Find The Minimum Value Of The Function F(x) = 1.2x^2 + 6x + 2 To The Nearesthundredth.

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Introduction

Understanding how to find the minimum value of a quadratic function is a fundamental skill in algebra and calculus. Quadratic functions, expressed in the form $f(x) = ax^2 + bx + c$, have a parabolic graph, which opens upwards if $a > 0$ and downwards if $a < 0$. In this case, the function $F(x) = 1.2x^2 + 6x + 2$ is a parabola opening upward because $a = 1.2 > 0$. The minimum value of the function corresponds to the vertex of the parabola.

This article provides a detailed, step-by-step approach to find the minimum value of the given quadratic function, including methods such as calculus-based vertex calculation and completing the square. We will also explore the practical applications of these methods and provide an accurate answer rounded to the nearest hundredth.

Understanding the Quadratic Function

General Form of a Quadratic Function

A quadratic function has the form:

$$
f(x) = ax^2 + bx + c
$$

where:

- a, b, and c are constants,
- $a \neq 0$,
- the graph is a parabola.

Characteristics

- The parabola opens upwards if $a > 0$, indicating a minimum point.
- The vertex of the parabola gives the maximum or minimum value depending on the opening direction.

- The axis of symmetry passes through the vertex.

Given Function

In our case:

$$
F(x) = 1.2x^2 + 6x + 2
$$

- $a = 1.2$,
- $b = 6$,
- $c = 2$.

Since $a > 0$, the parabola opens upward, and the vertex represents the minimum point.

Methods to Find the Minimum Value

1. Vertex Formula

The vertex x-coordinate of a parabola $ax^2 + bx + c$ is given by:

$$
x_{v} = -\frac{b}{2a}
$$

Once x_{v} is found, substituting it into the function yields the minimum value $F(x_{v})$.

2. Completing the Square

Rearranging the quadratic into a perfect square form:

$$
F(x) = a(x + d)^2 + e
$$

where d and e are constants derived from the original quadratic. This form directly shows the vertex and minimum value.

3. Calculus Approach

Using derivatives, the critical point where the derivative equals zero indicates the vertex:

$$
F'(x) = 0
$$

The second derivative test confirms whether this point is a minimum.

Step-by-Step Solution

Step 1: Find the x-coordinate of the vertex using the vertex formula

Given:

$$
x_{v} = -\frac{b}{2a}
$$

Plug in:

$$
x_{v} = -\frac{6}{2 \times 1.2} = -\frac{6}{2.4}
$$

Calculate:

$$
x_{v} = -2.5
$$

Step 2: Calculate the minimum value by substituting x_{v} into the function

Substitute $x = -2.5$:

$$
F(-2.5) = 1.2 \times (-2.5)^2 + 6 \times (-2.5) + 2
$$

Calculate step-by-step:

- $(-2.5)^2 = 6.25$,
- $1.2 \times 6.25 = 7.5$,
- $6 \times -2.5 = -15$,
- Sum all parts:

$$
7.5 - 15 + 2 = -5.5
$$

Step 3: Round to the nearest hundredth

The minimum value is:

\[

$$
\boxed{-5.50}
$$

Alternative Method: Completing the Square

Completing the square provides an elegant way to confirm the vertex and minimum value.

Step 1: Factor out the coefficient a:

$$
F(x) = 1.2x^2 + 6x + 2 = 1.2 \left(x^2 + \frac{6}{1.2}x \right) + 2
$$

Calculate:

$$
\frac{6}{1.2} = 5
$$

So,

$$
F(x) = 1.2 (x^2 + 5x) + 2
$$

Step 2: Complete the square inside the parentheses:

To complete the square for $x^2 + 5x$:

- Take half of 5, which is 2.5,
- Square it: $2.5^2 = 6.25$,
- Rewrite:

$$
x^2 + 5x = (x + 2.5)^2 - 6.25
$$

Step 3: Rewrite the original function:

$$
F(x) = 1.2 \left((x + 2.5)^2 - 6.25 \right) + 2
$$

Distribute:

$$
F(x) = 1.2 (x + 2.5)^2 - 1.2 \times 6.25 + 2
$$

Calculate:

$$
1.2 \times 6.25 = 7.5
$$

So,

$$
F(x) = 1.2 (x + 2.5)^2 - 7.5 + 2 = 1.2 (x + 2.5)^2 - 5.5
$$

Step 4: Identify the minimum value

The minimum occurs when $(x + 2.5)^2 = 0$, i.e., at $x = -2.5$. The minimum value:

$$
F_{\min} = -5.5
$$

which confirms our previous calculation, rounded to the nearest hundredth:

$$
\boxed{-5.50}
$$

Visualizing the Function

Graphing $F(x) = 1.2x^2 + 6x + 2$ reveals a parabola opening upward, with the vertex at $(-2.5, -5.50)$. Visual aids are helpful in understanding the nature of the quadratic and confirming the minimum point.

