If F(1) = 10 And 2 F (x) 5 For All X, What Is The Smallest Possible Value Of F(4)?

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Understanding the problem involving the function F(x) requires careful analysis of the given conditions: F(1) = 10 and the inequality 2 F(x) 5 for all x. The goal is to determine the smallest possible value of F(4) consistent with these constraints. This exploration involves examining the nature of the function, interpreting the inequalities, and applying mathematical principles to find the minimal value.

Interpreting the Problem Statement

The Given Conditions

The problem provides two key pieces of information:

- **F(1) = 10**: The value of the function at x = 1 is exactly 10.
- **2 F(x) 5 for all x**: An inequality involving F(x) which, based on standard mathematical notation, likely means either:
  - $2 F(x) \geq 5$, or
  - $2 F(x) \leq 5$.

Since the inequality symbol is missing in the original statement, it's crucial to interpret it in context. Typically, in such problems, the inequality might be:

- $2 F(x) \geq 5$, which implies $F(x) \geq 2.5$ for all x, or
- $2 F(x) \leq 5$, which implies $F(x) \leq 2.5$ for all x.

Given the phrase "2 F(x) 5 for all x," it's reasonable to assume the intended inequality is:

$2F(x) \geq 5$ for all x.

This assumption aligns with common problem structures and the logical context of finding the minimal value of $F(4)$.

Understanding the Constraints and Their Implications

Analyzing the Inequality $2F(x) \geq 5$

Assuming the inequality is:

$2F(x) \geq 5$ for all x,

then, by dividing both sides by 2:

$F(x) \geq 2.5$ for all x.

This means the function $F(x)$ is bounded below by 2.5 everywhere, including at $x=1$ and $x=4$.

Implication of $F(1) = 10$

Since $F(1) = 10$, which is greater than 2.5, the function satisfies the inequality at $x=1$.

The main challenge is to determine the smallest possible value of $F(4)$, given that the function must satisfy:

- $F(1) = 10$,
- $F(x) \geq 2.5$ for all x.

Finding the Smallest Possible Value of F(4)

Key Observations

- The function must be at least 2.5 everywhere.

- At x=1, F(1) = 10, which is well above the lower bound.
- The question is: Can F(4) be less than 10, and if so, what is the minimal value it can take under the constraints?

Possible Types of Functions

The problem does not specify the continuity, differentiability, or linearity of F(x). To find the minimal possible F(4), we consider the broadest class of functions that satisfy the conditions, including:

- Discontinuous functions,
- Piecewise functions,
- Continuous functions,
- Monotonic functions.

In many optimization problems involving inequalities, the minimal or maximal value of a function at a specific point, given constraints, can be achieved by functions that "just meet" the boundary conditions.

Constructing an Extremal Function

To minimize F(4), consider constructing a function that:

- Meets F(1) = 10,
- Satisfies F(x) ≥ 2.5 for all x,
- Is as low as possible at x=4.

One approach is to consider functions that decrease from x=1 to x=4 while maintaining the lower bound of 2.5.

Mathematical Approach to the Problem

Linear Functions as Candidates

Assuming F(x) is linear for simplicity, i.e.,

F(x) = m x + c,

we can attempt to find the minimal F(4).

Given:

- $F(1) = m(1) + c = 10$,
- $F(x) \geq 2.5$ for all x, which implies the line must stay above or on the line $y=2.5$.

To minimize $F(4)$:

- $F(4) = 4m + c$,
- with $c = 10 - m$ (from $F(1) = 10$).

The minimal $F(4)$ occurs when the linear function just touches the lower bound at the point where it can be minimized, but still satisfies the constraints.

However, a linear function that is decreasing from 10 at $x=1$ to a lower value at $x=4$, but never dips below 2.5, is feasible if the slope m is negative but satisfies the lower bound constraint.

Condition:
For all x in [1, 4], $F(x) \geq 2.5$.

Since the function is linear:

$F(x) = m x + c$,

and at $x=1$:

$F(1) = m + c = 10$,

at $x=4$:

$F(4) = 4m + c$.

The minimal $F(4)$:

- occurs when the function decreases as rapidly as possible without crossing below 2.5.

The lowest point at $x=4$ must satisfy:

$F(4) \geq 2.5$,

which leads to:

$4m + c \geq 2.5$.

Substitute $c = 10 - m$:

$4m + (10 - m) \geq 2.5$,

$3m + 10 \geq 2.5$,

$3m \geq -7.5$,

$m \geq -2.5$.

To minimize $F(4)$:

$F(4) = 4m + c = 4m + (10 - m) = 3m + 10$.

Since $m \geq -2.5$, the minimal $F(4)$:

$F(4) = 3(-2.5) + 10 = -7.5 + 10 = 2.5$.

This matches the lower bound exactly, implying the minimal $F(4)$ is 2.5.

Conclusion: The Smallest Possible Value of F(4)

Based on the linear function analysis, the minimal value of $F(4)$ under the constraints is 2.5.

Key points:

- The function can be chosen as a linear, decreasing function from (1, 10) to (4, 2.5),
- It satisfies the boundary condition at $x=1$,
- It maintains the inequality $F(x) \geq 2.5$ for all x in $[1, 4]$,
- No other function type (e.g., nonlinear) can produce a lower value at $x=4$ without violating the constraints.

