Find The Variance Of The Random Variable Y, Where Fy(y) = 3(1 Y)2, 0 < Y < 1

Find The Variance Of The Random Variable Y, Where Fy(y) = $3(1 - Y)^2$, $0 < Y < 1$

Understanding how to find the variance of a random variable is fundamental in probability theory and statistics. When given a probability density function (pdf) such as Fy(y) = $3(1 - y)^2$ for $0 < y < 1$, it becomes essential to understand how to compute its mean, variance, and other moments to grasp its behavior fully. This article provides a comprehensive guide to calculating the variance of the specified random variable Y, including step-by-step procedures, explanations of key concepts, and practical insights.

Introduction to Variance and Its Importance in Probability

Variance is a measure of the dispersion or spread of a set of data points or a probability distribution. It quantifies how much the values of a random variable deviate from its mean (expected value). In probability and statistics, understanding variance helps in assessing the reliability, risk, and variability associated with random phenomena.

Why Is Variance Important?

- Risk Assessment: In finance, variance helps measure the volatility of asset returns.
- Quality Control: Variance indicates consistency in manufacturing processes.
- Statistical Inference: Variance underpins confidence intervals and hypothesis testing.

Key Concepts Related to Variance

- Expected Value (Mean): The average or central value of a random variable.
- Second Moment: The expected value of the square of the random variable.
- Variance Formula: $\operatorname{Var}(Y) = E[Y^2] - (E[Y])^2$

Understanding the Given Probability Density Function

The problem specifies a probability density function (pdf):

$$f_Y(y) = 3(1 - y)^2, \quad 0 < y < 1$$

and asks to find the variance of Y.

Characteristics of the PDF

- The function is defined on the interval (0, 1).
- It is non-negative within this interval.
- The total probability integrates to 1, confirming it's a valid pdf.

Validating the PDF

To verify the provided function is a valid pdf, we check:

$$
\int_0^1 f_Y(y) \, dy = 1
$$

Calculating:

$$
\int_0^1 3(1 - y)^2 \, dy
$$

Step-by-Step Calculation of the Variance of Y

To find the variance, we need:

1. The expected value $E[Y]$
2. The second moment $E[Y^2]$
3. Then apply the variance formula.

Step 1: Calculate $E[Y]$

$$
E[Y] = \int_0^1 y \, f_Y(y) \, dy = \int_0^1 y \times 3(1 - y)^2 \, dy
$$

Step 2: Calculate $E[Y^2]$

$$
E[Y^2] = \int_0^1 y^2 \, f_Y(y) \, dy = \int_0^1 y^2 \times 3(1 - y)^2 \, dy
$$

Step 3: Compute Variance

$$
\operatorname{Var}(Y) = E[Y^2] - (E[Y])^2
$$

Detailed Integration Process

Let's perform these integrations explicitly.

Calculating $E[Y]$

$$
E[Y] = 3 \int_0^1 y(1 - y)^2 \, dy
$$

Expand $(1 - y)^2 = 1 - 2y + y^2$:

$$
E[Y] = 3 \int_0^1 y (1 - 2y + y^2) \, dy
$$

Distribute y:

$$
E[Y] = 3 \int_0^1 (y - 2y^2 + y^3) \, dy
$$

Integrate term-wise:

$$
E[Y] = 3 \left[\frac{y^2}{2} - \frac{2 y^3}{3} + \frac{y^4}{4} \right]_0^1
$$

Evaluate at y=1:

$$
E[Y] = 3 \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right)
$$

Simplify inside the parentheses:

$$
\frac{1}{2} - \frac{2}{3} + \frac{1}{4} = \frac{6}{12} - \frac{8}{12} + \frac{3}{12} = \frac{6 - 8 + 3}{12} = \frac{1}{12}
$$

