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Understanding the relationship between prime numbers and their least common multiple (LCM) is a fundamental concept in number theory. When given that the LCM of two prime numbers A and B (with A > B) is 783, it prompts us to explore the possible values of these primes and subsequently compute the expression 2ab - 3a. This article provides a detailed step-by-step analysis to determine the values of A and B, understand their constraints, and calculate the desired expression, all while emphasizing the importance of prime numbers and LCM in mathematical problem-solving.

Understanding Prime Numbers and LCM

Before delving into the problem specifics, it’s essential to review some fundamental concepts:

What Are Prime Numbers?

- Prime numbers are natural numbers greater than 1 that have no positive divisors other than 1 and themselves.
- Examples include 2, 3, 5, 7, 11, 13, 17, etc.
- Prime numbers are building blocks of integers because every integer greater than 1 can be expressed as a product of primes.

What Is the Least Common Multiple (LCM)?

- The LCM of two numbers is the smallest positive integer that is divisible by both numbers.
- The LCM is useful for finding common multiples and solving problems involving synchronization of periodic events.

Properties of Prime Numbers and LCM

- The LCM of two prime numbers is either their product if they are distinct or the number itself if they are the same.
- Since prime numbers have no common factors other than 1, the LCM of two different

primes p and q is p × q.

Analyzing the Given Data: LCM of Two Primes A and B is 783

Given:
- A and B are prime numbers
- A > B
- LCM(A, B) = 783

Key observations:
- Since A and B are prime, their LCM depends on whether they are equal or different.
- If A ≠ B, then LCM(A, B) = A × B (because they are both prime and distinct).
- If A = B, then LCM(A, B) = A (or B).

Considering the given LCM of 783:
- If A ≠ B, then A × B = 783.
- If A = B, then A = B = 783, which cannot be true because 783 is not prime.

Therefore:
- A ≠ B
- A × B = 783

Factoring 783 to Find Prime Numbers A and B

To find A and B, we need to factor 783 into its prime factors.

Prime Factorization of 783

Let's perform prime factorization of 783:

1. Check divisibility by small primes:

- Divisible by 3?
Sum of digits: 7 + 8 + 3 = 18, which is divisible by 3.
Thus, 783 is divisible by 3.

2. Divide 783 by 3:

783 ÷ 3 = 261

3. Factor 261:

Sum of digits: $2 + 6 + 1 = 9$, divisible by 3.
So, 261 is divisible by 3.

4. Divide 261 by 3:

$261 \div 3 = 87$

5. Factor 87:

Sum of digits: $8 + 7 = 15$, divisible by 3.
$87 \div 3 = 29$

6. 29 is a prime number.

Prime factorization of 783:

$783 = 3 \times 3 \times 3 \times 29 = 3^3 \times 29$

Now, the prime factors of 783 are 3 and 29.

Identifying the Prime Numbers A and B

From the prime factorization:

- Factors are 3 and 29.
- Since A and B are prime, and A > B, the possible choices are:

Option 1: A = 29, B = 3

Option 2: A = 3, B = 29 (but since A > B, this is invalid)

Thus, the only valid option is:

- A = 29
- B = 3

Calculating the Expression: 2ab - 3a

Given A = 29 and B = 3, the expression is:

$$ 2ab - 3a $$

Let's substitute the values:

$$ a = 29 $$
$$ b = 3 $$

Compute step-by-step:

1. Compute $2ab$:

$$ 2 \times 29 \times 3 = 2 \times 87 = 174 $$

2. Compute $3a$:

$$ 3 \times 29 = 87 $$

3. Calculate $2ab - 3a$:

$$ 174 - 87 = 87 $$

Therefore, the value of $2ab - 3a$ is 87.

