If The Figure Is Rotated 90 Clockwise About The Origin, What Are The Coordinates Of M?

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Understanding how geometric transformations affect coordinate points is fundamental in coordinate geometry. Among these transformations, rotation plays a vital role, especially when analyzing figures on the Cartesian plane. If a figure is rotated 90 degrees clockwise about the origin, the coordinates of its points change in a specific, predictable manner. This article aims to thoroughly explore this transformation and determine the new coordinates of a point labeled M after such a rotation.

Understanding Rotation About the Origin

What Is Rotation in Coordinate Geometry?

In coordinate geometry, rotation involves turning a figure around a fixed point, known as the pivot or the center of rotation. When rotating about the origin (0,0), the transformation rules are straightforward and can be mathematically expressed using coordinate formulas.

Rotation About the Origin: Basic Principles

- The rotation is performed around the origin (0, 0).
- The angle of rotation determines how far and in which direction the figure is turned.
- For a 90-degree clockwise rotation, each point's position changes based on a specific rule.

Mathematical Representation of a 90-Degree

Clockwise Rotation

Coordinate Transformation Formula

If a point P(x, y) is rotated 90 degrees clockwise about the origin, its new position P'(x', y') can be calculated using the following formula:

- P'(x', y') = (y, -x)

This rule means that:

- The x-coordinate of the original point becomes the y-coordinate of the new point.
- The y-coordinate becomes the negative of the original x-coordinate.

Visual Explanation of the Transformation

Imagine the Cartesian plane:

- Before rotation, the point M has coordinates (x, y).
- After a 90-degree clockwise rotation, the point moves to a new position.
- The transformation effectively swaps the coordinates and changes the sign of the original x-coordinate.

This transformation can be visualized on the coordinate plane:

- The point (x, y) moves to (y, -x).
- This is equivalent to rotating the figure around the origin in a clockwise direction.

Applying the Rotation to Find Coordinates of M

Given Coordinates of M

To find the new coordinates of point M after rotation, you need the original coordinates of M, say:

- M(x, y)

Once you have these, you apply the transformation formula.

Step-by-Step Process

1. Identify the original coordinates of M: (x, y).
2. Apply the transformation rule for 90-degree clockwise rotation:
- x' = y
- y' = -x
3. Write the new coordinates as (y, -x).

Example:

Suppose M has coordinates (3, 4).

- After rotation:
- x' = y = 4
- y' = -x = -3
- Therefore, the new coordinates of M are (4, -3).

Practical Applications and Examples

Example 1: Rotating a Specific Point

Let’s consider a specific point to understand the process better.

- Original point M(2, -5)
- Apply the transformation:
- x' = y = -5
- y' = -x = -2
- New coordinates of M: (-5, -2)

Example 2: Geometric Figures and M's Position

Suppose you are given a triangle with vertices:

- A(1, 2)
- B(4, 3)
- M(2, 5)

If the figure is rotated 90 degrees clockwise about the origin:

- Coordinates of M become:
- x' = y = 5
- y' = -x = -2

- M's new position: (5, -2)

Understanding these transformations allows for precise plotting and analysis of figures after rotation.

Visualizing Rotation: Graphical Interpretation

Using Graphs to Understand Rotation

Graphical tools and graph paper can help visualize the rotation process:

- Plot the original point M(x, y).
- Draw the axes and mark the point.
- Rotate the point around the origin, following the transformation rules.
- Confirm that the new point aligns with the calculated coordinates (y, -x).

Why Visualization Matters

Visualizing the rotation helps students and professionals:

- Confirm the correctness of calculations.
- Better understand the geometric transformation.
- Apply similar principles to more complex figures or multiple rotations.

Common Mistakes and Tips

Misapplication of the Transformation Rule

- Remember that for a 90-degree clockwise rotation, the rule is (x, y) → (y, -x).
- Confusing clockwise and counterclockwise rotations can lead to incorrect calculations.

Handling Coordinates with Negative Values

- When dealing with negative coordinates, carefully apply the rule to avoid sign errors.

Double-Check Calculations

- Always verify the original coordinates before applying the transformation.
- Visualize the rotation if possible for confirmation.

