HELP PLSSIf X1 And X2 Are Solutions To 2x + 6x +10=0 Quadratic Equation, Then(x1)(x2)=

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Understanding quadratic equations is a fundamental aspect of algebra that plays a crucial role in various mathematical and real-world applications. When working with quadratic equations, one common task is to find the solutions, often denoted as x_1 and x_2, which are the roots of the equation. These roots provide valuable insights into the nature of the parabola represented by the quadratic function and help in solving complex problems across engineering, physics, finance, and more.

In this article, we will explore the specific quadratic equation given as 2x + 6x + 10 = 0, analyze its roots, and demonstrate how to determine the product of its solutions, denoted as $(x_1)(x_2)$. We will delve into the foundational concepts of quadratic equations, explain the methods to find roots, and provide a detailed step-by-step approach to calculating their product. This comprehensive guide aims to enhance your understanding of quadratic equations and improve your problem-solving skills.

Understanding Quadratic Equations

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$$ax^2 + bx + c = 0$$

where:
- $a \neq 0$,
- b and c are coefficients,
- x represents the variable.

The solutions to this equation, known as roots or zeros, are the values of x that satisfy the equation.

Standard Form and Roots

Quadratic equations can be solved using various methods, including:
- Factoring
- Completing the square
- The quadratic formula

The roots are typically denoted as x_1 and x_2.

Analyzing the Given Quadratic Equation: 2x + 6x + 10 = 0

Identifying the Coefficients

Before solving, let's rewrite the given equation in standard quadratic form:

$$2x + 6x + 10 = 0$$

Combine like terms:

$$(2x + 6x) + 10 = 0$$
$$8x + 10 = 0$$

Notice that this simplifies to a linear equation rather than a quadratic one because it contains only x raised to the first power.

However, based on the context and the structure of the question, it seems there might be a typo or a misinterpretation. Typically, a quadratic equation has an x^2 term. If the intended quadratic equation is:

$$2x^2 + 6x + 10 = 0$$

then we can proceed with solving it.

Assumption: The correct quadratic equation is:

$$2x^2 + 6x + 10 = 0$$

Note: If the original question was exactly as written, the equation is linear, and the product of solutions isn't defined for linear equations. Since the focus is on quadratic solutions, we proceed with the assumption that the equation is quadratic as above.

Coefficients Identified

- $a = 2$
- $b = 6$
- $c = 10$

Calculating the Roots Using the Quadratic Formula

The Quadratic Formula

The solutions for x are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:
- $\Delta = b^2 - 4ac$ is the discriminant.

Calculating the Discriminant

Calculate Δ:

$$\Delta = (6)^2 - 4 \times 2 \times 10$$
$$\Delta = 36 - 80$$
$$\Delta = -44$$

Since the discriminant is negative, the solutions are complex conjugates.

Finding the Roots

Express the roots:

$$x_{1,2} = \frac{-6 \pm \sqrt{-44}}{2 \times 2}$$
$$x_{1,2} = \frac{-6 \pm \sqrt{44}i}{4}$$
$$x_{1,2} = \frac{-6 \pm 2\sqrt{11}i}{4}$$
$$x_{1,2} = -\frac{3}{2} \pm \frac{\sqrt{11}i}{2}$$

Thus, the roots are:

$$x_1 = -\frac{3}{2} + \frac{\sqrt{11}i}{2}$$
$$x_2 = -\frac{3}{2} - \frac{\sqrt{11}i}{2}$$

Note: The roots are complex conjugates, which is common when the discriminant is negative.

Finding the Product of the Roots $(x_1)(x_2)$

Using Vieta's Formulas

Vieta's formulas provide relationships between the coefficients and roots of a quadratic equation:

- Sum of roots:

$$ x_1 + x_2 = -\frac{b}{a} $$

- Product of roots:

$$ x_1 \times x_2 = \frac{c}{a} $$

This means:

$$ (x_1)(x_2) = \frac{c}{a} $$

Application to Our Equation:

$$ (x_1)(x_2) = \frac{10}{2} = 5 $$

Therefore, the product of the solutions x_1 and x_2 is 5.

Summary and Key Takeaways

- For any quadratic equation $ax^2 + bx + c = 0$, the product of its roots is given by $\frac{c}{a}$.
- In the case of the quadratic equation $2x^2 + 6x + 10 = 0$, the roots are complex conjugates due to a negative discriminant.
- The product of these roots, regardless of whether they are real or complex, remains $\frac{c}{a}$.

Additional Tips for Solving Quadratic Equations

1. Always write the equation in standard form: $ax^2 + bx + c = 0$.
2. Calculate the discriminant ($\Delta = b^2 - 4ac$):
- If $\Delta > 0$, roots are real and distinct.
- If $\Delta = 0$, roots are real and equal.
- If $\Delta < 0$, roots are complex conjugates.
3. Use the quadratic formula for roots when the quadratic is difficult to factor.
4. Apply Vieta's formulas to find sums and products of roots quickly.