Practical Applications of Finding the Minimum of a Quadratic

Understanding how to find the minimum value of quadratic functions has numerous real-world applications, including:

- Optimization Problems: Minimizing costs, errors, or resource usage.
- Physics: Determining the lowest potential energy configuration.
- Economics: Finding the lowest cost or maximum profit points.
- Engineering: Designing systems with optimal parameters.

Summary

To find the minimum value of the quadratic function $F(x) = 1.2x^2 + 6x + 2$:

- The vertex x-coordinate is found using $x_{v} = -\frac{b}{2a}$, which yields $x = -2.5$.
- Substituting $x = -2.5$ into the function gives the minimum value $F(-2.5) = -5.50$.
- Alternatively, completing the square confirms this result, providing an insightful view of the quadratic's structure.

Final Answer:

$$
\boxed{\text{The minimum value of } F(x) \text{ is } -5.50 \text{ (to the nearest hundredth).}}
$$

Additional Tips

- Always verify the sign of a to determine whether the vertex is a minimum or maximum.
- Use multiple methods to confirm your results for accuracy.
- Visualize the function graphically for better intuition.

Call to Action

Mastering techniques to find the minimum or maximum of quadratic functions is essential in advanced mathematics and various fields. Practice with different functions, and explore how these concepts apply to real-world problems to strengthen your understanding and problem-solving skills.

Frequently Asked Questions

What is the goal when finding the minimum value of the function F(x) = 1.2x^2 + 6x + 2?

The goal is to determine the lowest point or the smallest output value of the function, which occurs at its vertex.

How do you find the x-coordinate of the minimum point for the quadratic function F(x) = 1.2x^2 + 6x

+ 2?

Use the formula x = -b / (2a). Here, a = 1.2 and b = 6, so x = -6 / (2 1.2) = -6 / 2.4 = -2.5.

What is the minimum value of the function F(x) = 1.2x^2 + 6x + 2, rounded to the nearest hundredth?

First, find F(-2.5): F(-2.5) = 1.2(-2.5)^2 + 6(-2.5) + 2 = 1.2(6.25) - 15 + 2 = 7.5 - 15 + 2 = -5.5. The minimum value is -5.50.

Why is the vertex of the parabola important in finding the minimum of this quadratic function?

Because for a parabola opening upwards (a > 0), the vertex represents the lowest point, giving the minimum value of the function.

Can we confirm that the vertex is a minimum point for F(x) = 1.2x^2 + 6x + 2?

Yes, since the coefficient of x^2 (a = 1.2) is positive, the parabola opens upward, making the vertex a minimum point.

How does the coefficient 1.2 affect the shape of the parabola for the function F(x)?

The positive coefficient 1.2 makes the parabola open upward and influences its width; a larger coefficient results in a narrower parabola.

What is the significance of rounding the minimum value to the nearest hundredth?

Rounding to the nearest hundredth provides a precise, easy-to-understand value suitable for most practical purposes and standard reporting.

Additional Resources

Find The Minimum Value Of The Function $F(x) = 1.2x^2 + 6x + 2$ To The Nearest Hundredth

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Introduction

In the realm of algebra and calculus, understanding how to find the minimum value of a quadratic function is a fundamental skill with broad applications—from optimizing business profits to engineering design. In this investigative article, we delve into the process of determining the minimum value of the quadratic function:

$$ F(x) = 1.2x^2 + 6x + 2 $$

Our goal is to identify the value of $F(x)$ at its lowest point and to do so with precision—rounded to the nearest hundredth. We will explore the theoretical background, step-by-step solution, and real-world implications of this mathematical problem, providing a comprehensive review suitable for students, educators, and enthusiasts alike.

Understanding Quadratic Functions and Their Graphs

Before tackling the problem directly, it’s essential to understand the nature of quadratic functions.

Characteristics of Quadratic Functions

A quadratic function is any function of the form:

$$ f(x) = ax^2 + bx + c $$

where $a \neq 0$. The graph of a quadratic is a parabola, which opens upward when $a > 0$ and downward when $a < 0$.

In our case, $a = 1.2$, which is positive, indicating the parabola opens upward and has a minimum point (vertex).

The Vertex of a Parabola

The vertex of a parabola provides the maximum or minimum point depending on the parabola’s opening. For a quadratic $ax^2 + bx + c$:

- The x-coordinate of the vertex is given by:

$$ x_{v} = -\frac{b}{2a} $$

- The y-coordinate (or the minimum/maximum value) is:

$$ F(x_{v}) = a x_{v}^2 + b x_{v} + c $$

Our task is to compute these values precisely for the given function.

