If Line Joining (1,2) And (7,6) Is Perpendicular To Line Joining (3,4) And (11,x)

If Line Joining (1,2) And (7,6) Is Perpendicular To Line Joining (3,4) And (11,x)

Understanding the geometric relationships between lines in coordinate geometry is fundamental in mathematics, especially in the study of slopes, angles, and their properties. A common problem involves determining the conditions under which two lines are perpendicular, given their endpoints. In this article, we will explore the scenario:

If the line joining the points (1, 2) and (7, 6) is perpendicular to the line joining the points (3, 4) and (11, x), what is the value of x?

This question invites us to analyze the slopes of the two lines, set the perpendicularity condition, and solve for the unknown coordinate parameter. Let's delve into the concepts step-by-step to understand how to approach such problems.

Understanding Coordinates and Slope in Geometry

Before solving the problem, it's essential to review some key concepts:

1. Points in the Coordinate Plane

- A point in the plane is represented by an ordered pair (x, y).
- The points involved here are:
- First line: (1, 2) and (7, 6)
- Second line: (3, 4) and (11, x)

2. Slope of a Line

- The slope (m) measures the steepness of a line.
- For two points (x_1, y_1) and (x_2, y_2), the slope is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

- The slope indicates the direction of the line:
- Positive slope: increasing from left to right.
- Negative slope: decreasing from left to right.
- Zero slope: horizontal line.

- Undefined slope: vertical line.

Calculating the Slopes of the Given Lines

Let's compute the slopes of both lines:

1. Slope of Line Joining (1, 2) and (7, 6)

$$
m_1 = \frac{6 - 2}{7 - 1} = \frac{4}{6} = \frac{2}{3}
$$

2. Slope of Line Joining (3, 4) and (11, x)

$$
m_2 = \frac{x - 4}{11 - 3} = \frac{x - 4}{8}
$$

Condition for Perpendicular Lines

In coordinate geometry, two lines are perpendicular if and only if the product of their slopes is -1:

$$
m_1 \times m_2 = -1
$$

Applying this to our slopes:

$$
\frac{2}{3} \times \frac{x - 4}{8} = -1
$$

Now, let's solve for x.

Solving for the Unknown Coordinate x

Starting with the perpendicularity condition:

$$
\frac{2}{3} \times \frac{x - 4}{8} = -1
$$

Multiply both sides by 8 to clear the denominator:

$$
\frac{2}{3} \times (x - 4) = -8
$$

Multiply both sides by 3 to clear the denominator:

$$
2 \times (x - 4) = -24
$$

Expand the left side:

$$
2x - 8 = -24
$$

Add 8 to both sides:

$$
2x = -16
$$

Divide both sides by 2:

$$
x = -8
$$

Conclusion: The value of x for which the lines are perpendicular is -8.

Visualizing the Problem

Visual aids can enhance understanding:

Graphical Representation

- Plot the points:
- (1, 2), (7, 6)
- (3, 4), (11, -8)
- Draw both lines to see their slopes and verify perpendicularity visually.

Understanding the Slopes

- The first line has a positive slope ($2/3$).
- The second line, with $x = -8$, will have a slope:

$$
m_2 = \frac{-8 - 4}{8} = \frac{-12}{8} = -\frac{3}{2}
$$

- Multiplying:

$$
\frac{2}{3} \times -\frac{3}{2} = -1
$$

which confirms the perpendicularity condition.

Applications of Perpendicular Lines in Real Life and Mathematics

Understanding perpendicular lines is crucial in various fields:

1. Engineering and Architecture

- Ensuring structural elements are perpendicular for stability and design.

2. Computer Graphics and Design

- Creating accurate orthogonal projections and layouts.

3. Navigation and Robotics

- Calculating trajectories and movement paths.

4. Mathematical Proofs and Problem Solving

- Establishing relationships between geometric figures and solving for unknowns.

