Find The Length Of The Third Side. If Necessary, Write In Simplistic Radical Form.

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Understanding how to determine the length of the third side of a triangle is a fundamental skill in geometry. Whether you're solving for unknown sides in a triangle, working on a math homework problem, or preparing for an exam, mastering this concept will deepen your comprehension of geometric principles and enhance your problem-solving abilities. Sometimes, the calculations involve radicals, and presenting your answer in its simplest radical form can be both elegant and precise.

In this comprehensive guide, we'll explore various methods to find the length of the third side of a triangle, including the use of the Pythagorean theorem, Law of Cosines, and Law of Sines. We'll also discuss how to simplify radicals to their simplest form, ensuring your solutions are both accurate and neatly presented.

Understanding Triangle Types and When to Use Each Method

Before diving into calculations, it's important to recognize the type of triangle you're dealing with:

- Right Triangle: One angle is 90°. The Pythagorean theorem applies here.
- Oblique Triangle: No right angle. Use Law of Cosines or Law of Sines.
- Equilateral Triangle: All sides equal; if two sides are known, the third is the same.
- Isosceles Triangle: Two sides are equal; use properties accordingly.
- Scalene Triangle: All sides are different; generally, Law of Cosines or Sines are used.

Key Concepts and Formulas for Finding the Third Side

Depending on the information provided, different formulas are applicable:

- Pythagorean Theorem: For right-angled triangles.
- Law of Cosines: For non-right triangles when two sides and the included angle are known, or when all sides are known.
- Law of Sines: For cases with known angles and sides but not the included angle.

Using the Pythagorean Theorem

The Pythagorean theorem states that in a right triangle:

$$
c^2 = a^2 + b^2
$$

where:

- c is the hypotenuse (longest side)
- a and b are the other two sides

Example:

Suppose you have a right triangle with legs $a = 3$ units and $b = 4$ units. Find the hypotenuse c:

$$
c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

Note: When the radical simplifies to a perfect square, the radical can be written as an integer.

Applying the Law of Cosines

The Law of Cosines is invaluable for non-right triangles, especially when:

- Two sides and the included angle are known (SAS case)
- All three sides are known (SSS case)

Law of Cosines Formula:

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

Where:

- a, b, c are the sides of the triangle
- C is the angle opposite side c

Finding the third side:

Given two sides and the included angle, rearranged to find the third side:

$$
c = \sqrt{a^2 + b^2 - 2ab \cos C}
$$

Example:

Given:

- $a = 7$ units
- $b = 10$ units
- $C = 60^\circ$

Calculate side c:

$$
c = \sqrt{7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ}
$$

Since $\cos 60^\circ = 0.5$,

$$
c = \sqrt{49 + 100 - 2 \times 7 \times 10 \times 0.5} = \sqrt{149 - 70} = \sqrt{79}
$$

Simplify radical:

Since 79 is prime, it cannot be simplified further. The length of the third side is:

\[

$$
c = \sqrt{79}
$$

Using the Law of Sines

The Law of Sines is best when:

- Two angles and a side are known (AAS or ASA cases), or
- Two sides and a non-included angle are known (SSA case)

Law of Sines Formula:

$$
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
$$

Example:

Suppose:

- $A = 30^\circ$
- $B = 45^\circ$
- Side $a = 10$ units

Find side c:

First, find C:

$$
C = 180^\circ - A - B = 180^\circ - 30^\circ - 45^\circ = 105^\circ
$$

Now, find side c:

$$
\frac{a}{\sin A} = \frac{c}{\sin C}
\Rightarrow c = \frac{\sin C \times a}{\sin A}
$$

Calculate:

$$
c = \frac{\sin 105^\circ \times 10}{\sin 30^\circ}
$$

Since $\sin 105^\circ \approx 0.9659$ and $\sin 30^\circ = 0.5$:

$$
c \approx \frac{0.9659 \times 10}{0.5} = \frac{9.659}{0.5} = 19.318
$$

Answer: $c \approx 19.318$ units.

When to Write in Simplistic Radical Form

Radicals are often used when the square root cannot be simplified to an integer or a fraction.

Simplifying radicals makes your answer cleaner and easier to interpret.

Steps to Simplify Radicals:

1. Prime Factorization: Break down the radicand into prime factors.
2. Identify Perfect Squares: Group factors into pairs or groups that form perfect squares.
3. Reduce the Radical: Take out the perfect squares from under the radical as integers.

