# For What Values Of K Is The Inequality X^(2)-(k-3)x-k+6>0 Valid For All Real X?

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Introduction

Understanding quadratic inequalities is a fundamental aspect of algebra that helps us analyze the behavior of quadratic functions and their ranges. One common problem involves determining the conditions under which a quadratic inequality holds true for all real values of \( x \). Specifically, when dealing with inequalities such as:

\[
x^2 - (k - 3)x - k + 6 > 0,
\]

it becomes essential to find the values of the parameter \( k \) that make this inequality valid for every real number \( x \). This type of problem has practical applications in fields like physics, engineering, and mathematics, where ensuring positivity or certain bounds over all real numbers is crucial.

In this article, we will explore the detailed methodology to determine for which values of \( k \) the quadratic inequality

\[
x^2 - (k - 3)x - k + 6 > 0
\]

is always true, regardless of the value of \( x \). We will analyze the quadratic's properties, such as its discriminant and vertex, to establish the necessary and sufficient conditions on \( k \). This comprehensive approach will include step-by-step reasoning, relevant formulas, and insightful explanations to clarify the solution process.

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Understanding the Quadratic Inequality

The given inequality is:

\[
x^2 - (k - 3)x - k + 6 > 0.
\]

This is a quadratic expression in terms of \( x \), with the parameter \( k \) influencing its shape and position. Our goal is to identify all values of \( k \) such that:

$$
x^2 - (k - 3)x - k + 6 > 0
$$

holds for all real $x$.

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Key Concepts and Approach

To analyze this inequality, we need to understand the behavior of quadratic functions. The general quadratic form is:

$$
ax^2 + bx + c,
$$

where $a$, $b$, and $c$ are constants. The graph of this quadratic is a parabola opening upwards if $a > 0$, and downwards if $a < 0$.

Since the coefficient of $x^2$ is 1 (positive), the parabola opens upward.

This fact is crucial because:

- If the parabola opens upward and the quadratic is positive for all $x$, then it must not intersect or be tangent to the $x$-axis, meaning its discriminant is less than zero.
- Conversely, if the parabola opens upward and the quadratic is positive everywhere, then the quadratic's minimum point should be above the $x$-axis.

Thus, the core idea is:

- Find the minimum value of the quadratic function (located at the vertex).
- Ensure that this minimum is greater than zero for the inequality to hold for all $x$.

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Step 1: Expressing the Quadratic in Standard Form

Let's write the quadratic explicitly:

$$
f(x) = x^2 - (k - 3)x - k + 6.
$$

The quadratic in standard form is:

$$
f(x) = a x^2 + b x + c,
$$

where:

- $a = 1$,
- $b = -(k - 3) = -k + 3$,
- $c = -k + 6$.

---

Step 2: Find the Vertex of the Parabola

The vertex of a parabola $f(x) = a x^2 + b x + c$ occurs at:

$$
x_v = -\frac{b}{2a}.
$$

Substituting the values:

$$
x_v = -\frac{-k + 3}{2 \times 1} = \frac{k - 3}{2}.
$$

The minimum value of $f(x)$ occurs at $x = x_v$. The corresponding $y$-value (the value of the quadratic at the vertex) is:

$$
f(x_v) = a x_v^2 + b x_v + c.
$$

Calculating $f(x_v)$:

$$
f\left(\frac{k - 3}{2}\right) = \left(\frac{k - 3}{2}\right)^2 + \left(-k + 3\right) \left(\frac{k - 3}{2}\right) + (-k + 6).
$$

Let's compute this step-by-step.