Additional Related Concepts and Problems

To deepen your understanding, consider exploring related topics:

1. Finding the Equation of a Line

- Given two points, derive the line's equation using point-slope form.

2. Distance Between Two Points

- Use the distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

3. Midpoint of a Line Segment

- Find the midpoint between two points:

$$
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
$$

4. Parallel Lines and Slopes

- Lines are parallel if their slopes are equal.

Common Mistakes to Avoid

- Incorrect slope calculation: Always subtract the y-coordinates from top to bottom and x-coordinates from left to right.
- Forgetting the negative reciprocal condition: Remember, perpendicular lines have slopes that multiply to -1.
- Ignoring special cases: Vertical and horizontal lines have undefined and zero slopes, respectively, which need special handling.

Summary and Key Takeaways

- The slope of the line joining (1, 2) and (7, 6) is $\frac{2}{3}$.

- The slope of the line joining (3, 4) and (11, x) is $\frac{x - 4}{8}$.
- For the lines to be perpendicular, their slopes satisfy:

$$
\frac{2}{3} \times \frac{x - 4}{8} = -1
$$

- Solving yields $x = -8$.

Understanding how to calculate slopes and apply the perpendicularity condition is essential in coordinate geometry, enabling solving a wide range of problems involving lines, angles, and geometric relationships.

Practice Problems for Further Learning

1. Find the value of x if the points (2, 3) and (x, 7) form a line perpendicular to the line joining (4, 5) and (6, 9).
2. Determine whether the points (1, 2) and (4, 5) form a line perpendicular to the line joining (3, 4) and (x, 10). Find x.
3. Given points $A(0, 0)$ and $B(4, y)$, and points $C(1, 2)$ and $D(5, 6)$, find y such that lines AB and CD are perpendicular.

By mastering these concepts and problem-solving approaches, you can confidently analyze and determine relationships between lines in coordinate geometry, enhancing your mathematical reasoning and application skills.

Frequently Asked Questions

How can we determine the value of x when two lines are perpendicular?

Calculate the slopes of both lines and set their product to -1, then solve for x.

What is the slope of the line passing through points (1,2) and (7,6)?

The slope is (6-2)/(7-1) = 4/6 = 2/3.

What is the slope of the line passing through points (3,4) and

(11,x)?

The slope is (x - 4) / (11 - 3) = (x - 4) / 8.

How do you find the perpendicularity condition between two lines?

The slopes of the two lines must satisfy m1 m2 = -1.

What is the equation to find x given the slopes are perpendicular?

Set (2/3) ((x - 4)/8) = -1 and solve for x.

What is the value of x when solving the perpendicularity condition?

x = -14.

Why is it necessary to find the slopes when dealing with perpendicular lines?

Because the slope relationship (product equals -1) determines perpendicularity.

Can the points be used directly to verify perpendicularity without calculating slopes?

No, because the slopes are needed to confirm the lines are perpendicular.

What is the significance of the slope formula in coordinate geometry?

It helps determine the steepness and orientation of a line, essential for analyzing perpendicularity.

Additional Resources

If Line Joining (1,2) And (7,6) Is Perpendicular To Line Joining (3,4) And (11,x) – An In-Depth Geometric Analysis

Understanding geometric relationships, especially those involving slopes and perpendicular lines, is fundamental in coordinate geometry. The problem statement, "If line joining (1,2) and (7,6) is perpendicular to line joining (3,4) and (11,x)," presents a classic scenario where algebra and geometry intersect. In this guide, we'll explore how to analyze, derive, and solve for the unknown coordinate x by applying principles of slope calculation, perpendicularity conditions, and algebraic manipulation.

Introduction to the Problem

At its core, this problem involves two line segments:

- Line Segment 1: Connecting points $(1,2)$ and $(7,6)$
- Line Segment 2: Connecting points $(3,4)$ and $(11,x)$

The key question is: "Under what conditions are these two lines perpendicular?"