Summary and Final Answer

Step-by-step summary:

- Recognized that the LCM of two primes, A and B, is the product $A \times B$ (since they are distinct primes).
- Factorized 783 to find its prime factors: $3^3 \times 29$.
- Determined the primes involved: 3 and 29.
- Assigned the larger prime to A and the smaller to B, respecting the condition $A > B$.
- Calculated the expression $2ab - 3a$ with the identified values.

Final Answer:

$$
\boxed{87}
$$

Additional Insights and Related Concepts

Importance of Prime Factorization in Number Theory
Prime factorization is a foundational technique that simplifies the process of solving various mathematical problems, including finding LCMs and GCDs (Greatest Common Divisors). It provides a clear view of the building blocks of numbers.

Applications in Real-World Problems
- Synchronizing periodic events
- Cryptography and security algorithms
- Simplifying fractions and ratios
- Solving Diophantine equations

Practice Questions
1. If the LCM of two primes is 945, find the primes.
2. For two primes A and B with A > B, if their LCM is 1001, determine A and B.
3. Compute $3ab - 2a$ given the primes involved in similar problems.

Conclusion

Understanding the relationship between prime numbers and their least common multiple is crucial in solving many mathematical problems. In this specific case, the prime factorization of 783 led us to identify the prime numbers 3 and 29, allowing us to compute the expression $2ab - 3a$ as 87. This problem exemplifies how prime factorization and properties of primes facilitate efficient problem-solving in number theory. Whether for academic exercises or practical applications, mastering these concepts enhances mathematical reasoning and analytical skills.

Frequently Asked Questions

If the LCM of two prime numbers A and B (A > B) is 783, what are the possible values of A and B?

The prime numbers are 29 and 3, since $783 = 3 \times 3 \times 3 \times 29$, but as they are primes, the pair must be 29 and 3.

Given two prime numbers A and B with A > B and their LCM is 783, how do we determine the value of 2ab - 3a?

First, identify the primes A and B (from the prime factorization or constraints). Then, substitute the values into the expression 2ab - 3a, where A = a and B = b, and compute accordingly.

What is the value of 2ab - 3a if A=29 and B=3, given that their LCM is 783?

Substitute a=29 and b=3 into the expression: $2\times29\times3 - 3\times29 = 2\times87 - 87 = 174 - 87 = 87$.

Are there any other possible pairs of prime numbers A and B with A > B and LCM 783?

No, because 783 factors as $3 \times 3 \times 3 \times 29$, and the only prime factors are 3 and 29, so the only prime pair with LCM 783 and A > B is (29, 3).

How does the prime factorization of 783 help in solving for A and B in this problem?

The prime factorization ($3^3 \times 29$) indicates the possible prime factors involved. Since the numbers are prime, the pair must be 3 and 29, which helps determine their values directly and compute the required expression.

What is the final answer for 2ab - 3a when A=29 and B=3?

The value of 2ab - 3a is 87.

Additional Resources

If LCM Of Two Prime Numbers A And B (a > b) Is 783, Then The Value Of 2ab - 3a Is? This problem presents an intriguing scenario involving prime numbers, least common multiples (LCM), and algebraic expressions. It combines fundamental concepts from number theory with algebraic manipulation, making it a rich topic for mathematical exploration. Understanding the relationship between prime numbers and their least common multiple is essential to solving this problem efficiently, and it offers a good opportunity to delve into prime factorization, properties of prime numbers, and algebraic computation.

Understanding the Problem Statement

The problem states that the LCM of two prime numbers A and B, where A > B, is 783. The goal is to find the value of the expression 2ab - 3a, where a and b are these prime numbers.

Key points to consider:
- Both A and B are prime numbers.

- A > B.
- LCM(A, B) = 783.
- Find 2ab - 3a.

This problem involves multiple concepts, including prime numbers, LCM, and algebraic expressions, which need to be analyzed step by step.

Fundamental Concepts Involved

Prime Numbers

Prime numbers are natural numbers greater than 1 that have no divisors other than 1 and themselves. Examples include 2, 3, 5, 7, 11, etc. They are the building blocks of number theory, especially in factorization.