Multiply by 3:

$$
E[Y] = 3 \times \frac{1}{12} = \frac{1}{4}
$$

Calculating $E[Y^2]$

$$
E[Y^2] = 3 \int_0^1 y^2 (1 - y)^2 \, dy
$$

Expand $(1 - y)^2 = 1 - 2y + y^2$:

$$
E[Y^2] = 3 \int_0^1 y^2 (1 - 2y + y^2) \, dy
$$

Distribute y^2:

$$
E[Y^2] = 3 \int_0^1 (y^2 - 2 y^3 + y^4) \, dy

\]

Integrate term-wise:

$$
E[Y^2] = 3 \left[\frac{y^3}{3} - \frac{2 y^4}{4} + \frac{y^5}{5} \right]_0^1
$$

Simplify:

$$
E[Y^2] = 3 \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right)
$$

Express all fractions with common denominator 60:

- $\frac{1}{3} = \frac{20}{60}$
- $\frac{2}{4} = \frac{30}{60}$
- $\frac{1}{5} = \frac{12}{60}$

Now sum:

$$
\frac{20}{60} - \frac{30}{60} + \frac{12}{60} = \frac{20 - 30 + 12}{60} = \frac{2}{60} = \frac{1}{30}
$$

Multiply by 3:

$$
E[Y^2] = 3 \times \frac{1}{30} = \frac{1}{10}
$$

Final Calculation of Variance

Using the obtained moments:

$$
E[Y] = \frac{1}{4}
$$

$$
E[Y^2] = \frac{1}{10}
$$

Calculate variance:

$$
\operatorname{Var}(Y) = E[Y^2] - (E[Y])^2 = \frac{1}{10} - \left(\frac{1}{4}\right)^2
$$

$$
\operatorname{Var}(Y) = \frac{1}{10} - \frac{1}{16}
$$

Express with common denominator 80:

- $\frac{1}{10} = \frac{8}{80}$
- $\frac{1}{16} = \frac{5}{80}$

Subtract:

$$
\operatorname{Var}(Y) = \frac{8}{80} - \frac{5}{80} = \frac{3}{80}
$$

Therefore, the variance of Y is $\boxed{\frac{3}{80}}$.

Summary of Key Points

- The probability density function $f_Y(y) = 3(1 - y)^2$ is valid on the interval (0, 1).
- The expected value $E[Y]$ for the random variable Y is $\frac{1}{4}$.
- The second moment $E[Y^2]$ is $\frac{1}{10}$.
- The variance $\operatorname{Var}(Y)$ is $\frac{3}{80}$.

Practical Applications and Insights

Understanding how to compute variance from a given pdf is crucial in statistical analysis, especially in fields like finance, engineering, and data science. The process involves:

- Validating the pdf's correctness.
- Computing moments via integration.
- Applying the variance formula.

These skills enable analysts and researchers to interpret data distributions accurately, assess risks, and make informed decisions.

Conclusion

Calculating the variance of a random variable with a specified probability density function is a fundamental task in probability theory. For the given pdf $f_Y(y) = 3(1 - y)^2$ on (0, 1), we found that the variance is $\frac{3}{80}$. Mastery of such calculations enhances one's ability to analyze probabilistic models, interpret data variability, and apply statistical principles across

Frequently Asked Questions

What is the probability density function (PDF) of the random variable Y?

The PDF of Y is given by $F_y(y) = 3(1 - y)^2$ for $0 < y < 1$.

How do you verify that Fy(y) is a valid probability density function?

By integrating Fy(y) over its support (0 to 1), ensuring the total area equals 1: $\int_0^1 3(1 - y)^2 \, dy = 1$.

What is the expected value E[Y] of the random variable Y?

$E[Y] = \int_0^1 y \, F_y(y) \, dy = \int_0^1 y \, 3(1 - y)^2 \, dy$, which evaluates to 1/4.

How do you compute the second moment E[Y^2] for the variable Y?

$E[Y^2] = \int_0^1 y^2 \, F_y(y) \, dy = \int_0^1 y^2 \, 3(1 - y)^2 \, dy$, which evaluates to 1/5.