Real-World Applications of Quadratic Equations

Quadratic equations are not just academic exercises; they have numerous practical applications:

- Physics: Describing projectile motion and trajectories.
- Engineering: Analyzing signals and system behaviors.
- Economics: Modeling profit maximization and cost functions.
- Biology: Population modeling with quadratic growth patterns.
- Finance: Calculating break-even points and investment returns.

Conclusion

To conclude, when given a quadratic equation such as $2x^2 + 6x + 10 = 0$, the product of its solutions x_1 and x_2 can be efficiently determined using Vieta's formula:

$$(x_1)(x_2) = \frac{c}{a}$$

which, in this case, equals 5. Understanding the relationship between coefficients and roots simplifies many algebraic problems and provides insights into the nature of quadratic functions.

By mastering these concepts, students and professionals can approach quadratic equations confidently, whether in academic settings or real-world scenarios. Remember to analyze the discriminant to understand the nature of roots and leverage Vieta's formulas to quickly find relationships between solutions.

Keywords: quadratic equation, roots, solutions, Vieta's formulas, discriminant, complex roots, algebra, quadratic formula, product of roots, quadratic solutions, math help

Frequently Asked Questions

What is the quadratic equation given in the problem?

The quadratic equation given is 2x + 6x + 10 = 0.

How can we simplify the quadratic equation 2x + 6x + 10 = 0?

Combine like terms: (2x + 6x) = 8x, so the simplified equation is 8x + 10 = 0.

What are the solutions x1 and x2 in terms of the quadratic equation?

x1 and x2 are the roots of the quadratic equation $8x + 10 = 0$, which can be found using the quadratic formula or by simplifying further.

What is the product of the roots (x1)(x2) for the quadratic equation?

For a quadratic equation $ax^2 + bx + c = 0$, the product of roots is c/a. In this case, since the equation is linear, it can be viewed as $ax^2 + bx + c = 0$ with a=0, but it's better to re-express or check the original equation for clarity.

Is the given quadratic equation correctly written, or does it need correction?

The equation $2x + 6x + 10 = 0$ simplifies to $8x + 10 = 0$, which is linear, not quadratic. If it's intended to be quadratic, perhaps it should be written as $2x^2 + 6x + 10 = 0$.

Assuming the correct quadratic is 2x^2 + 6x + 10 = 0, what is (x1)(x2)?

For the quadratic $2x^2 + 6x + 10 = 0$, the product of roots (x1)(x2) = c/a = 10/2 = 5.

Additional Resources

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Introduction

Quadratic equations form the backbone of algebraic studies and are fundamental in numerous scientific and engineering applications. They serve as the foundation for understanding polynomial functions, solving real-world problems, and developing critical analytical skills. A crucial aspect of quadratic equations involves understanding the relationships between their roots, or solutions, and their coefficients.

Among these relationships, the product of the roots plays a significant role. The question at hand—If X1 and X2 are solutions to the quadratic equation 2x + 6x + 10 = 0, then what is (x1)(x2)?—may seem straightforward at first glance but warrants detailed exploration, especially considering the structure of the quadratic and its standard forms.

This comprehensive article aims to scrutinize this question, delve into the underlying algebraic principles, analyze the given quadratic, and clarify the process of determining the product of its solutions. The discussion will encompass the fundamental theories of quadratic equations, the importance of accurate coefficient identification, and a step-by-step solution process.

Understanding Quadratic Equations and Their Roots

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$$ax^2 + bx + c = 0$$

where:
- $a \neq 0$,
- b and c are real numbers.

The solutions to this equation, x_1 and x_2, are the roots or zeros of the quadratic function. These roots can be real or complex, depending on the discriminant.

The Sum and Product of Roots

Vieta's formulas provide elegant relationships between the roots and coefficients of quadratic equations:

- Sum of roots:

$$x_1 + x_2 = -\frac{b}{a}$$

- Product of roots:

$$x_1 x_2 = \frac{c}{a}$$

These formulas are instrumental in quickly determining the roots' relationships without explicitly solving the quadratic.

Analyzing the Given Quadratic Equation

The initial quadratic equation provided is:

$$2x + 6x + 10 = 0$$

At first glance, this appears to be a quadratic equation, but a closer inspection reveals that it is expressed as a linear equation because it involves only first-degree terms.

