVEN THREE POINTS?

TO FIND THE QUADRATIC EQUATION, SUBSTITUTE EACH POINT INTO THE EQUATION $Y = Ax^2 + Bx + C$ TO CREATE A SYSTEM OF THREE EQUATIONS, THEN SOLVE FOR A, B, AND C.

WHAT ARE THE STEPS TO DETERMINE THE COEFFICIENTS A, B, AND C FOR THE POINTS (3,10), (5,36), AND (-2,15)?

FIRST, PLUG EACH POINT INTO $Y = Ax^2 + Bx + C$: FOR (3,10), (5,36), AND (-2,15). THIS RESULTS IN THREE EQUATIONS. THEN, SOLVE THESE EQUATIONS SIMULTANEOUSLY TO FIND A, B, AND C.

CAN I USE MATRIX METHODS TO FIND THE QUADRATIC EQUATION FROM THE THREE POINTS?

YES, YOU CAN SET UP THE SYSTEM AS A MATRIX AND USE METHODS LIKE GAUSSIAN ELIMINATION OR MATRIX INVERSION TO SOLVE FOR A, B, AND C EFFICIENTLY.

WHAT IS THE QUADRATIC EQUATION FOR THE POINTS (3,10), (5,36), AND (-2,15)?

THE QUADRATIC EQUATION PASSING THROUGH THESE POINTS IS $y = 3x^2 - 4x + 4$.

HOW CAN I VERIFY THAT THE FOUND QUADRATIC EQUATION FITS ALL THREE POINTS?

SUBSTITUTE EACH X-VALUE INTO THE DERIVED EQUATION AND CHECK IF THE OUTPUT MATCHES THE GIVEN Y-VALUE FOR EACH POINT.

IS IT POSSIBLE TO FIND THE QUADRATIC EQUATION WITHOUT SOLVING A SYSTEM OF EQUATIONS?

GENERALLY, SOLVING A SYSTEM IS THE MOST STRAIGHTFORWARD METHOD; ALTERNATIVE APPROACHES LIKE POLYNOMIAL INTERPOLATION CAN ALSO BE USED, BUT SOLVING THE SYSTEM IS MOST COMMON FOR THREE POINTS.

WHAT TOOLS CAN I USE TO FIND THE QUADRATIC EQUATION FROM THREE POINTS QUICKLY?

YOU CAN USE GRAPHING CALCULATORS, ALGEBRA SOFTWARE LIKE WOLFRAMALPHA, DESMOS, OR ALGEBRAIC TOOLS LIKE EXCEL OR GEOGEBRA TO COMPUTE THE COEFFICIENTS EFFICIENTLY.

WHAT IS THE IMPORTANCE OF THE QUADRATIC EQUATION IN REAL-WORLD APPLICATIONS?

QUADRATIC EQUATIONS MODEL MANY REAL-WORLD PHENOMENA SUCH AS PROJECTILE MOTION, PROFIT MAXIMIZATION, AND AREA OPTIMIZATION, MAKING THEM ESSENTIAL IN VARIOUS FIELDS.

ADDITIONAL RESOURCES

FIND THE EQUATION $y = Ax^2 + Bx + C$ FOR THE FUNCTION: (3,10), (5,36), AND (-2,15)

UNDERSTANDING HOW TO DETERMINE THE QUADRATIC EQUATION THAT PASSES THROUGH A SET OF POINTS IS A FUNDAMENTAL SKILL IN ALGEBRA AND ANALYTICAL GEOMETRY. IN THIS ARTICLE, WE WILL EXPLORE THE PROCESS OF FINDING THE QUADRATIC FUNCTION OF THE FORM $y = Ax^2 + Bx + C$ THAT PRECISELY FITS THREE GIVEN POINTS: (3,10), (5,36), AND (-2,15). WE WILL DELVE INTO THE MATHEMATICAL PRINCIPLES BEHIND THIS TASK, STEP-BY-STEP METHODS TO SOLVE FOR THE COEFFICIENTS, AND PRACTICAL TIPS FOR APPLYING THIS PROCESS TO SIMILAR PROBLEMS.

INTRODUCTION TO QUADRATIC FUNCTIONS AND THEIR SIGNIFICANCE

QUADRATIC FUNCTIONS ARE POLYNOMIAL FUNCTIONS OF DEGREE TWO, CHARACTERIZED BY THEIR PARABOLIC GRAPHS. THEY ARE WIDELY USED IN VARIOUS FIELDS, INCLUDING PHYSICS FOR PROJECTILE MOTION ANALYSIS, ECONOMICS FOR COST-PROFIT MODELING, AND ENGINEERING FOR OPTIMIZATION PROBLEMS.

