Given The Function F(x) = 4x3 + 13x, What Is The Gradient Of The Function When X = 2 ?

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Understanding the gradient of a function at a specific point is fundamental in calculus, especially when analyzing the behavior of functions, rates of change, and slopes of curves. In this article, we will explore how to find the gradient of the function $F(x) = 4x^3 + 13x$ at the point where x equals 2. We will cover the concepts of derivatives, how to compute the derivative of the function, and how to evaluate it at the given point. This comprehensive guide aims to clarify the process and provide you with a solid understanding of how to approach such problems.

What Is the Gradient of a Function?

Definition of Gradient

The gradient of a function at a particular point is a measure of the instantaneous rate of change or the slope of the tangent line to the function at that point. In the context of single-variable functions, this is represented mathematically by the derivative of the function.

- For a function F(x), the gradient at a point x = a is denoted as F'(a) or dF/dx at x = a.
- The gradient tells us whether the function is increasing or decreasing at that point and how steep the curve is.

Why Is the Gradient Important?

Understanding the gradient is essential because:

- It helps in analyzing the behavior of functions.
- It is used in optimization problems to find maximum or minimum points.
- It provides insights into the rate at which the function's output changes with respect to its input.
- It is foundational in physics for understanding velocity and acceleration.

Understanding the Function $F(x) = 4x^3 + 13x$

Overview of the Function

The function $F(x) = 4x^3 + 13x$ is a polynomial function of degree 3, which means its graph is a cubic curve with potential points of inflection, maxima, and minima.

- The term $4x^3$ dominates the behavior of the function at large values of x.
- The term 13x influences the slope at various points but is less dominant at large x.

Graphical Behavior

- When x is large and positive, the function tends to increase rapidly due to the cubic term.
- When x is negative, the cubic term causes the function to decrease sharply.
- The function is smooth and continuous, with a well-defined tangent at every point.

Calculating the Derivative of F(x)

Why Derive the Function?

To find the gradient at a specific point, we need to find the derivative of F(x). The derivative provides the slope of the tangent line at any point.

Rules Used for Differentiation

- Power Rule: $d/dx\, [x^n] = n\, x^{n-1}$
- Sum Rule: $d/dx\, [u + v] = du/dx + dv/dx$

Step-by-Step Derivation

Given $F(x) = 4x^3 + 13x$,

1. Derive $4x^3$:
- $d/dx\, [4x^3] = 4\, d/dx\, [x^3] = 4\; 3x^2 = 12x^2$
2. Derive 13x:
- $d/dx\, [13x] = 13\, d/dx\, [x] = 13\; 1 = 13$

Therefore, the derivative F'(x) is:
\[F'(x) = 12x^2 + 13 \]

Evaluating the Gradient at x = 2

Substitute x = 2 into the derivative

To find the gradient when x = 2, plug in 2 into F'(x):

$$ F'(2) = 12(2)^2 + 13 $$

Calculate step-by-step:

1. Square 2:
- $2^2 = 4$
2. Multiply by 12:
- $12 4 = 48$
3. Add 13:
- $48 + 13 = 61$

So, the gradient of the function when x = 2 is 61.

Interpreting the Result

What Does a Gradient of 61 Mean?

- The slope of the tangent line to the curve at x = 2 is 61.
- The function is increasing rapidly at this point, indicating a steep positive slope.
- The high value suggests that around x = 2, the function's output is changing quite significantly with small changes in x.

Implications in Real-World Contexts

The gradient can relate to various real-world scenarios, such as:

- In physics, if F(x) represented position over time, the gradient would be velocity at x = 2.
- In economics, it could represent the rate of change of profit or cost with respect to some variable.

Additional Related Concepts

Finding the Equation of the Tangent Line at x = 2

Once the gradient is known, you might want to find the equation of the tangent line to the curve at that point.