Summary: Key Takeaways

- Rotation about the origin involves transforming points based on a specific rule.
- For a 90-degree clockwise rotation, the coordinate transformation is (x, y) → (y, -x).
- Applying this rule to point M(x, y) yields the new position (y, -x).
- Visualization and careful calculation are essential for accurate results.
- Understanding this transformation is fundamental in fields like computer graphics, engineering, and mathematics.

Conclusion

In summary, when a figure is rotated 90 degrees clockwise about the origin, each point's coordinates are transformed according to a specific rule: (x, y) → (y, -x). To find the new coordinates of point M, simply substitute its original coordinates into this formula. This process is straightforward and essential for solving problems involving geometric transformations, helping students and professionals analyze and manipulate figures with precision. Mastery of this concept enhances spatial reasoning and is foundational in many applications across science, technology, engineering, and mathematics.

Keywords: rotation about the origin, 90-degree clockwise rotation, coordinate transformation, point M, geometric transformation, Cartesian plane, rotation formula, coordinate geometry, rotation example, graphing rotation

Frequently Asked Questions

How do you find the new coordinates of point M after a 90-degree clockwise rotation about the origin?

To rotate a point (x, y) 90 degrees clockwise about the origin, the new coordinates become (y, -x).

What is the general rule for rotating any point (x, y) 90 degrees clockwise around the origin?

The rule is to transform (x, y) into (y, -x) for a 90-degree clockwise rotation.

If the original coordinates of M are (3, 4), what are the coordinates after a 90-degree clockwise rotation?

The new coordinates would be (4, -3).

Why does the coordinate transformation for a 90-degree clockwise rotation involve swapping and negating coordinates?

Because rotating 90 degrees clockwise effectively swaps x and y, then reflects the new x-coordinate to the negative side, resulting in (y, -x).

Can the same rotation rule be applied for points in all quadrants?

Yes, the rule (x, y) to (y, -x) applies universally, regardless of the original quadrant of the point.

How does rotating a figure 90 degrees clockwise affect the shape's orientation and coordinates?

The figure's orientation changes by 90 degrees clockwise, and each point's coordinates are transformed according to the rule (x, y) to (y, -x).

Is the rotation about the origin the same as rotating about any other point?

No, rotating about the origin uses the rule (x, y) to (y, -x), but rotation about other points requires translation to and from the origin before and after rotation.

What are common applications of rotating points or figures 90 degrees clockwise in mathematics?

Such rotations are used in coordinate geometry, computer graphics, robotics, and solving geometric transformations efficiently.

If the original coordinates of M are (-2, 5), what will be its coordinates after a 90-degree clockwise rotation?

The new coordinates will be (5, 2).

Additional Resources

If The Figure Is Rotated 90 Clockwise About The Origin, What Are The Coordinates Of M?

Understanding how geometric figures transform under rotation is a fundamental concept in coordinate geometry, often encountered in mathematics, engineering, and computer graphics. A common scenario involves rotating a figure about the origin (0,0), which alters the positions of its points. One widespread question is: If a point M on a figure is rotated 90 degrees clockwise about the origin, what will be its new coordinates? This question not only tests your grasp of coordinate transformations but also offers insight into the geometric intuition behind rotations.

In this article, we will explore this question in depth, starting from the foundational principles of rotation in the coordinate plane, moving through the mathematical derivations, and culminating with practical examples and tips. Whether you're a student tackling a homework problem or a professional working with geometric transformations, this guide aims to clarify the process and ensure you can confidently determine the new coordinates of any point after a 90-degree clockwise rotation about the origin.

Understanding Rotation in the Coordinate Plane

Before we delve into the specifics of a 90-degree rotation, it is essential to understand the general rules governing rotations in the Cartesian coordinate system.

What Is Rotation About the Origin?

Rotation about the origin involves turning a figure or point around the point (0,0) by a specified angle. The direction of rotation matters:

- Counterclockwise rotation: positive angles (e.g., +90°, +180°)

- Clockwise rotation: negative angles (e.g., -90°, -180°)

In our case, the rotation is 90 degrees clockwise.