THE GENERAL FORM OF A QUADRATIC FUNCTION IS:

$$y = Ax^2 + Bx + C$$

HERE, A, B, AND C ARE CONSTANTS THAT DETERMINE THE PARABOLA'S SHAPE AND POSITION. KNOWING THESE COEFFICIENTS ALLOWS US TO UNDERSTAND THE UNDERLYING RELATIONSHIP BETWEEN X AND Y, PREDICT VALUES, AND ANALYZE THE FUNCTION'S BEHAVIOR.

IN OUR CONTEXT, GIVEN A SET OF POINTS THAT THE PARABOLA PASSES THROUGH, OUR GOAL IS TO FIND THE SPECIFIC VALUES

OF A, B, AND C THAT SATISFY ALL THREE POINTS SIMULTANEOUSLY. THIS PROCESS INVOLVES SOLVING A SYSTEM OF EQUATIONS DERIVED FROM SUBSTITUTING EACH POINT INTO THE GENERAL QUADRATIC FORMULA.

SETTING UP THE SYSTEM OF EQUATIONS

GIVEN THE THREE POINTS:

- (3, 10)
- (5, 36)
- (-2, 15)

WE SUBSTITUTE EACH INTO THE QUADRATIC EQUATION $Y = Ax^2 + Bx + C$, RESULTING IN THREE EQUATIONS:

1. For (3, 10):

$$10 = A(3)^2 + B(3) + C$$

2. For (5, 36):

$$36 = A(5)^2 + B(5) + C$$

3. For (-2, 15):

$$15 = A(-2)^2 + B(-2) + C$$

SIMPLIFYING EACH:

1. $10 = 9A + 3B + C$

2. $36 = 25A + 5B + C$

3. $15 = 4A - 2B + C$

THIS FORMS A SYSTEM OF THREE EQUATIONS WITH THREE UNKNOWNNS, WHICH WE CAN SOLVE USING METHODS SUCH AS SUBSTITUTION, ELIMINATION, OR MATRIX TECHNIQUES.

SOLVING THE SYSTEM OF EQUATIONS

LET'S PROCEED STEP-BY-STEP TO FIND A, B, AND C.

STEP 1: WRITE THE EQUATIONS EXPLICITLY

EQUATION (1): $9A + 3B + C = 10$

EQUATION (2): $25A + 5B + C = 36$

EQUATION (3): $4A - 2B + C = 15$

STEP 2: ELIMINATE C

SUBTRACT EQUATION (1) FROM EQUATION (2):

$$(25A - 9A) + (5B - 3B) + (C - C) = 36 - 10$$

THIS SIMPLIFIES TO:

$$16A + 2B = 26$$

DIVIDE THROUGH BY 2 TO SIMPLIFY:

$$8A + B = 13 \quad \text{EQUATION (4)}$$

SIMILARLY, SUBTRACT EQUATION (3) FROM EQUATION (1):

$$(9A - 4A) + (3B - (-2B)) + (C - C) = 10 - 15$$

SIMPLIFIES TO:

$$5A + 5B = -5$$

DIVIDE BOTH SIDES BY 5:

$$A + B = -1 \quad \text{EQUATION (5)}$$

STEP 3: SOLVE FOR A AND B

FROM EQUATION (5):

$$A + B = -1$$

EXPRESS B IN TERMS OF A:

$$B = -1 - A$$

SUBSTITUTE B INTO EQUATION (4):

$$8A + (-1 - A) = 13$$

SIMPLIFY:

$$8A - 1 - A = 13$$

$$7A - 1 = 13$$

ADD 1 TO BOTH SIDES:

$$7A = 14$$

DIVIDE BOTH SIDES BY 7:

$$A = 2$$

NOW, FIND B:

$$B = -1 - A = -1 - 2 = -3$$

STEP 4: FIND C USING ONE OF THE ORIGINAL EQUATIONS

USE EQUATION (1):

$$9A + 3B + C = 10$$

SUBSTITUTE $A = 2$ AND $B = -3$:

$$92 + 3(-3) + C = 10$$

$$18 - 9 + C = 10$$

SIMPLIFY:

$$9 + C = 10$$

SUBTRACT 9 FROM BOTH SIDES:

$$C = 1$$

FINAL EQUATION OF THE QUADRATIC FUNCTION

HAVING DETERMINED THE COEFFICIENTS A, B, AND C, WE CAN NOW WRITE THE SPECIFIC QUADRATIC EQUATION:

$$Y = 2x^2 - 3x + 1$$

THIS EQUATION DESCRIBES THE PARABOLA PASSING THROUGH THE THREE GIVEN POINTS.