Example:

Suppose the calculation yields:

\[
c = \sqrt{180}
\]

Prime factorization:

\[
180 = 2^2 \times 3^2 \times 5
\]

Group perfect squares:

\[
\sqrt{2^2 \times 3^2 \times 5} = \sqrt{(2 \times 3)^2 \times 5} = 6 \sqrt{5}
\]

Final simplified radical:

\[

c = 6 \sqrt{5}

\]

Summary of Steps to Find the Third Side

When solving for the third side of a triangle, follow these steps:

1. Identify triangle type and known information.
2. Choose the appropriate formula: Pythagorean theorem, Law of Cosines, or Law of Sines.
3. Plug in the known values carefully.
4. Perform calculations accurately, considering the order of operations.
5. Simplify radicals if the answer contains square roots that can be reduced.
6. Express the final answer in simplest radical form if possible.

Additional Tips for Accurate and Clear Solutions

- Always double-check your measurements and calculations.
- Use a calculator for trigonometric functions, ensuring it's in the correct mode.
- When simplifying radicals, verify that you've fully factored the radicand.
- Keep units consistent throughout calculations.
- When presenting your final answer, write radicals in their simplest form for clarity and precision.

Practice Problems for Mastery

1. A triangle has sides of 8 units and 15 units with an included angle of 60°. Find the length of the third side in radical form.

2. In a right triangle, one leg measures 9 units, and the hypotenuse measures 15 units. Find the other leg, writing your answer in simplest radical form.

3. Two sides of a triangle are 7 units and 10 units, and the included angle is 45°. Determine the third side, simplifying your radical answer.

4. Given $A = 45^\circ$, $B = 60^\circ$, side $a = 12$ units, find side c.

Conclusion

Finding the length of the third side of a triangle is a crucial skill that combines understanding of various geometric principles and algebraic manipulation. Whether you're working with right triangles, oblique triangles, or applying the Law of Cosines or Sines, the key is to select the appropriate method based on the known information. When radicals are involved, simplifying them to their simplest form not only makes your work look neat but also enhances your mathematical clarity.

By practicing these techniques and mastering the process of radical simplification, you'll be well-equipped to solve a wide range of triangle problems confidently and accurately. Remember, precision and clarity are

Frequently Asked Questions

How do I find the length of the third side of a triangle when two sides and the included angle are known?

Use the Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos(C)$. Solve for c and simplify the radical if possible.

What is the process to find the third side of a triangle using the Law of Cosines?

Identify the known sides and included angle, then substitute into $c^2 = a^2 + b^2 - 2ab \cos(C)$. Take the square root of the simplified expression to find the length.

Can I write the length of the third side in radical form if the calculation results in a non-perfect square?

Yes, if the radical cannot be simplified further, leave it in radical form, such as $\sqrt{n}$, where n is not a perfect square.

When given two sides and an angle opposite one of them, how do I determine the third side?

Use the Law of Cosines with the known sides and included angle to find the third side: $c = \sqrt{a^2 + b^2 - 2ab \cos(C)}$. Simplify the radical as needed.

Why is it important to write the third side length in radical form if it cannot be simplified?

Writing in radical form precisely expresses the exact length without decimal approximation, maintaining mathematical accuracy and clarity.

Additional Resources

Find The Length Of The Third Side. If Necessary, Write In Simplistic Radical Form.

Understanding how to determine the length of the third side of a triangle is a fundamental aspect of geometry. Whether you're a student preparing for exams, a teacher crafting lesson plans, or an enthusiast refreshing your knowledge, mastering this concept is essential. This comprehensive guide will delve into the various methods to find the third side, explain key principles, and show how to write the answer in simplified radical form when necessary.

Introduction to Triangle Sides and the Importance of the Third Side

A triangle consists of three sides and three angles. Knowing two sides and the included angle, or two angles and a side, allows us to find the unknown lengths using different methods. The third side's length is crucial for various applications, including construction, navigation, and problem-solving in mathematics.

Key points:

- The third side's length can often be found using the Law of Cosines or the Law of Sines.
- When the sides involve square roots, it is often necessary to simplify radicals for a cleaner answer.
- The problem's context determines which method to use and whether the answer should be in radical form.

Methods for Finding the Length of the Third Side

Different scenarios dictate different approaches. Here, we explore the most common methods.

1. Using the Law of Cosines

The Law of Cosines is essential when:

- You know two sides and the included angle.
- You know all three sides but want to find an angle.

Law of Cosines formula:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Where:

- a and b are known sides.
- C is the included angle between sides a and b.
- c is the side opposite angle C, which we want to find.