Visualizing the Rotation

Imagine holding a piece of paper with a point M marked on it. Rotating the paper 90 degrees clockwise around the center (origin) moves the point to a new position, M'.

Mathematical Foundations of Rotation

The core of understanding coordinate transformations under rotation relies on the rotation formulas derived from trigonometry.

Rotation Formulas

For a point M(x, y), rotating it by an angle θ about the origin results in a new point M'(x', y') given by:

- $x' = x \cos \theta - y \sin \theta$
- $y' = x \sin \theta + y \cos \theta$

These formulas come from applying the rotation matrix:

$$
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
=
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
$$

Applying the Formulas for a 90-Degree Clockwise Rotation

Since the question specifies a 90-degree clockwise rotation, the angle θ is -90° (negative because clockwise).

Trigonometric Values at -90°

Recall the following:

- $\cos(-90^\circ) = 0$
- $\sin(-90^\circ) = -1$

Substituting these into the rotation formulas:

$$
x' = x \times 0 - y \times (-1) = y
$$
$$
y' = x \times (-1) + y \times 0 = -x
$$

Thus, the transformation simplifies to:

- $x' = y$
- $y' = -x$

This means that every point (x, y) maps to $(y, -x)$ after a 90-degree clockwise rotation.

Step-by-Step Guide to Find the Coordinates of M

Suppose you are given the original coordinates of point $M(x, y)$, and you need to find the coordinates of M' after the rotation.

Step 1: Identify the original coordinates of M.

Step 2: Apply the rotation formulas:

- $x' = y$
- $y' = -x$

Step 3: Write the new coordinates as (x', y').

Example:

If $M(3, 4)$, then

- $x' = 4$
- $y' = -3$

So, M' (after rotation) is at $(4, -3)$.

Visual Examples and Intuitive Understanding

Visualizing the effect of a 90-degree clockwise rotation helps solidify the concept:

- The point (x, y) initially lies somewhere in the plane.
- After rotation, it moves to the position (y, -x).
- Geometrically, this corresponds to a quarter-turn clockwise, swapping the roles of the coordinates and changing the sign of the original x-coordinate.

Diagram Description:

Imagine the coordinate axes with point M at (x, y). Drawing the rotated point M' at (y, -x) illustrates how the point "rotates" around the origin, moving from its initial position to the new one.

Special Cases and Points of Interest

- Origin (0,0): Rotating the origin leaves it unchanged, since (0,0) maps to (0,0).
- Points on axes:
- On the x-axis (y=0): (x, 0) rotates to (0, -x).
- On the y-axis (x=0): (0, y) rotates to (y, 0).
- Points on the line y = x: (x, x) rotate to (x, -x), which lies on the line y = -x.
- Points on the line y = -x: (x, -x) rotate to (-x, -x), which lies on the line y = -x.

Practical Applications and Implications

Understanding how to find the coordinates after rotation is crucial in various fields:

- Computer Graphics: Rendering objects with rotations.
- Robotics: Calculating the orientation of robotic arms.
- Navigation: Adjusting coordinates based on directional changes.
- Mathematics Education: Developing spatial reasoning skills.

In real-world scenarios, this transformation allows engineers and programmers to manipulate objects precisely and predict their new positions after rotations.

Summary and Key Takeaways

- Rotating a point 90 degrees clockwise about the origin transforms any point (x, y) into (y, -x).
- The rotation formulas are derived from the rotation matrix and

trigonometric identities.
- The simplicity of the formulas makes it easy to compute the new coordinates without complex calculations.
- Recognizing special points and their images helps deepen geometric intuition.

Final Thoughts

Mastering coordinate transformations like the 90-degree clockwise rotation empowers students and professionals to analyze and manipulate geometric figures effectively. The key lies in understanding the underlying formulas and visualizing the geometric movement. Whether applied in solving math problems, designing graphics, or programming robots, knowing how to determine the new position of a point after rotation is an essential skill that bridges theoretical mathematics and practical application.

Next time you're faced with a problem involving rotating figures or points about the origin, remember the core transformation:

$(x, y) \rightarrow (y, -x)$

This simple yet powerful rule unlocks a range of possibilities in understanding the dynamic nature of the coordinate plane.

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