- To do this, find the point on the function at x = 2:
$$ F(2) = 4(2)^3 + 13(2) = 4 8 + 26 = 32 + 26 = 58 $$
- The point is (2, 58).
- The tangent line equation:
$$ y - y_1 = m (x - x_1) $$
where $m = 61$, $x_1 = 2$, $y_1 = 58$:
$$ y - 58 = 61 (x - 2) $$
Simplified:
$$ y = 61x - 122 + 58 = 61x - 64 $$

Significance of the Tangent Line

The tangent line approximates the function's behavior near $x = 2$ and provides a linear approximation of the function at that point.

Practical Applications of the Gradient Calculation

Optimization Problems

- Finding maxima or minima of functions involves setting the derivative to zero and analyzing the gradient.
- In this case, since the gradient at $x=2$ is positive, the function is increasing there.

Physics and Engineering

- Velocity calculations, rate of change of physical quantities, and slope analysis in engineering designs.

Economics and Business

- Marginal analysis, where the derivative indicates marginal cost, revenue, or profit at specific points.

Summary

- The gradient of the function $F(x) = 4x^3 + 13x$ at any point x is given by its derivative, $F'(x) = 12x^2 + 13$.
- Evaluating at $x = 2$ yields a gradient of 61.
- This indicates a steep, increasing slope at $x = 2$, with the function rising quickly at this point.
- The process involves differentiating the function, substituting the specific x-value, and interpreting the result in context.

Conclusion

Calculating the gradient at a specific point is a straightforward yet powerful application of calculus. By understanding and applying the derivative, you can analyze the rate at which functions change and make informed decisions in various fields such as physics, economics, and engineering. The example of $F(x) = 4x^3 + 13x$ at $x = 2$ demonstrates how derivatives provide immediate insight into the behavior of functions at particular points, enabling better analysis and application of mathematical concepts in real-world scenarios.

Frequently Asked Questions

What is the given function in the problem?

The function is $F(x) = 4x^3 + 13x$.

How do you find the gradient (derivative) of the function F(x)?

You differentiate $F(x)$ with respect to x, applying power rule to each term.

What is the derivative of F(x) = 4x^3 + 13x?

The derivative, $F'(x)$, is $12x^2 + 13$.

How do you evaluate the gradient at x = 2?

Substitute $x = 2$ into the derivative: $F'(2) = 12(2)^2 + 13$.

What is the value of the gradient when x = 2?

$F'(2) = 12(4) + 13 = 48 + 13 = 61$.

Why is the gradient important in understanding the function?

The gradient indicates the slope or rate of change of the function at a specific point.

Can the gradient be negative or positive? What does that indicate?

Yes, the gradient can be negative or positive, indicating whether the function is decreasing or increasing at that point.

Is the gradient at x=2 a constant or does it vary with x?

The gradient varies with x; at $x=2$, it has a specific value, but it changes at other points.

How does understanding the gradient help in graphing the function?

Knowing the gradient at different points helps sketch the slope of the tangent lines and the overall shape of the graph.

Additional Resources

Given The Function F(x) = 4x^3 + 13x, What Is The Gradient Of The Function When X = 2?

In the world of calculus, understanding how a function behaves at specific points is fundamental. Whether you're a student delving into derivatives for the first time or a professional applying calculus to real-world problems, grasping how to determine the gradient—or the slope—of a function at a given point is essential. Today, we'll explore this concept using the function $F(x) = 4x^3 + 13x$, focusing on calculating its gradient when $x = 2$. This process involves differentiating the function and then evaluating the derivative at the specified point, providing insights into the function's rate of change at that moment.

Understanding the Concept of Gradient in Calculus

Before diving into the calculations, it's important to clarify what the gradient of a function signifies.

What Is the Gradient?

- Definition: In calculus, the gradient of a function at a particular point refers to the slope of the tangent line to the curve at that point. It indicates how rapidly the function's value changes as the input variable changes.
- In One Dimension: For single-variable functions, the gradient is simply the derivative, representing the instantaneous rate of change.
- Geometrical Interpretation: Imagine drawing a tangent line at a point on the curve. The gradient tells you whether the curve is increasing or decreasing at that point, and how steep that increase or decrease is.

Why Is it Important?