Additional Considerations and Generalizations

Beyond Linear Functions

If the function F(x) is not restricted to be linear, could F(4) be smaller than 2.5?

- Since the minimal value F(4) can take without violating the constraints is 2.5, any nonlinear function that dips below 2.5 at x=4 would violate the inequality.
- Therefore, the minimal value remains 2.5 regardless of the function's shape, provided it adheres to the constraints.

Impact of Function Continuity or Differentiability

If the problem states that F(x) must be continuous or differentiable, the conclusion still holds:

- The "best" (minimal at x=4) function is a straight line decreasing from F(1)=10 to F(4)=2.5,
- No smooth or continuous function can have F(4) below 2.5 without violating the inequality.

Summary and Final Answer

In summary, the problem asks: If F(1) = 10 and 2 F(x) ≥ 5 for all x, what is the smallest possible value of F(4)?

The analysis shows:

- The lower bound for F(x) is 2.5 everywhere.
- The minimal value of F(4) consistent with all conditions is 2.5.
- This minimum can be achieved by a linear function decreasing from 10 at x=1 to 2.5 at x=4.

Therefore, the smallest possible value of F(4) is:

2.5

Final Thoughts

This problem exemplifies fundamental concepts in inequalities, function analysis, and optimization. It highlights the importance of understanding constraints and their implications for the extremal values of functions. Whether through linear modeling or more complex functions, the key is to identify the boundary conditions and construct functions that "just meet" the constraints to find minimal or maximal values.

If you're preparing for exams, practicing similar problems involving inequalities and function bounds will strengthen your analytical skills and improve your problem-solving confidence.

Frequently Asked Questions

Given that F(1) = 10 and 2F(x) ≥ 5 for all x, what is the smallest possible value of F(4)?

The smallest possible value of F(4) is 2.5.

How does the inequality 2F(x) ≥ 5 influence the minimum of F(4)?

It implies that F(x) ≥ 2.5 for all x, including x=4, setting a lower bound of 2.5 for F(4).

Is it possible for F(4) to be less than 2.5 given the conditions?

No, since 2F(4) ≥ 5, F(4) must be at least 2.5.

What role does the value F(1) = 10 play in determining F(4)?

F(1) = 10 constrains the function at x=1, but does not restrict the minimum value at x=4 beyond the inequality 2F(x) ≥ 5.

Can F(x) be a constant function satisfying the given conditions?

No, because if F(x) were constant, then 2F(x) ≥ 5 implies F(x) ≥ 2.5, but with F(1)=10, the constant function would be F(x)=10, which is consistent. So yes, a constant function at F(x)=10 satisfies all conditions.

Is there a specific type of function that achieves the smallest F(4) value?

Yes, a function that satisfies F(1)=10 and F(x) ≥ 2.5 for all x, such as a constant function F(x)=10, meets the conditions, but the minimal F(4) is 2.5 if F(x) can vary.

Could F(4) be exactly 2.5 while F(1)=10?

Yes, F(4) can be exactly 2.5, as long as the function satisfies the conditions elsewhere.

What is the key takeaway about the smallest possible value of F(4)?

The key takeaway is that F(4) cannot be less than 2.5 due to the inequality, so the smallest possible value is 2.5.

Additional Resources

Understanding the Problem: If $F(1) = 10$ And $2 F(x) \leq 5$ For All x, What Is The Smallest Possible Value Of F(4)?

In the realm of mathematical functions, especially those involving inequalities and constraints, problems often challenge us to find the minimal or maximal values under given conditions. One such intriguing problem involves the function $F(x)$ with specific initial conditions and inequalities: If $F(1) = 10$ and $2F(x) \leq 5$ for all x, then determining the smallest possible value of F(4) becomes a fascinating exercise in understanding the interplay between function values and inequalities. This type of problem is common in calculus and algebra, often requiring careful analysis of the constraints and logical deductions to arrive at an optimal solution.

The Core of the Problem

Let's restate the problem clearly:

- The function $F(x)$ satisfies $F(1) = 10$.
- For all x, the inequality $2F(x) \leq 5$ holds.
- The question: What is the smallest possible value of F(4)?

This problem involves understanding what the constraints imply about $F(x)$ at different points, especially at $x = 4$.

Breaking Down the Constraints

1. The Fixed Value: $F(1) = 10$

This is straightforward. At $x = 1$, the function's value is explicitly given:

```plaintext
F(1) = 10
```

This fixed point will serve as an anchor for our analysis.

2. The Inequality: $2F(x) \leq 5$ for all x

This is a universal inequality applying to every x in the domain (which we assume includes at least the points of interest: 1 and 4). Let's analyze what this inequality means:

```plaintext
2F(x) ≤ 5
```
Dividing both sides by 2:

```plaintext
F(x) ≤ 5/2
```

This inequality places an upper bound on $F(x)$ for all x:

```plaintext
F(x) ≤ 2.5
```

Key Observation: Contradiction with $F(1) = 10$

Here lies an apparent contradiction:

- We know $F(1) = 10$.
- But from the inequality, $F(1) \leq 2.5$.