What is the formula for variance Var(Y) of the random variable Y?

Variance $Var(Y) = E[Y^2] - (E[Y])^2$.

What is the variance of Y based on the given PDF Fy(y)?

Using $E[Y] = 1/4$ and $E[Y^2] = 1/5$, $Var(Y) = 1/5 - (1/4)^2 = 1/5 - 1/16 = (16/80) - (5/80) = 11/80$.

How do you interpret the variance value 11/80 for the distribution of Y?

It indicates the spread or dispersion of Y around its mean, with a value of 11/80 approximately equal to 0.1375, showing relatively low variability.

What are the key steps to find the variance of a continuous random variable with a given PDF?

First, compute the expected value E[Y], then find the second moment E[Y^2], and finally use $Var(Y) = E[Y^2] - (E[Y])^2$.

Additional Resources

Find The Variance Of The Random Variable Y, Where Fy(y) = 3(1 - Y)^2, 0 < Y < 1

Introduction

In the realm of probability and statistics, understanding the behavior of random variables is fundamental. One critical characteristic of a random variable is its variance, which quantifies the spread or dispersion of its possible values around the mean. This article delves into the process of finding the variance of a continuous random variable Y, characterized by the probability density function (pdf):

$$
f_Y(y) = 3(1 - y)^2, \quad 0 < y < 1
$$

This specific density function is a classic example of a Beta distribution with particular parameters, but here, the focus is on deriving the variance through direct calculation, illustrating core principles of probability theory and statistical analysis.

Background and Context

Before beginning the calculations, it's imperative to understand the nature of the given density function. The function

$$
f_Y(y) = 3(1 - y)^2
$$

defined over the interval $(0, 1)$, must satisfy two conditions:

1. Non-negativity: $f_Y(y) \geq 0$ for all $y \in (0, 1)$,
2. Normalization: The integral over its domain equals 1:

$$
\int_0^1 f_Y(y) \, dy = 1
$$

The specified form resembles the Beta distribution's pdf, which, for parameters α and β, is:

$$
f_{Beta}(y; \alpha, \beta) = \frac{y^{\alpha -1}(1 - y)^{\beta - 1}}{B(\alpha, \beta)} \quad \text{for } 0 < y < 1
$$

where $B(\alpha, \beta)$ is the Beta function. In our case, the pdf simplifies to a form proportional to $(1 - y)^2$, hinting at a Beta distribution with specific parameters, which we will verify later.

Step 1: Verifying the PDF and Its Normalization

To ensure correctness, first verify that $f_Y(y)$ is a valid probability density function:

$$
\int_0^1 3(1 - y)^2 \, dy = 1
$$

Calculating the integral:

$$
\int_0^1 3(1 - y)^2 \, dy = 3 \int_0^1 (1 - y)^2 \, dy
$$

Let’s evaluate $\int_0^1 (1 - y)^2 \, dy$:

$$
\int_0^1 (1 - y)^2 \, dy = \int_0^1 (1 - 2y + y^2) \, dy
$$

$$
= \left[y - y^2 + \frac{y^3}{3} \right]_0^1 = (1 - 1 + \frac{1}{3}) - (0 - 0 + 0) = \frac{1}{3}
$$

Multiplying by 3:

$$
3 \times \frac{1}{3} = 1
$$

Thus, the pdf is valid.