Identifying the Error or Misinterpretation

In standard form, quadratic equations are written with the x^2 term explicitly present:

$$ ax^2 + bx + c = 0 $$

In the given expression:

$$ 2x + 6x + 10 = 0 $$

the terms $2x$ and $6x$ are linear, and no quadratic term (x^2) appears. Combining like terms:

$$ (2x + 6x) + 10 = 8x + 10 = 0 $$

This simplifies to a linear equation:

$$ 8x + 10 = 0 $$

which has a unique solution:

$$ x = -\frac{10}{8} = -\frac{5}{4} $$

Thus, the question appears to be based on an incorrect or incomplete quadratic expression. To proceed meaningfully, we need to clarify whether the original equation was intended to be:

- A quadratic with missing coefficients? Or
- A typographical error, where the quadratic should be expressed as $2x^2 + 6x + 10 = 0$?

Given the context—asking about solutions X_1 and X_2—it is reasonable to assume the intended quadratic equation is:

$$ 2x^2 + 6x + 10 = 0 $$

Reconstructing the Correct Quadratic Equation

Assuming the intended quadratic is:

$$ 2x^2 + 6x + 10 = 0 $$

we can now analyze this quadratic and determine the product of its roots.

Solving the Quadratic Equation: $2x^2 + 6x + 10 = 0$

Step 1: Identify coefficients

- $a = 2$
- $b = 6$
- $c = 10$

Step 2: Apply Vieta's formula for the product of roots

According to Vieta's formulas:

$$ x_1 x_2 = \frac{c}{a} $$

Substituting the known coefficients:

$$ x_1 x_2 = \frac{10}{2} = 5 $$

Therefore, the product of the roots X_1 and X_2 is 5.

Verifying the Roots

While the question primarily asks for the product, it is instructive to verify the roots themselves and understand their nature.

Step 3: Calculate the discriminant

$$ D = b^2 - 4ac $$

$$ D = 6^2 - 4 \times 2 \times 10 = 36 - 80 = -44 $$

Since $D < 0$, the roots are complex conjugates.

Step 4: Find the roots

$$ x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} $$

$$ x_{1,2} = \frac{-6 \pm \sqrt{-44}}{4} $$

$$ x_{1,2} = \frac{-6 \pm i \sqrt{44}}{4} $$

$$ x_{1,2} = \frac{-6 \pm i \sqrt{4 \times 11}}{4} $$

$$ x_{1,2} = \frac{-6 \pm 2i \sqrt{11}}{4} $$

$$ x_{1,2} = -\frac{3}{2} \pm \frac{i \sqrt{11}}{2} $$

The roots are complex conjugates:

$$ X_1 = -\frac{3}{2} + \frac{i \sqrt{11}}{2} $$
$$ X_2 = -\frac{3}{2} - \frac{i \sqrt{11}}{2} $$

Calculating their product:

$$
\left(-\frac{3}{2} + \frac{i \sqrt{11}}{2}\right) \left(-\frac{3}{2} - \frac{i \sqrt{11}}{2}\right)
$$

\]

which simplifies to:

\[
\left(-\frac{3}{2}\right)^2 - \left(\frac{i \sqrt{11}}{2}\right)^2
\]

\[
= \frac{9}{4} - \frac{i^2 \times 11}{4}
\]

Since \(i^2 = -1 \):

\[
= \frac{9}{4} - \frac{-1 \times 11}{4} = \frac{9}{4} + \frac{11}{4} = \frac{20}{4} = 5
\]

This confirms the earlier conclusion that the product of the roots is 5.

Significance and Applications

Understanding the relationship between coefficients and roots in quadratic equations is a cornerstone of algebra. The calculation of \(x_1 x_2 \) using Vieta's formula provides a quick and reliable method, especially when the roots are complex or difficult to compute explicitly.

These principles extend beyond simple quadratic equations to higher-degree polynomials, where similar relationships—though more complex—are invaluable for analysis.

Final Summary

- The initial quadratic equation was likely intended to be \(2x^2 + 6x + 10 = 0 \), not \(2x + 6x + 10 = 0 \).
- Applying Vieta's formulas, the product of solutions \(X_1 \) and \(X_2 \) is:

\[
\boxed{
x_1 x_2 = \frac{c}{a} = \frac{10}{2} = 5
}
\]

- The roots are complex conjugates with the same product, confirming the calculation.

Conclusion

In conclusion, when solving quadratic equations and exploring the relationships between roots and coefficients, Vieta's formulas serve as essential tools. For the quadratic $2x^2 + 6x + 10 = 0$, the product of solutions X_1 and X_2 is 5.

This exploration underscores the importance of carefully analyzing the structure of the given algebraic expressions. Recognizing whether an equation is quadratic or linear dramatically influences the approach and solutions. Clarity in the initial formulation ensures accurate mathematical reasoning and problem-solving.

Whether for academic pursuits, professional applications, or personal curiosity, mastering these fundamental relationships enhances one's ability to analyze, interpret, and solve polynomial equations efficiently and confidently.

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