Find The Points On The Curve Y = X3 + 3x2 9x + 10 Where The Tangent Is Horizontal.

Find The Points On The Curve $Y = X^3 + 3x^2 - 9x + 10$ Where The Tangent Is Horizontal

Understanding how to find points on a curve where the tangent line is horizontal is a fundamental concept in calculus. In this article, we will explore how to identify these specific points on the curve defined by the equation $Y = X^3 + 3x^2 - 9x + 10$. Whether you're a student preparing for exams or a math enthusiast seeking clarity, this step-by-step guide will help you master the process of locating points with horizontal tangents on this cubic curve.

Understanding the Concept of Horizontal Tangents

What Is a Tangent Line?

A tangent line to a curve at a specific point is a straight line that just "touches" the curve at that point without crossing it locally. The slope of this tangent line gives us the rate at which the y-value changes with respect to x at that point.

When Is a Tangent Line Horizontal?

A tangent line is horizontal when its slope is zero. Geometrically, this means the curve has a flat tangent at that point, indicating a local maximum, minimum, or a point of inflection where the curve flattens out momentarily.

Why Find Points with Horizontal Tangents?

Finding these points helps in analyzing the behavior of the graph, identifying extrema (highest and lowest points), and understanding the nature of the function's shape. These points are also crucial in optimization problems and in understanding the curvature of the graph.

Step-by-Step Process to Find Points with Horizontal Tangents on the Curve $Y = X^3 + 3x^2 - 9x + 10$

Step 1: Find the First Derivative of the Function

The first derivative of the function $Y = X^3 + 3x^2 - 9x + 10$ provides the slope of the tangent line at any point x.

- Compute dY/dx:

- d/dx of $X^3 = 3X^2$
- d/dx of $3x^2 = 6x$
- d/dx of $-9x = -9$
- d/dx of $10 = 0$

Therefore,
$dY/dx = 3X^2 + 6x - 9$

Step 2: Set the Derivative Equal to Zero to Find Horizontal Tangents

Since the tangent is horizontal when the slope is zero:

$3X^2 + 6x - 9 = 0$

Divide through by 3 to simplify:

$X^2 + 2x - 3 = 0$

Step 3: Solve the Quadratic Equation for x

The quadratic equation:

$X^2 + 2x - 3 = 0$

can be solved using the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

where $a = 1$, $b = 2$, $c = -3$.

Calculating discriminant:

$\Delta = (2)^2 - 4(1)(-3) = 4 + 12 = 16$

Now, find the roots:

$x = [-2 \pm \sqrt{16}] / 2$

$x = [-2 \pm 4] / 2$

This yields two solutions:

- $x = (-2 + 4) / 2 = 2 / 2 = 1$
- $x = (-2 - 4) / 2 = -6 / 2 = -3$

Step 4: Find Corresponding y-values for Each x

To find the y-coordinates corresponding to these x-values, substitute x = 1 and x = -3 into the original function.

For x = 1:

$Y = (1)^3 + 3(1)^2 - 9(1) + 10$
$Y = 1 + 3 - 9 + 10$
$Y = (1 + 3 + 10) - 9 = 14 - 9 = 5$

For x = -3:

$Y = (-3)^3 + 3(-3)^2 - 9(-3) + 10$
$Y = -27 + 3(9) + 27 + 10$
$Y = -27 + 27 + 27 + 10$
$Y = (0 + 27 + 10) = 37$

Summary of the Points with Horizontal Tangents

Based on the calculations above, the points on the curve where the tangent line is horizontal are:

- **(1, 5)**
- **(-3, 37)**

These points indicate where the curve flattens out, potentially corresponding to local maxima, minima, or points of inflection.

Additional Insights: Analyzing the Nature of These Points

Second Derivative Test

To determine whether these points are maxima, minima, or inflection points, analyze the second derivative.

- The first derivative is: $dY/dx = 3X^2 + 6x - 9$
- The second derivative is:
$d^2Y/dx^2 = d/dx\,[3X^2 + 6x - 9] = 6X + 6$

At x = 1:

$d^2Y/dx^2 = 6(1) + 6 = 6 + 6 = 12 > 0$
Since the second derivative is positive, the point (1, 5) is a local minimum.

At $x = -3$:
$d^2Y/dx^2 = 6(-3) + 6 = -18 + 6 = -12 < 0$
Since the second derivative is negative, the point (-3, 37) is a local maximum.

Summary:
- (1, 5) is a local minimum point.
- (-3, 37) is a local maximum point.