- Understanding the gradient helps in analyzing the behavior of the function—whether it's rising or falling.
- It plays a vital role in optimization problems, physics (like velocity and acceleration), and in various engineering applications.

Step 1: Deriving the General Derivative of the Function $F(x) = 4x^3 + 13x$

Calculating the gradient at a specific point begins with finding the derivative of the function.

Differentiation Rules Applied

- Power Rule: The derivative of x^n with respect to x is $n x^{n-1}$.
- Constant Multiple Rule: The derivative of $a \cdot f(x)$ is $a \cdot f'(x)$.

Applying these rules:

$$
F(x) = 4x^3 + 13x
$$

- Derivative of $4x^3$:

$$
\frac{d}{dx} (4x^3) = 4 \times 3x^{2} = 12x^2
$$

- Derivative of $13x$:

$$
\frac{d}{dx} (13x) = 13
$$

Putting it together:

$$
F'(x) = 12x^2 + 13
$$

This derivative function $F'(x)$ gives the gradient of the original function at any point x.

Step 2: Evaluating the Derivative at $x = 2$

Now, to find the gradient when $x = 2$, substitute 2 into the derivative:

$$
F'(2) = 12 \times (2)^2 + 13
$$

Calculating step-by-step:

- Square 2:

$$
(2)^2 = 4
$$

- Multiply by 12:

$$
12 \times 4 = 48
$$

- Add 13:

$$
48 + 13 = 61
$$

Result:

$$
\boxed{F'(2) = 61}
$$

This means the slope of the tangent to the curve $F(x) = 4x^3 + 13x$ at $x=2$ is 61.

Deep Dive: Significance of the Result

Achieving a gradient of 61 at $x=2$ has several implications:

- Steepness of the Curve: A gradient of 61 indicates a very steep incline at that point. The function is increasing rapidly as x approaches 2.
- Rate of Change: For every unit increase in x near 2, the function's value increases approximately by 61 units.
- Behavior of the Function: Since the derivative is positive and large at $x=2$, the function is in a phase of rapid growth at that point.

Additional Insights: How The Gradient Changes

Understanding the derivative's behavior over the domain offers broader insights:

The Second Derivative

- Computing the second derivative $F''(x)$:

$$
F''(x) = \frac{d}{dx} (12x^2 + 13) = 24x
$$

- At $x=2$:

$$
F''(2) = 24 \times 2 = 48
$$

A positive second derivative ($48 > 0$) indicates that the function is concave up at $x=2$, meaning the slope is increasing as x increases. Therefore, the tangent line at $x=2$ is not only steep but also on an upward curve that is becoming steeper.

Practical Applications of Gradient Computations

Calculating the gradient at specific points isn't just an academic exercise; it has real-world applications:

- Physics: Determining velocity (rate of change of position) at a specific moment.
- Economics: Analyzing marginal cost or revenue at a certain production level.
- Engineering: Assessing stress or strain rates within materials.
- Data Science: Gradient-based optimization algorithms, like gradient descent, depend heavily on understanding such derivatives to find minima or maxima.

Summary: From Function to Gradient

- The original function $F(x) = 4x^3 + 13x$ describes a cubic curve.
- Differentiating yields $F'(x) = 12x^2 + 13$, representing the slope at any point.
- Evaluating at $x=2$ gives a gradient of 61, indicating a steep, upward-sloping tangent line at that point.
- The positive second derivative suggests the function's slope is increasing at $x=2$, contributing to the function's convex shape there.

Final Thoughts

Understanding how to compute and interpret the gradient of a function at specific points is a cornerstone of calculus with far-reaching implications across various scientific and engineering disciplines. Using straightforward differentiation rules, we can analyze the behavior of complex functions, predict their future trends, and make informed decisions based on their rates of change. In our example, the function $F(x) = 4x^3 + 13x$ reveals a rapidly increasing trend at $x=2$, with a steep gradient of 61, illustrating the power of calculus to unlock insights about the dynamics of mathematical models.

Whether you're exploring polynomial functions or more complex systems, mastering the art of derivative evaluation provides a vital tool for decoding the language of change that underpins much of the natural and designed world.

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