If A = - I And B = 3 , Then Find The Value Of The A ^ 2 * B In Fully Simplified Form .

If A = - I And B = 3 , Then Find The Value Of The A ^ 2 B In Fully Simplified Form .

Introduction to the Problem

Understanding the problem involves recognizing the nature of the matrices involved and applying algebraic operations accordingly. The problem provides two variables:

- A: Given as $-I$, where I is the identity matrix.
- B: Given as a scalar, 3.

The goal is to evaluate the expression $A^2 B$ in its fully simplified form. To do this, one must understand matrix multiplication, scalar multiplication, and how these operations interact with the identity matrix and scalar constants.

Understanding the Components

The Identity Matrix I

- The identity matrix I is a square matrix with ones on the diagonal and zeros elsewhere.
- For a 2x2 matrix, I is represented as:
$$
I = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$
- For an $n \times n$ identity matrix, the same pattern applies.

The Matrix \(A = -I\)

- \(A\) is the negative of the identity matrix:
\[
A = -I = -1 \times I
\]
- Which, for a 2x2 case, is:
\[
A = \begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\]
- This matrix scales any vector it multiplies by \(-1\).

Scalar \(B = 3\)

- \(B\) is a scalar value, which can be multiplied with matrices via scalar multiplication.

Step-by-Step Solution Process

Step 1: Compute \(A^2\)

- Recall that \(A = -I\).
- Compute \(A^2 = A \times A\).

Since \(A = -I\), the product becomes:

\[
A^2 = (-I) \times (-I)
\]

- Scalar multiplication distributes over matrix multiplication:

\[
A^2 = (-1 \times I) \times (-1 \times I) = (-1) \times (-1) \times (I \times I)
\]

- Because scalar multiplication is associative and commutative with matrices:

\[
A^2 = (1) \times (I \times I) = I \times I

$$
\quad
$$

- The identity matrix multiplied by itself:

$$
I \times I = I
$$

- Therefore:

$$
A^2 = I
$$

Step 2: Multiply A^2 by scalar B

- The expression we need to evaluate is:

$$
A^2 B
$$

- Since $A^2 = I$, and $B = 3$, the scalar multiplication is straightforward:

$$
A^2 B = I \times 3
$$

- Multiplying a scalar with a matrix involves multiplying each element of the matrix by the scalar:

$$
I \times 3 = \begin{bmatrix}
1 \times 3 & 0 \\
0 & 1 \times 3
\end{bmatrix}
= \begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
$$

Final Answer and Explanation

The simplified form of the expression $A^2 B$ when $A = -I$ and $B = 3$ is the scalar multiple of the identity matrix:

$$
\boxed{
\begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
}
$$

This matrix is often denoted as $3I$, representing a scalar multiple of the identity matrix.

Additional Insights and Generalization

Matrix Power Properties

- For any scalar multiple of the identity matrix, kI, where k is a scalar, the following property holds:

$$
(kI)^n = k^n I
$$

- Applying this property to our problem:

$$
A^2 = (-I)^2 = (-1)^2 I = 1 \times I = I
$$

- Hence, the calculation is consistent across different powers.

Implications for Larger Matrices

- The same logic applies to matrices of higher dimensions, such as 3x3, 4x4, etc., where the identity matrix is appropriately larger.
- The key is recognizing that squaring $-I$ results in I, regardless of size, because:

$$
(-I)^2 = (-1)^2 \times I^2 = 1 \times I = I
$$

- Therefore, the calculation is universally valid.

Summary

- Given $A = -I$, the square A^2 equals I.
- Multiplying A^2 by scalar $B = 3$ yields $3I$.
- Expressed as a matrix, the result is:

$$
\boxed{
\begin{bmatrix}
3 & 0 \\
0 & 3
\end{bmatrix}
}
$$

- This result confirms that the operation simplifies neatly, leveraging properties of the identity matrix and scalar multiplication.

Conclusion

In conclusion, understanding the behavior of the identity matrix under various operations simplifies many matrix algebra problems. Recognizing that $(-I)^2 = I$ allows straightforward computation of expressions like $A^2 B$, regardless of the matrix size, provided A is a scalar multiple of the identity matrix. The scalar B scales the identity matrix directly, resulting in a matrix where all diagonal entries are equal to the scalar, and off-diagonal entries are zero. This fundamental property underscores the elegance and consistency of matrix algebra.

Frequently Asked Questions

If A = -I and B = 3, what is the value of A squared

multiplied by B?

Since A = -I, then A^2 = (-I)^2 = (-1)^2 I^2 = 1 I = I. Therefore, A^2 B = I 3 = 3.

Given A = -I and B = 3, how do you simplify A^2 B?

A^2 = (-I)^2 = I. Multiplying by B, we get I 3 = 3. So, the simplified value is 3.

Calculate the value of (A^2) B for A = -I and B = 3.

A^2 = (-I)^2 = I. Multiplying by B, I 3 = 3. Hence, the value is 3.

What is the fully simplified form of A^2 B if A = -I and B = 3?

Substituting A and B, A^2 = I, so A^2 B = I 3 = 3. The fully simplified form is 3.

If A = -I and B = 3, find the value of A^2 B in simplest terms.

Since A = -I, A^2 = (-I)^2 = I. Therefore, A^2 B = I 3 = 3.