Visualizing the Curve and Its Critical Points

Creating a graph of the function $Y = X^3 + 3x^2 - 9x + 10$ can help visualize the points of interest. The graph will show the curve rising, reaching a maximum at (-3, 37), then descending to a minimum at (1, 5), before rising again. The horizontal tangents at these points correspond to the peaks and valleys of the graph, providing visual confirmation of the calculus-based analysis.

Practical Applications of Finding Horizontal Tangents

- **Optimization Problems:** Determining maximum profit or minimum cost by analyzing the critical points on a demand or cost curve.
- **Engineering and Physics:** Analyzing the points where a system's rate of change momentarily halts, such as in motion or structural stress analysis.
- **Economics:** Finding equilibrium points where the rate of change of economic indicators is zero.

Conclusion

Finding points on a curve where the tangent is horizontal involves calculating the first derivative, setting it to zero, solving the resulting quadratic equation, and then evaluating the original function at those critical points. For the curve $Y = X^3 + 3x^2 - 9x + 10$, the points with horizontal tangents are at (1, 5) and (-3, 37). The second derivative test further reveals that these points correspond to a local minimum and maximum, respectively. Mastering this process enhances your understanding of the behavior of complex functions and equips you with powerful tools for calculus analysis and real-world problem-solving.

If you want to deepen your understanding of calculus concepts like derivatives, critical points, and curve analysis, consider exploring related topics such as the second derivative test, concavity, and inflection points. Practice with different functions to develop confidence in applying these techniques across various mathematical contexts.

Frequently Asked Questions

How do you find the points on the curve y = x^3 + 3x^2 - 9x + 10 where the tangent is horizontal?

To find points where the tangent is horizontal, first compute the derivative y' = 3x^2 + 6x - 9, then set y' = 0 and solve for x. The solutions give the x-coordinates of the points with horizontal tangents.

What is the derivative of y = x^3 + 3x^2 - 9x + 10?

The derivative y' = 3x^2 + 6x - 9.

How do you solve for x when the tangent to the curve y = x^3 + 3x^2 - 9x + 10 is horizontal?

Set the derivative y' = 0, so 3x^2 + 6x - 9 = 0, then divide both sides by 3 to get x^2 + 2x - 3 = 0, and solve the quadratic equation.

What are the solutions to the quadratic equation x^2 + 2x - 3 = 0?

The solutions are x = 1 and x = -3, found using the quadratic formula or factoring.

How do you find the corresponding y-values for the points where the tangent is horizontal?

Substitute x = 1 and x = -3 into the original curve y = x^3 + 3x^2 - 9x + 10 to find the respective y-coordinates.

What are the points on the curve where the tangent is horizontal?

The points are (1, y(1)) and (-3, y(-3)). Calculating y(1) gives 1 + 3 - 9 + 10 = 5, and y(-3) gives -27 + 27 + 27 + 10 = 37. So, the points are (1, 5) and (-3, 37).

Why is it important to find points where the tangent is horizontal on a curve?

These points indicate local maxima, minima, or inflection points, which are critical for understanding the behavior of the graph and optimizing functions.

Can you generalize the process to find points with horizontal tangents for any polynomial curve?

Yes. For any polynomial curve y = f(x), find its derivative f'(x), set f'(x) = 0, solve for x, and then find the corresponding y-values. These points indicate where the tangent line is horizontal.

Additional Resources

Understanding the Points on the Curve $y = x^3 + 3x^2 - 9x + 10$ Where the Tangent Is Horizontal

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Introduction

In the world of calculus and analytic geometry, the study of curves and their properties is fundamental for understanding the behavior of functions. Among these properties, the points where the tangent to a curve is horizontal hold special significance. These points often correspond to local maxima, minima, or points of inflection, and they are crucial in optimization problems, physics, engineering, and computer graphics.

This article delves into the mathematical journey of finding the specific points on the curve:

$$
y = x^3 + 3x^2 - 9x + 10
$$

where the tangent line is horizontal. We will explore the process step-by-step, employing derivatives, critical point analysis, and algebraic solutions to identify these points accurately.

The Significance of Horizontal Tangents

Before diving into calculations, it’s important to understand why points with horizontal tangents are significant:

- Critical Points: They often mark critical points of the function, indicating potential local maxima or minima.
- Shape Analysis: Understanding where the tangent is horizontal helps in sketching the graph accurately.
- Optimization: Identifying these points is essential in solving real-world problems involving maximum profit, minimum cost, etc.

Mathematical Foundations

To find points where the tangent is horizontal, we need to analyze the derivative of the function \(y

$= f(x)$.