Application:

Suppose you know sides a, b, and the included angle C. Plug the known values into the formula and solve for c:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

Example:

Given $a = 7$, $b = 10$, and $C = 60^\circ$:

$$c = \sqrt{7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ}$$

$$c = \sqrt{49 + 100 - 140 \times 0.5}$$

$$c = \sqrt{149 - 70}$$

$$c = \sqrt{79}$$

The answer: $c = \sqrt{79}$ (already in radical form, simplified since 79 has no perfect square factors).

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2. Using the Law of Sines

The Law of Sines applies when:

- You know either two angles and one side (AAS or ASA cases).
- You want to find an unknown side after determining an angle.

Law of Sines formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Application:

Suppose you know sides a, b, and angles A, B, and want to find side c:

$$c = \frac{\sin C}{\sin A} \times a$$

or

$$ c = \frac{\sin C}{\sin B} \times b $$

Example:

Given $a=8$, $A=30^\circ$, $B=45^\circ$, find side c.

First, find C:

$$ C = 180^\circ - A - B = 180^\circ - 30^\circ - 45^\circ = 105^\circ $$

Now, compute c:

$$ c = \frac{\sin 105^\circ}{\sin 30^\circ} \times 8 $$

Calculate sine values:

$$ \sin 105^\circ = \sin (180^\circ - 75^\circ) = \sin 75^\circ \approx 0.9659 $$
$$ \sin 30^\circ = 0.5 $$

So,

$$ c \approx \frac{0.9659}{0.5} \times 8 = 1.9318 \times 8 = 15.4544 $$

The approximate answer: $c \approx 15.45$, but if the exact radical form is desired, express $\sin 75^\circ$ as:

$$ \sin 75^\circ = \sin (45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ $$

Using known values:

$$ \sin 45^\circ = \frac{\sqrt{2}}{2} $$

$$ \cos 30^\circ = \frac{\sqrt{3}}{2} $$

$$ \cos 45^\circ = \frac{\sqrt{2}}{2} $$

$$ \sin 30^\circ = \frac{1}{2} $$

So,

$$ \sin 75^\circ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4} $$

Now, the exact value for c:

$$ c = \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{1}{2}} \times 8 = \left(\frac{\sqrt{6} + \sqrt{2}}{4} \times 2 \right) \times 8 $$

Simplify:

$$ c = (\frac{\sqrt{6} + \sqrt{2}}{2}) \times 8 = (\sqrt{6} + \sqrt{2}) \times 4 $$

Final exact form:

$$ c = 4 (\sqrt{6} + \sqrt{2}) $$

This radical form clearly expresses the exact length.

Special Cases and Key Considerations

Understanding when to use each method is crucial. Here are some special cases and considerations:

Case 1: Sides and Included Angle Known (Use Law of Cosines)

- When you know two sides and the included angle, the Law of Cosines provides a straightforward way to find the third side.
- This scenario is common in non-right triangles.

Case 2: Two Angles and a Side Known (Use Law of Sines)

- When you know two angles and one side, first find the third angle, then use Law of Sines to find the missing side.
- This is called the ASA or AAS case.

Case 3: All Three Sides Known (Use Law of Cosines or Heron's Formula)

- When all sides are given, the Law of Cosines can find angles or verify side lengths.
- Heron's formula can compute the area, which might help indirectly.

Case 4: Right Triangles (Use Pythagoras Theorem)

- When the triangle is a right triangle, the Pythagoras theorem simplifies calculations:

$$ c^2 = a^2 + b^2 $$

- The hypotenuse c is the side opposite the right angle.

Expressing the Third Side in Simplistic Radical Form

Expressing the side length in radical form often involves simplifying square roots or other radicals.

Why Simplify Radicals?

- Simplified radical forms are more exact and easier to interpret.
- They avoid decimal approximations, which can introduce rounding errors.
- Certain radicals can be simplified further by factoring out perfect squares.

Steps to Simplify Radicals

1. Factor the Radicand: Break down the number inside the radical into prime factors.
2. Identify Perfect Squares: Find factors that are perfect squares.
3. Extract the Square Roots: Take out perfect squares from under the radical.
4. Reduce the Radical: Write the simplified radical with the factors outside and remaining inside.

Example:

Simplify $\sqrt{180}$:

- Prime factorization: $180 = 2^2 \times 3^2 \times 5$
- Extract perfect squares:

$$\sqrt{180} = \sqrt{2^2 \times 3^2 \times 5} = \sqrt{(2 \times 3)^2 \times 5}$$

$$ = 6 \sqrt{5} $$

So, the simplified radical form of $\sqrt{180}$ is $6 \sqrt{5}$.

Applying this to the third side:

Suppose the calculation yields $c = \sqrt{79}$. Since 79 is prime, it cannot

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