Least Common Multiple (LCM)

The LCM of two numbers is the smallest number that is a multiple of both. For prime numbers, the LCM depends on whether the primes are equal or different:
- If the primes are different, their LCM is typically their product, because they share no common factors other than 1.
- If the primes are the same, the LCM is just that prime.

Properties of Prime Numbers and LCM

- For two distinct primes p and q, LCM(p, q) = p × q.
- For the same prime p, LCM(p, p) = p.

The problem's key is to determine the primes A and B such that their LCM is 783.

Breaking Down the Problem

Step 1: Factorize 783

To understand which primes are involved, we need to factor 783 into its prime factors.

Prime factorization of 783:
- 783 ÷ 3 = 261 (since 7 + 8 + 3 = 18, divisible by 3)
- 261 ÷ 3 = 87

- 87 ÷ 3 = 29

Thus, 783 = 3 × 3 × 3 × 29 = 3^3 × 29

So, 783 = 3^3 × 29.

Since the prime number A and B are involved in this factorization, and the LCM is 783, the primes involved must be among the prime factors of 783, i.e., 3 and 29.

Step 2: Determine the Prime Numbers A and B

Given that A and B are prime numbers and that their LCM is 783, which factors into 3^3 × 29, the possible prime candidates are 3 and 29.

Because the LCM of two primes is either their product (if they are distinct) or the prime itself (if they are equal), and given that the LCM is 783, the only consistent possibility is:

- A = 29 (since 29 is prime)
- B = 3 (since 3 is prime)

This matches the prime factors and satisfies A > B, as 29 > 3.

Note: The presence of 3^3 in the factorization indicates that the prime 3 appears with multiplicity in the LCM, which is consistent with 3 being one of the primes involved.

Calculating the Desired Expression

Having identified A = 29 and B = 3, the next step is to compute the value of 2ab - 3a.

Step 3: Plugging in the Values

- a = 29
- b = 3

Calculate:

2ab - 3a

= 2 × 29 × 3 - 3 × 29

= (2 × 29 × 3) - (3 × 29)

= (2 × 87) - 87

= 174 - 87

= 87

Result: The value of 2ab - 3a is 87.

Summary and Final Remarks

This problem elegantly combines prime number properties, prime factorization, and algebraic substitution to arrive at a solution. The key steps involved recognizing the prime factors of 783, understanding the implications of the LCM for prime numbers, and then performing straightforward algebraic calculations.

Features and Insights from the Problem

- Clear relationship between prime factors and LCM: When dealing with prime numbers, the LCM is often directly related to their product unless they are equal.
- Prime factorization simplifies problem-solving: Breaking down 783 into prime factors immediately indicates the potential prime candidates.
- Algebraic substitution is straightforward after identifying primes: Once the primes are known, the calculation becomes a simple matter of substitution.

Pros and Cons of the Approach

Pros:
- Simplicity: The problem reduces to prime factorization and substitution.
- Logical clarity: Step-by-step reasoning minimizes errors.
- Educational value: Reinforces understanding of prime numbers, LCM, and algebra.

Cons:
- Potential for oversight: If one does not consider prime factorization thoroughly, might miss the correct primes.
- Limited scope: Problems with more complex LCMs or non-prime factors require more advanced methods.

Conclusion

The problem of finding the value of 2ab - 3a given the LCM of two prime numbers is an excellent example of applying basic number theory concepts to problem-solving. By factorizing 783 into its prime components, recognizing the primes involved, and substituting into the algebraic expression, the solution is obtained efficiently. The final answer, 87, showcases how foundational concepts in mathematics can lead to elegant solutions.

Understanding such problems enhances mathematical reasoning and prepares learners for more complex topics involving number theory, algebra, and their intersections. The approach outlined here emphasizes methodical analysis, clear reasoning, and practical application of fundamental principles, making it a valuable exercise for students and enthusiasts alike.

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