VERIFYING THE SOLUTION

VERIFICATION ENSURES THE ACCURACY OF OUR CALCULATIONS:

- For (3, 10):

$$Y = 2(3)^2 - 3(3) + 1 = 29 - 9 + 1 = 18 - 9 + 1 = 10 \quad \square$$

- For (5, 36):

$$Y = 2(5)^2 - 3(5) + 1 = 225 - 15 + 1 = 50 - 15 + 1 = 36 \quad \square$$

- For (-2, 15):

$$Y = 2(-2)^2 - 3(-2) + 1 = 24 + 6 + 1 = 8 + 6 + 1 = 15 \quad \square$$

ALL POINTS SATISFY THE EQUATION, CONFIRMING ITS CORRECTNESS.

IMPLICATIONS AND APPLICATIONS

UNDERSTANDING HOW TO DERIVE THE QUADRATIC EQUATION FROM A SET OF POINTS HAS BROAD IMPLICATIONS:

- DATA MODELING: ENGINEERS AND SCIENTISTS OFTEN FIT QUADRATIC FUNCTIONS TO EMPIRICAL DATA TO MODEL RELATIONSHIPS AND PREDICT FUTURE VALUES.
- OPTIMIZATION: BUSINESSES UTILIZE QUADRATIC MODELS TO MAXIMIZE PROFIT OR MINIMIZE COSTS, REQUIRING PRECISE EQUATIONS.
- PHYSICS: TRAJECTORY ANALYSIS OF PROJECTILES OFTEN INVOLVES QUADRATIC EQUATIONS, WHERE INITIAL CONDITIONS ARE KNOWN POINTS ON THE TRAJECTORY.
- EDUCATION: MASTERING THIS PROCESS ENHANCES PROBLEM-SOLVING SKILLS AND MATHEMATICAL REASONING.

CONCLUSION AND PRACTICAL TIPS

FINDING THE QUADRATIC EQUATION FOR A SET OF POINTS INVOLVES TRANSLATING GEOMETRIC INFORMATION INTO ALGEBRAIC EQUATIONS, THEN SYSTEMATICALLY SOLVING FOR THE COEFFICIENTS. HERE ARE SOME PRACTICAL TIPS:

- ORGANIZE YOUR EQUATIONS CLEARLY: WRITE DOWN EACH SUBSTITUTION EXPLICITLY TO AVOID ERRORS.
- USE ELIMINATION OR SUBSTITUTION SYSTEMATICALLY: CHOOSE THE METHOD THAT SIMPLIFIES THE PROCESS BASED ON THE PROBLEM.
- VERIFY YOUR SOLUTION: SUBSTITUTE THE FOUND COEFFICIENTS BACK INTO THE ORIGINAL EQUATIONS TO CONFIRM ACCURACY.
- LEVERAGE TECHNOLOGY: GRAPHING CALCULATORS, ALGEBRA SOFTWARE, OR SPREADSHEETS CAN EXPEDITE CALCULATIONS, ESPECIALLY WITH LARGER DATASETS.

BY MASTERING THIS TECHNIQUE, STUDENTS AND PROFESSIONALS CAN ANALYZE COMPLEX RELATIONSHIPS, MODEL REAL-WORLD PHENOMENA, AND DEVELOP A DEEPER UNDERSTANDING OF QUADRATIC FUNCTIONS.

IN SUMMARY, THROUGH SYSTEMATIC SUBSTITUTION AND ALGEBRAIC MANIPULATION, WE HAVE DERIVED THE QUADRATIC FUNCTION $y = 2x^2 - 3x + 1$ THAT PASSES THROUGH THE POINTS (3,10), (5,36), AND (-2,15). THIS PROCESS EXEMPLIFIES THE POWER OF ALGEBRAIC METHODS IN SOLVING PRACTICAL MATHEMATICAL PROBLEMS AND HIGHLIGHTS FOUNDATIONAL SKILLS ESSENTIAL FOR ADVANCED STUDIES AND APPLICATIONS.

[Find The Equation Y Ax 2 Bx C For The Function 3 10 5 36 And 2 15](#)

Find The Equation Y Ax 2 Bx C For The Function 3 10 5 36 And 2 15

Related Articles

- [What Is The Name Of The Flower That Is Predominantly Seen Around Christmas Time?](#)
- [The Spanish Armada Defeated Admiral Howard And The English Fleet A\) True. B\) False.](#)
- [In Considering Its Marketing Strategy A New Venture Will Typically Concentrate On](#)

[Back to Home](#)