Derivative and Slope of Tangent Line

The derivative $y' = \frac{dy}{dx}$ gives the slope of the tangent line at any point x.

- When $y' = 0$, the tangent line is horizontal.
- When $y' \neq 0$, the tangent line has a non-zero slope.

Thus, our key task is to compute y' and solve for x when $y' = 0$.

Step-by-Step Solution

1. Differentiating the Function

Given:

$$
y = x^3 + 3x^2 - 9x + 10
$$

Applying power rule for differentiation:

$$
\frac{dy}{dx} = 3x^2 + 6x - 9
$$

2. Setting the Derivative Equal to Zero

To find points where the tangent line is horizontal:

$$
3x^2 + 6x - 9 = 0
$$

Divide through by 3 to simplify:

$$
x^2 + 2x - 3 = 0
$$

3. Solving the Quadratic Equation

The quadratic:

$$
x^2 + 2x - 3 = 0
$$

can be factored or solved using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:

- $a = 1$
- $b = 2$
- $c = -3$

Calculating discriminant:

$$
\Delta = 2^2 - 4 \times 1 \times (-3) = 4 + 12 = 16
$$

Thus,

$$
x = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}
$$

which gives two solutions:

$$
x_1 = \frac{-2 + 4}{2} = \frac{2}{2} = 1
$$

$$
x_2 = \frac{-2 - 4}{2} = \frac{-6}{2} = -3
$$

4. Finding Corresponding y-Values

Now, substitute $x = 1$ and $x = -3$ into the original function to find the respective points.

For $x = 1$:

$$
y = (1)^3 + 3(1)^2 - 9(1) + 10 = 1 + 3 - 9 + 10 = 5
$$

Point: $(1, 5)$

For $x = -3$:

$$
y = (-3)^3 + 3(-3)^2 - 9(-3) + 10 = -27 + 3 \times 9 + 27 + 10 = -27 + 27 + 27 + 10 = 37
$$

Point: $(-3, 37)$

Comprehensive Overview of the Findings

Point	Corresponding x	Corresponding y	Interpretation
Point 1	$x = 1$	$y = 5$	Point where the tangent line is horizontal.
Point 2	$x = -3$	$y = 37$	Another point with a horizontal tangent.

Visualizing the Curve and Critical Points

While this article focuses on the analytical process, visual intuition is invaluable. Plotting the function, either manually or via graphing tools, reveals:

- The points $(1, 5)$ and $(-3, 37)$ on the curve.
- The tangent lines at these points are flat (horizontal).
- The nature of these points (maxima, minima, or inflection points) can be further analyzed via second derivatives or the first derivative test.

Confirming the Nature of These Points

To determine whether these points are local maxima, minima, or points of inflection, the second derivative test is typically employed.

1. Compute the Second Derivative

Given:

$$
y' = 3x^2 + 6x - 9
$$

Differentiate again:

$$
y'' = 6x + 6
$$

2. Evaluate at Critical Points

- At $x = 1$:

$$
y''(1) = 6(1) + 6 = 12 > 0
$$

Since $y''(1) > 0$, the point $(1, 5)$ is a local minimum.

- At $x = -3$:

$$
y''(-3) = 6(-3) + 6 = -18 + 6 = -12 < 0
$$

Since $y''(-3) < 0$, the point $(-3, 37)$ is a local maximum.

Summary of Key Findings

- Points with Horizontal Tangents:

- $(1, 5)$, which is a local minimum.
- $(-3, 37)$, which is a local maximum.

- These points are critical in understanding the overall shape and behavior of the curve.

Broader Implications and Applications

Understanding where the tangent is horizontal on a curve like $y = x^3 + 3x^2 - 9x + 10$ is not merely an academic exercise. It has practical applications across various fields:

- Economics: Identifying maximum profit points.
- Physics: Finding equilibrium points in systems.
- Engineering: Optimizing design parameters.
- Computer Graphics: Rendering curves accurately by identifying key points.

Final Remarks

Identifying points with horizontal tangents involves a systematic process—differentiation, solving for critical points, and analyzing the nature of these points. In our example, the function's critical points revealed both a maximum and a minimum, providing a comprehensive understanding of its behavior.

This approach exemplifies the power of calculus in dissecting complex functions and offers a robust framework applicable to a wide array of mathematical and real-world problems.

References & Further Reading

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- Larson, Ron. Calculus. 11th Edition.
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Embark on your calculus journey with confidence, knowing that mastering the identification of horizontal tangent points is a vital step in understanding the intricate dance of curves and their critical features!

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