This question is fundamental for students and professionals working in fields like computational geometry, computer graphics, and engineering design, where understanding the orientation and relationships of lines is crucial.

Step 1: Understanding the Concept of Perpendicular Lines

Perpendicular lines are lines that intersect at a right angle (90 degrees). In coordinate geometry, the critical property to analyze is the slope of each line:

- If the slopes of two lines are negative reciprocals of each other, then the lines are perpendicular.

Mathematically, if m_1 and m_2 are the slopes of two lines, then:

$$
m_1 \times m_2 = -1
$$

This relationship forms the foundation for solving the given problem.

Step 2: Calculating the Slope of the First Line

Given points $(1,2)$ and $(7,6)$:

$$
m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{7 - 1} = \frac{4}{6} = \frac{2}{3}
$$

Thus, the slope of line joining (1,2) and (7,6) is $\frac{2}{3}$.

Step 3: Deriving the Slope of the Second Line

Given points $(3,4)$ and $(11,x)$:

$$
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$$
m_2 = \frac{x - 4}{11 - 3} = \frac{x - 4}{8}
$$

This expression involves the unknown x, which we aim to solve for.

Step 4: Applying the Perpendicularity Condition

Since the lines are perpendicular, their slopes satisfy:

$$
m_1 \times m_2 = -1
$$

Substituting the known and unknown values:

$$
\frac{2}{3} \times \frac{x - 4}{8} = -1
$$

Step 5: Solving for x

Now, solve the equation:

$$
\frac{2}{3} \times \frac{x - 4}{8} = -1
$$

Multiply both sides by 8 to clear the denominator:

$$
\frac{2}{3} \times (x - 4) = -8
$$

Next, multiply both sides by 3 to clear the fraction:

$$
2 \times (x - 4) = -24
$$

Expand the left side:

$$
2x - 8 = -24
$$

Add 8 to both sides:

$$
2x = -16
$$

Finally, divide both sides by 2:

$$
x = -8
$$

Therefore, the value of x that makes the line segments perpendicular is -8.

Summary of the Key Steps

- Calculated the slope of the first line.
- Expressed the slope of the second line with the unknown x.
- Applied the perpendicularity condition $m_1 \times m_2 = -1$.
- Solved the resulting algebraic equation for x.

Additional Insights and Applications

1. Coordinate Geometry in Design and Engineering

Understanding how to determine perpendicular lines is essential in CAD (Computer-Aided Design), robotics, and structural engineering where precise angles and orientations are crucial.

2. Graphical Interpretation

Plotting the points and lines can provide an intuitive understanding of the problem. Visualizing the lines and their intersection at right angles reinforces the algebraic findings.

3. Extensions to Other Geometric Problems

- Finding the equation of perpendicular bisectors.
- Determining distances and midpoints.
- Analyzing slopes of multiple lines for concurrency or parallelism.

Common Pitfalls and Clarifications

- Miscalculating slopes: Always double-check numerator and denominator signs.
- Incorrectly assuming x as positive: The algebra shows that x can be negative; do not assume its sign beforehand.
- Forgetting the negative reciprocal condition: Remember, the critical property for perpendicular lines is that their slopes multiply to -1.

Final Remarks

This problem offers a clear illustration of how algebra and geometry work hand-in-hand to solve for unknowns in coordinate systems. The key takeaway is the importance of understanding the slope relationship for perpendicularity and applying algebraic techniques to find the unknown coordinate x. Whether you're tackling academic exercises or real-world engineering problems, mastering these foundational concepts equips you with a powerful toolset for analyzing spatial relationships.

In conclusion, the line joining (1,2) and (7,6) is perpendicular to the line joining (3,4) and (11, x) when $x = -8$. This solution exemplifies how geometric principles translate into algebraic equations, enabling precise solutions in coordinate geometry.

If Line Joining 1 2 And 7 6 Is Perpendicular To Line Joining 3 4 And 11 X

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