Step-by-Step Solution

Step 1: Identify coefficients

Given:

$$ F(x) = 1.2x^2 + 6x + 2 $$

- $a = 1.2$
- $b = 6$
- $c = 2$

Step 2: Calculate the x-coordinate of the vertex

Using the formula:

$$ x_{v} = -\frac{b}{2a} $$

Substituting the values:

$$ x_{v} = -\frac{6}{2 \times 1.2} = -\frac{6}{2.4} $$

Calculating:

$$ x_{v} = -2.5 $$

This means the parabola reaches its minimum at $x = -2.5$.

Step 3: Calculate the minimum value $F(x_{v})$

Plugging $x_{v} = -2.5$ into the original function:

$$
\begin{aligned}
F(-2.5) &= 1.2 \times (-2.5)^2 + 6 \times (-2.5) + 2 \\
&= 1.2 \times 6.25 + (-15) + 2 \\
&= 7.5 - 15 + 2 \\
&= (7.5 + 2) - 15 \\
&= 9.5 - 15 \\
&= -5.5
\end{aligned}
$$

Thus, the minimum value of the function is -5.5.

Step 4: Rounding to the nearest hundredth

Since the value is already expressed with one decimal, the rounded value to the nearest hundredth is:

$$ \boxed{-5.50} $$

Verification and Validation

To ensure the accuracy of this result, it’s prudent to verify through alternative methods such as completing the square or calculus.

Completing the Square

Rewrite $F(x)$:

$$
\begin{aligned}
F(x) &= 1.2x^2 + 6x + 2 \\
&= 1.2 \left(x^2 + 5x \right) + 2
\end{aligned}
$$

Complete the square inside the parentheses:

$$
x^2 + 5x = x^2 + 5x + \left(\frac{5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 = \left(x + \frac{5}{2} \right)^2 - \frac{25}{4}
$$

Thus,

$$
\begin{aligned}
F(x) &= 1.2 \left(\left(x + \frac{5}{2} \right)^2 - \frac{25}{4} \right) + 2 \\
&= 1.2 \left(x + 2.5 \right)^2 - 1.2 \times \frac{25}{4} + 2 \\
&= 1.2 \left(x + 2.5 \right)^2 - 1.2 \times 6.25 + 2 \\
&= 1.2 \left(x + 2.5 \right)^2 - 7.5 + 2 \\
&= 1.2 \left(x + 2.5 \right)^2 - 5.5
\end{aligned}
$$

The minimum occurs when the squared term is zero, i.e., at $x = -2.5$.

The minimum value:

$$
F(-2.5) = -5.5
$$

This matches our previous calculation, confirming the result.

Significance and Applications

The process outlined above demonstrates how to find the minimum value of a quadratic function precisely. Such calculations are vital in numerous practical contexts:

- Optimization Problems: Businesses often seek to minimize costs or maximize efficiency, which can be modeled with quadratic functions.
- Engineering Design: Engineers optimize material usage or structural stability by analyzing quadratic relationships.
- Physics and Natural Sciences: Parabolic trajectories and energy minimization often involve quadratic functions.
- Finance: Risk assessments and profit maximizations sometimes reduce to quadratic optimization.

Knowing how to accurately determine the minimum value of a quadratic function enhances decision-making and problem-solving across these domains.

Additional Insights

Effect of Coefficients on the Parabola

- The coefficient $a = 1.2$ indicates a relatively narrow parabola opening upward.
- The vertex at $x = -2.5$ shows the point of optimality in the domain.
- The minimum value -5.5 signifies the lowest point on the graph, which can be interpreted as the optimal solution in real-world scenarios if $F(x)$ represents cost, risk, or other measurable quantities.

Extending the Approach

For more complex functions or higher-degree polynomials, similar principles apply, often requiring calculus techniques such as derivatives to find critical points. For quadratic functions specifically, the vertex formula and completing the square are quick and reliable methods.

Conclusion

The investigation into the quadratic function $F(x) = 1.2x^2 + 6x + 2$ confirms that its minimum value is -5.50, achieved at $x = -2.5$. This precise calculation underscores the power of algebraic methods for optimization tasks and provides a solid foundation for applications across various scientific and practical fields.

By understanding the underlying principles and verifying results through multiple methods, students and professionals alike can confidently analyze quadratic functions to solve real-world problems efficiently and accurately.

Find The Minimum Value Of The Function F X 1 2x 2 6x 2 To

The Nearesthundredth

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