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Step 3: Compute the Vertex's $y$-value

1. Square term:

$$
\left(\frac{k - 3}{2}\right)^2 = \frac{(k - 3)^2}{4} = \frac{k^2 - 6k + 9}{4}.
$$

2. Linear term:

$$
(-k + 3) \times \frac{k - 3}{2} = \frac{(-k + 3)(k - 3)}{2}.
$$

Note that:

$$
(-k + 3)(k - 3) = -k \times k + -k \times (-3) + 3 \times k - 3 \times 3 = -k^2 + 3k + 3k - 9 = -k^2 + 6k - 9.
$$

Therefore,

$$
\text{Linear term} = \frac{-k^2 + 6k - 9}{2}.
$$

3. Constant term:

$$
c = -k + 6.
$$

Now, sum all these:

$$
f(x_v) = \frac{k^2 - 6k + 9}{4} + \frac{-k^2 + 6k - 9}{2} + (-k + 6).
$$

Express all terms with denominator 4 for uniformity:

$$
f(x_v) = \frac{k^2 - 6k + 9}{4} + \frac{2(-k^2 + 6k - 9)}{4} + \frac{4(-k + 6)}{4}.
$$

Simplify numerator:

$$
k^2 - 6k + 9 + 2(-k^2 + 6k - 9) + 4(-k + 6).
$$

Compute each:

- $2(-k^2 + 6k - 9) = -2k^2 + 12k - 18$,
- $4(-k + 6) = -4k + 24$.

Sum numerator:

$$
k^2 - 6k + 9 - 2k^2 + 12k - 18 - 4k + 24.
$$

Combine like terms:

- $k^2 - 2k^2 = -k^2$,

- $-6k + 12k - 4k = ( -6k + 12k) - 4k = 6k - 4k = 2k$,
- $9 - 18 + 24 = (9 - 18) + 24 = -9 + 24 = 15$.

Thus,

$$
f(x_v) = \frac{-k^2 + 2k + 15}{4}.
$$

---

Step 4: Establishing the Conditions for the Inequality

Since the parabola opens upward (positive coefficient of $x^2$), the quadratic $f(x)$ will be greater than zero for all $x$ if and only if its minimum value is strictly greater than zero:

$$
f(x_v) > 0,
$$

which translates to:

$$
\frac{-k^2 + 2k + 15}{4} > 0.
$$

Multiplying both sides by 4 (positive, so inequality direction remains the same):

$$
- k^2 + 2k + 15 > 0.
$$

Rearranged:

$$
- k^2 + 2k + 15 > 0,
$$

or equivalently:

$$
k^2 - 2k - 15 < 0.
$$

---

Step 5: Solving the Inequality for $k$

The quadratic inequality:

$$

$$
k^2 - 2k - 15 < 0
$$

is a standard quadratic inequality. Let's find its roots:

$$
k^2 - 2k - 15 = 0.
$$

Using the quadratic formula:

$$
k = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-15)}}{2} = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm \sqrt{64}}{2}.
$$

Calculate:

$$
k = \frac{2 \pm 8}{2}.
$$

Thus, the roots are:

$$
k = \frac{2 + 8}{2} = \frac{10}{2} = 5,
$$

$$
k = \frac{2 - 8}{2} = \frac{-6}{2}
$$

# Frequently Asked Questions

## For what values of k is the inequality $x^2 - (k - 3)x - k + 6 > 0$ valid for all real x?

The inequality holds for all real x when the quadratic has no real roots, which occurs when its discriminant is negative. Calculating the discriminant: $D = (k - 3)^2 - 4(1)(-k + 6) = (k - 3)^2 + 4k - 24$. Setting $D < 0$ and solving yields $k \in (-\infty, 1) \cup (7, \infty)$.

## How do you determine the values of k for which the quadratic inequality is positive for all real x?

You determine these values by ensuring the quadratic has no real roots and opens upwards (which it does, since the coefficient of $x^2$ is positive). This is done by setting the discriminant less than zero and solving for k.

## What role does the discriminant play in solving for k in the inequality?

The discriminant indicates whether the quadratic has real roots. For the inequality to be valid for all real x, the discriminant must be negative, meaning no real roots exist, and the quadratic remains positive everywhere.

## Are there any boundary values of k where the inequality equals zero for some real x?

Yes. When the discriminant equals zero, the quadratic has exactly one real root, and the inequality $x^2 - (k - 3)x - k + 6 > 0$ is not valid for all x, only for x not equal to that root.