Since $F(1) = 10$ explicitly, but $F(1) \leq 2.5$ must hold for all x, this suggests a problem in the initial interpretation.

Conclusion:

The only way to reconcile $F(1) = 10$ with $F(x) \leq 2.5$ for all x is if $x \neq 1$. But the problem states for all x, so the constraints are incompatible unless we interpret the inequality differently.

Clarifying the Inequality

The phrase "2 F(x) ≤ 5 for all x" could be interpreted as:

- Either the inequality holds for all x in the domain excluding the point x = 1, where F(1) = 10.
- Or the inequality is only intended for x ≠ 1, and the point x=1 is a special case.

Often, in such problems, the constraints are meant to apply except at certain points, or the inequality is not strict at the point where the function is explicitly defined.

Suppose the problem is:

- F(1) = 10
- For all x ≠ 1, 2F(x) ≤ 5

In that case, at x = 1, F(1) = 10 is an exception, and the inequality constrains F(x) elsewhere.

Alternatively, if the problem intends the inequality to hold for all x, then the explicit value F(1) = 10 contradicts the inequality, which would make the problem impossible.

Reconciling the Conditions

Given the standard interpretation, the most logical assumption is:

- The inequality 2 F(x) ≤ 5 applies for all x ≠ 1.
- At x = 1, F(1) = 10 is given explicitly, possibly as an exception.

In this scenario, the key constraints are:

- F(1) = 10
- For x ≠ 1, F(x) ≤ 2.5

Implications for F(4)

Since x = 4 is not equal to 1, the inequality applies:

```plaintext
```

$F(4) \leq 2.5$
```

Our goal is to find the smallest possible value of F(4), given these constraints.

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Is There a Lower Bound?

The problem asks for the smallest possible value of F(4). Since the inequality constrains $F(4) \leq 2.5$, and nothing suggests F(4) cannot be arbitrarily negative, unless additional constraints are given.

In typical mathematical problems, unless specified, functions are often assumed to be real-valued and unbounded below unless constrained.

But, in many cases, the question about the smallest possible value of a function at a point, given an upper bound, implies no lower bound other than the constraints of the function itself.

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Could F(4) be arbitrarily negative?

- If no constraints prevent F(4) from becoming very negative, then F(4) can tend toward $-\infty$.
- But the problem most likely expects that the minimal value is constrained by the inequality $F(4) \leq 2.5$, and no other constraints are given.

Thus, the minimal value of F(4) could, in theory, be $-\infty$, unless the problem specifies $F(x) \geq$ some value.

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Common Assumptions and Additional Constraints

In typical function analysis, the following are common:

- F(x) is bounded below or above.
- The function is continuous or satisfies certain properties.
- The problem emphasizes "smallest possible value", implying an infimum.

Given the current problem statement, the key is:

- $F(4) \leq 2.5$ (from inequality).
- No explicit lower bound is provided for F(4).
```

Final Conclusion: The Smallest Possible Value of $F(4)$

If the only constraints are:

1. $F(1) = 10$
2. $2F(x) \leq 5$ for all $x \neq 1$

then:

- $F(4) \leq 2.5$

and since no lower bounds are specified, $F(4)$ can be made arbitrarily negative.

Therefore, the smallest possible value of $F(4)$ is:

Negative infinity ($-\infty$)

In the context of real-valued functions, this means:

> $F(4)$ can be made arbitrarily small, approaching negative infinity.

Additional Considerations and Clarifications

If the problem intended to suggest that $F(x)$ must be bounded below, or the domain imposes other constraints, then the answer could differ.

Suppose the problem is:

- $F(1) = 10$
- $2F(x) \leq 5$ for all x

and the function $F(x)$ is continuous or bounded, then:

- Since $F(1) = 10$, which exceeds 2.5, the inequality $2F(1) \leq 5$ does not hold at $x=1$ unless the inequality is not applied at $x=1$.

Hence, the only consistent interpretation is:

- The inequality applies for all $x \neq 1$, with $F(1) = 10$ as a fixed point outside the inequality.

Summary and Final Answer

- Given the constraints: $F(1) = 10$ and $F(x) \leq 2.5$ for all $x \neq 1$,
- The smallest possible value of $F(4)$ is:

Negative infinity ($-\infty$).

Because there's no lower bound specified, and $F(4)$ can be made arbitrarily negative, respecting the inequality $F(4) \leq 2.5$.

Key Takeaways

- When analyzing functions with inequalities, always clarify whether the constraints apply at all points or except certain points.
- The fixed point condition $F(1) = 10$ can conflict with bounds like $F(x) \leq 2.5$ unless the inequality is not valid at $x=1$.
- If no explicit lower bounds are given, the infimum (greatest lower bound) can be $-\infty$.

Final Note

In most typical problems, additional conditions such as continuity, boundedness, or specific function classes would restrict the possible values of $F(4)$ further. Without such restrictions, the theoretical minimal value is $-\infty$. If the problem intends for $F(4)$ to be bounded, then more information would be necessary to determine a finite minimal value

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