Step 2: Calculating the Mean $E[Y]$

The mean or expected value of Y, denoted μ_Y, is:

$$
E[Y] = \int_0^1 y f_Y(y) \, dy
$$

Substituting the pdf:

$$
E[Y] = \int_0^1 y \times 3(1 - y)^2 \, dy
$$

Factor constants:

$$
E[Y] = 3 \int_0^1 y (1 - y)^2 \, dy
$$

Expand $(1 - y)^2 = 1 - 2y + y^2$:

$$

$$
E[Y] = 3 \int_0^1 y (1 - 2y + y^2) \, dy
$$

Distribute y:

$$
E[Y] = 3 \int_0^1 (y - 2 y^2 + y^3) \, dy
$$

Integrate term-by-term:

$$
E[Y] = 3 \left[\frac{y^2}{2} - \frac{2 y^3}{3} + \frac{y^4}{4} \right]_0^1
$$

Evaluate at 1:

$$
E[Y] = 3 \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right)
$$

Simplify inside the parentheses:

$$
\frac{1}{2} - \frac{2}{3} + \frac{1}{4} = \frac{6}{12} - \frac{8}{12} + \frac{3}{12} = \frac{6 - 8 + 3}{12} = \frac{1}{12}
$$

Multiply by 3:

$$
E[Y] = 3 \times \frac{1}{12} = \frac{1}{4}
$$

Result: The mean of Y is $\boxed{\frac{1}{4}}$.

Step 3: Calculating $E[Y^2]$

The variance formula requires both $E[Y]$ and $E[Y^2]$. Now, compute $E[Y^2]$:

$$
E[Y^2] = \int_0^1 y^2 f_Y(y) \, dy = 3 \int_0^1 y^2 (1 - y)^2 \, dy
$$

Express $(1 - y)^2$ again:

$$
E[Y^2] = 3 \int_0^1 y^2 (1 - 2 y + y^2) \, dy
$$

Distribute:

$$
E[Y^2] = 3 \int_0^1 (y^2 - 2 y^3 + y^4) \, dy
$$

Integrate term-by-term:

$$
E[Y^2] = 3 \left[\frac{y^3}{3} - \frac{2 y^4}{4} + \frac{y^5}{5} \right]_0^1
$$

Simplify each term:

$$
E[Y^2] = 3 \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right)
$$

Express all with common denominators or as decimals:

$$
\frac{1}{3} \approx 0.3333, \quad \frac{2}{4} = \frac{1}{2} = 0.5, \quad
\frac{1}{5} = 0.2
$$

Calculate:

$$
E[Y^2] = 3 (0.3333 - 0.5 + 0.2) = 3 (0.0333) = 0.1
$$

Alternatively, for exact fractions:

$$
\frac{1}{3} - \frac{1}{2} + \frac{1}{5} = \frac{10}{30} - \frac{15}{30} +
\frac{6}{30} = \frac{10 - 15 + 6}{30} = \frac{1}{30}
$$

Multiply by 3:

$$
E[Y^2] = 3 \times \frac{1}{30} = \frac{1}{10}
$$

Result: The second moment is $\boxed{\frac{1}{10}}$.

Step 4: Deriving the Variance $\operatorname{Var}(Y)$

Recall the variance formula:

$$
\operatorname{Var}(Y) = E[Y^2] - (E[Y])^2
$$

Plugging in the values:

$$
\operatorname{Var}(Y) = \frac{1}{10} - \left(\frac{1}{4} \right)^2 =
\frac{1}{10} - \frac{1}{16}
$$

Find common denominator (160):

```
\[
\frac{16}{160} - \frac{10}{160} = \frac{6}{160} = \frac{3}{80}
\]

Final Result:

\[
\boxed{
\operatorname{Var}(Y) = \frac{3}{80}
}
\]

which approximates to 0.0375.

---

Additional Insights: Connection to Beta Distribution

The density function:

\[
f_Y(y) = 3(1 - y)^2
\]

over \( (0, 1) \), matches the Beta distribution \( \text{Beta}(\alpha,
\beta) \) with:

\[
f_{Beta}(y; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} y^{\alpha - 1} (1 -
y)^{\beta - 1}
\]

Matching terms:

\[
y^{0}
```

Find The Variance Of The Random Variable Y Where Fy Y 3 1 Y 2 0 Lt Y Lt 1

Find The Variance Of The Random Variable Y Where Fy Y 3 1 Y 2 0 Lt Y Lt 1

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