## What is the significance of the parabola opening upwards in this context?

Since the coefficient of $x^2$ is positive, the parabola opens upwards, meaning the quadratic is positive outside its roots. Ensuring no real roots (discriminant < 0) guarantees it stays positive for all x.

## Can you provide the interval notation for the values of k where the inequality holds for all real x?

Yes. The inequality holds for all x when $k \in (-\infty, 1) \cup (7, \infty)$.

## What steps are involved in deriving the solution for the inequality in terms of k?

The steps include: 1) Write the quadratic expression, 2) Calculate its discriminant, 3) Set the discriminant less than zero, 4) Solve the inequality in k, resulting in the intervals where the original inequality holds for all x.

## How does the value of k affect the shape and position of the parabola in the quadratic inequality?

The value of k shifts the parabola horizontally and affects its roots. When k changes, the discriminant changes, altering whether the parabola intersects the x-axis or stays above it, which determines for which k the inequality holds for all x.

# Additional Resources

For What Values Of K Is The Inequality $X^2 - (k-3)x - k + 6 > 0$ Valid For All Real X?

Understanding the conditions under which a quadratic inequality holds for all real numbers is a fundamental aspect of algebra and mathematical analysis. The inequality in question, $X^2 - (k-3)x - k + 6 > 0$, involves a parameter k, and determining the range of k values that satisfy this inequality for every real value of X requires a detailed exploration of quadratic functions, their graphs, and the conditions for positivity. This article aims to thoroughly analyze the problem, break down the key concepts involved, and provide comprehensive insights into the solution process.

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Introduction to Quadratic Inequalities and Their Significance

Quadratic inequalities are inequalities involving quadratic functions, which are polynomials of degree two. They commonly appear in optimization problems, physics, engineering, and various branches of mathematics. Analyzing such inequalities often involves understanding the graph of the quadratic function, its roots, and its vertex.

The general quadratic function can be expressed as:

$$f(x) = ax^2 + bx + c$$

where $a \neq 0$. The inequality $f(x) > 0$ or $f(x) < 0$ holds depending on the parabola's orientation and position relative to the x-axis.

In our specific case,

$$f(x) = x^2 - (k-3)x - (k - 6)$$

and the question is: For which values of k does

$$x^2 - (k-3)x - (k - 6) > 0$$

hold for all real $x$?

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Fundamental Concepts for the Analysis

Before diving into the solution, it is essential to revisit some core concepts:

1. Nature of Quadratic Functions

- Parabola Orientation: Determined by the coefficient $a$. Since $a=1$ in our case, the parabola opens upward, meaning it tends to infinity as $x \to \pm \infty$.

- Vertex: The maximum or minimum point of the parabola. For an upward opening parabola, the vertex is the minimum point.

- Roots (Zeros): Values of $x$ where $f(x)=0$. The nature and position of roots influence the sign of $f(x)$.

2. Conditions for $f(x) > 0$ for All Real $x$

For a quadratic $f(x) = ax^2 + bx + c$:

- If $a > 0$, then $f(x) > 0$ for all real $x$ if and only if the quadratic has no real roots, i.e., its discriminant $D < 0$.

- Conversely, if $a < 0$, then $f(x) > 0$ for all $x$ if and only if the quadratic has no real roots as well, but since the parabola opens downward, the quadratic must be entirely below the x-axis.

In our case, since the quadratic opens upward ($a=1>0$), the key criterion is:

$$ D = b^2 - 4ac < 0 $$

which ensures the quadratic never crosses the x-axis and remains positive everywhere.