Additional Resources

If A = - I And B = 3 , Then Find The Value Of The A ^ 2 B In Fully Simplified Form

In the realm of algebra and matrix operations, understanding how to manipulate and simplify expressions involving matrices and scalars is fundamental. Today, we delve into a specific problem that combines matrix algebra with scalar multiplication, providing clarity on how to handle such expressions systematically. The problem statement is straightforward but rich in concepts: If A = - I and B = 3, then find the value of A^2 B in fully simplified form. This exploration will guide you through the steps involved, from understanding the matrices involved to performing the necessary calculations and simplifications.

Understanding the Given Matrices and Scalars

Before we jump into calculations, it’s essential to understand the components of the problem:

The Identity Matrix, I

- The notation I typically denotes the identity matrix.
- An identity matrix is a square matrix with ones on the diagonal and zeros elsewhere.
- For example, in 2x2 form:

```
I = | 1 0 |
| 0 1 |
```

- The identity matrix acts as the multiplicative identity in matrix algebra, meaning A I = A for any compatible matrix A.

The Matrix A = -I

- Given that A = -I, this means A is simply the negative of the identity matrix.
- For 2x2 matrices, this can be written as:

```
A = -I = | -1 0 |
| 0 -1 |
```

- This matrix has all diagonal entries as -1 and zeros elsewhere.

The Scalar B = 3

- B is a scalar, a real number, in this case, 3.
- When multiplying a matrix by a scalar, each element of the matrix is multiplied by that scalar.

Step-by-Step Calculation of A^2 B

The goal is to compute A^2 B and express the result in fully simplified form.

Step 1: Compute A^2

- Since A = -I, the square of A is:

```
A^2 = A A
```

- For matrices, multiplication follows the standard rules, but because A is a scalar multiple of the identity, the calculation simplifies significantly.

Step 2: Recognize the Properties of Identity Matrices

- Recall that:

```
I I = I
```

- Also, multiplying an identity matrix by a scalar:

```
k I = the scalar multiplied to each entry of I
```

- Therefore, for A = -I:

```
A^2 = (-I) (-I)
```

```
= (-1) I (-1) I
```

```
= (-1) (-1) I I
```

- Since scalar multiplication is associative and commutative, this simplifies to:

```
= (1) I I
```

```
= I I
```

- And, as we know, I I = I.

Step 3: Simplify A^2

- From the above reasoning:

```
A^2 = I
```

- This result holds for any size of the identity matrix, assuming A is a scalar multiple of I.

Step 4: Multiply A^2 by B

- Now, multiply the result by scalar B = 3:

```
A^2 B = I 3
```

- Since scalar multiplication distributes over matrices:

```
I 3 = 3 I
```

- For a 2x2 identity matrix:

```
3 I = | 3 0 |
| 0 3 |
```

Final Result:

$A^2 B = 3 I$

Expressed explicitly, the fully simplified form of the expression is:

```
| 3 0 |
| 0 3 |
```

which is a scalar multiple of the identity matrix.

Deep Dive: Why Does A^2 Equal I?

The crux of this problem hinges on understanding why $A^2 = I$ when $A = -I$. Let’s explore this in more detail, as it’s a critical insight in matrix algebra.

Properties of the Identity Matrix

- The identity matrix I is the multiplicative identity in matrix algebra:

```
I A = A I = A
```

- For any scalar k, the matrix k I satisfies:

```
```

```
(k I)^2 = k^2 I
```

Calculating (A)^2 when A = -I

- Since A = -I, then:

```
A^2 = (-I) (-I)
```

- Using scalar multiplication properties:

```
= (-1) I (-1) I
= (-1) (-1) I I
= 1 I I
= I
```

- We used the fact that I I = I and scalar multiplication:

```
(-1) (-1) = 1
```

This confirms that A^2 = I for A = -I, regardless of the size of the identity matrix, as long as the algebraic rules are followed.

Broader Implications and Applications

Understanding this specific calculation isn’t just a theoretical exercise—it has practical implications across various fields such as computer graphics, quantum mechanics, and systems engineering.

In Linear Algebra

- Recognizing that (-I)^2 = I helps simplify complex matrix expressions.
- It illustrates how scalar multiples of identity matrices behave under powers, which is essential in solving matrix equations and systems.

In Physics

- In quantum mechanics, operators often involve identity matrices scaled by constants.
- Knowing how these operators behave under powers simplifies calculations involving time evolution and state transformations.

In Computer Graphics

- Transformation matrices often involve scaling, rotation, and reflection matrices, many of which are scalar multiples of identity matrices.
- Efficient calculations depend on understanding these properties to optimize graphics rendering.

Summary and Final Thoughts

The problem posed—If A = - I and B = 3, then find the value of A^2 B in fully simplified form—serves as an excellent example of fundamental matrix algebra principles. The key steps involved recognizing the properties of the identity matrix and scalar multiplication, which led to the straightforward conclusion that:

```
A^2 B = 3 I
```

This result is not only mathematically elegant but also highlights the importance of understanding matrix properties for simplifying complex expressions. It demonstrates how combining basic principles—like the behavior of identity matrices and scalar multiplication—can unlock solutions to seemingly intricate problems.

In essence, this problem underscores a broader lesson: grasping foundational properties in algebra can significantly streamline problem-solving processes across mathematics and its applied disciplines. Whether dealing with matrices in advanced physics or transformations in computer graphics, these principles remain universally applicable and invaluable for both students and professionals alike.

In conclusion, the fully simplified form of the expression A^2 B when A = -I and B = 3 is:

3 I, or explicitly,

```
| 3 0 |
| 0 3 |
```

This concise result encapsulates the beauty and power of matrix algebra, reinforcing the importance of understanding fundamental properties to simplify and solve complex mathematical expressions efficiently.

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