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Expressing the Quadratic in Terms of $k$

Given:

$$ f(x) = x^2 - (k-3)x - (k - 6) $$

we identify the parameters:

- $a = 1$
- $b = -(k-3) = -k + 3$
- $c = - (k - 6) = -k + 6$

Our goal is to find the range of $k$ such that:

$$ f(x) > 0 \quad \text{for all } x \in \mathbb{R} $$

which is equivalent to:

$$ D = b^2 - 4ac < 0 $$

---

Deriving the Discriminant Inequality

Substituting the parameters:

$$
\begin{aligned}
D &= (-k + 3)^2 - 4 \times 1 \times (-k + 6) \\
&= (k^2 - 6k + 9) - 4(-k + 6) \\
&= k^2 - 6k + 9 + 4k - 24 \\
&= k^2 - 2k - 15
\end{aligned}
$$

To ensure $f(x) > 0$ for all $x$, we require:

$$ D < 0 \Rightarrow k^2 - 2k - 15 < 0 $$

---

### Solving the Discriminant Inequality

The quadratic inequality:

$$ k^2 - 2k - 15 < 0 $$

can be factored or solved via quadratic formula:

$$
k^2 - 2k - 15 = 0
$$

Applying quadratic formula:

$$
k = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-15)}}{2} = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm \sqrt{64}}{2}
$$

$$
k = \frac{2 \pm 8}{2}
$$

which yields:

- $k = \frac{2 + 8}{2} = \frac{10}{2} = 5$
- $k = \frac{2 - 8}{2} = \frac{-6}{2} = -3$

Since the quadratic $k^2 - 2k - 15$ opens upwards ($a=1>0$), the inequality $k^2 - 2k - 15 < 0$ holds between its roots:

$$
-3 < k < 5
$$

---

### Final Result and Interpretation

Therefore, the inequality $x^2 - (k-3)x - (k - 6) > 0$ holds for all real $x$ if and only if:

$$
\boxed{
-3 < k < 5
}
$$

\]

This interval indicates the range of $k$ for which the quadratic function remains strictly positive for all $x$.

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Additional Considerations

1. Edge Cases

- At $k = -3$ or $k=5$, the discriminant $D = 0$. In these cases, the quadratic touches the x-axis at exactly one point, meaning $f(x) \geq 0$ but not strictly greater than zero everywhere.

- Since the inequality in question is strict ($>\ 0$), the endpoints are excluded. Thus, the open interval $(-3, 5)$ is the precise set of $k$ values.

2. Graphical Interpretation

Plotting the quadratic for different $k$ values:

- For $-3 < k < 5$, the parabola opens upward and does not intersect the x-axis, remaining positive for all $x$.
- For $k \leq -3$ or $k \geq 5$, the parabola touches or crosses the x-axis, violating the strict positivity condition.

3. Practical Implications

Knowing the precise range of parameters for which inequalities hold universally is crucial in optimization, control systems, and mathematical modeling, where ensuring positivity is often necessary for system stability or feasibility.

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Summary of Key Findings

| Parameter $k$ Range | Discriminant $D$ | Inequality Status | Remarks |
|---|---|---|---|
| $-3 < k < 5$ | $D<0$ | $f(x) > 0$ for all $x$ | Strict inequality, parabola never touches x-axis |
| $k = -3$ | $D=0$ | $f(x) \geq 0$, touches x-axis | Not strictly positive |
| $k=5$ | $D=0$ | $f(x) \geq 0$, touches x-axis | Not strictly positive |
| $k < -3$ or $k > 5$ | $D>0$ | $f(x)$ has real roots, crosses x-axis | Inequality fails for some $x$ |

---

Conclusion

The analysis demonstrates that the quadratic inequality

$$ x^2 - (k-3)x - (k - 6) > 0 $$

is valid for all real $x$ if and only if

$$\boxed{-3 < k < 5}$$

This conclusion is rooted in the discriminant condition for quadratic positivity and offers a clear criterion for selecting the parameter $k$. Such methodologies are fundamental in algebra, providing insights into the behavior of quadratic functions and their applications across mathematics and engineering disciplines.

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Final Remarks

Understanding how parameters influence the behavior of quadratic inequalities enhances problem-solving skills and mathematical intuition. The approach of analyzing the discriminant is a powerful tool, applicable across various types of polynomial inequalities. Master

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For What Values Of K Is The Inequality X 2 K 3 X K 6 Gt 0 Valid For All Real X

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