Ifsin(x)=1/3andsec(y)=17/15, Wherexandylie Between 0 And/2, Evaluate The Expression

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Understanding the relationships between trigonometric functions and their values is fundamental in mathematics, especially when dealing with angles within specific intervals. In this article, we will explore the problem: If sin(x) = 1/3 and sec(y) = 17/15, where x and y are between 0 and 2, evaluate the given expression. We will break down the process step-by-step, covering the necessary concepts, calculations, and reasoning to arrive at the solution.

Understanding the Problem

Before diving into calculations, it's important to comprehend what is given and what needs to be found:

- sin(x) = 1/3, where x ∈ [0, 2] (assuming radians)
- sec(y) = 17/15, where y ∈ [0, 2]
- The goal is to evaluate a certain trigonometric expression involving x and y (the expression will be specified later).

Note: Since the original problem does not specify the exact expression to evaluate, we will assume it involves common trigonometric functions of x and y, such as sin, cos, tan, or combinations thereof. For demonstration, we will evaluate the expression: sin(x) + cos(y), which is a typical example in such problems.

Step 1: Clarify the Domain and Functions

Understanding the Domain of x and y

- Both x and y are between 0 and 2. Usually, in trigonometry, angles are measured in radians unless specified otherwise.
- The interval [0, 2] radians approximately corresponds to [0°, 114.59°], since 2 radians ≈ 114.59°.
- This range covers parts of the first quadrant and possibly into the second, depending on the specific context.

Given Values and Their Implications

- sin(x) = 1/3 ≈ 0.3333. Since sin(x) is positive in the first and second quadrants, x could be in [0, π] (or [0, 3.1416]) — but constrained further by the domain.
- sec(y) = 17/15 ≈ 1.1333. Since secant is 1/cosine, and secant is positive here, y is in quadrants where cosine is positive (quadrant I and IV).

Step 2: Find the Corresponding Cosine and Sine Values

Calculating cos(x)

Given sin(x) = 1/3, we can find cos(x) using the Pythagorean identity:

$$
\sin^2 x + \cos^2 x = 1
$$

$$
\left(\frac{1}{3}\right)^2 + \cos^2 x = 1
$$

$$
\frac{1}{9} + \cos^2 x = 1
$$

$$
\cos^2 x = 1 - \frac{1}{9} = \frac{8}{9}
$$

$$
\cos x = \pm \sqrt{\frac{8}{9}} = \pm \frac{\sqrt{8}}{3} = \pm \frac{2\sqrt{2}}{3}
$$

Since x is between 0 and 2 radians (~0° to 114.59°), and sin(x) > 0, cos(x) is positive in the first quadrant and negative in the second quadrant.

- For x in [0, π/2], cos(x) > 0
- For x in [π/2, π], cos(x) < 0

Given the domain [0, 2], which includes angles up to approximately 114.59°, x could be in either the first or second quadrant.

However, without additional information, we often consider the principal value in the first quadrant for simplicity unless stated otherwise.

Thus, the most straightforward assumption is:

$$
\boxed{
\cos x = \frac{2\sqrt{2}}{3}
}
$$

Calculating cos(y) from sec(y) = 17/15

Since sec(y) = 1 / cos(y):

$$
\cos y = \frac{1}{\sec y} = \frac{15}{17}
$$

Because sec(y) > 0, and y is between 0 and 2 radians, y is in a quadrant where cosine is positive — namely, quadrants I or IV.

Now, find sin(y) using the Pythagorean identity:

$$
\sin^2 y + \cos^2 y = 1
$$

$$
\sin^2 y + \left(\frac{15}{17}\right)^2 = 1
$$

$$
\sin^2 y + \frac{225}{289} = 1
$$

$$
\sin^2 y = 1 - \frac{225}{289} = \frac{289}{289} - \frac{225}{289} = \frac{64}{289}
$$

$$
\sin y = \pm \frac{8}{17}
$$

Again, considering y in [0, 2], and since sec(y) > 0, y could be in quadrants I or IV.

- In quadrant I, sin(y) > 0 → sin y = +8/17

- In quadrant IV, sin(y) < 0 → sin y = -8/17

Most problems assume the principal value in quadrant I unless otherwise specified. So, for simplicity, let:

\[
\sin y = \frac{8}{17}
\]

Step 3: Evaluate the Target Expression

Assuming the expression to evaluate is sin(x) + cos(y), substituting the calculated values:

\[
\sin(x) + \cos y = \frac{1}{3} + \frac{15}{17}
\]

Find a common denominator:

\[
\frac{1}{3} = \frac{17}{51}
\]
\[
\frac{15}{17} = \frac{51}{51} \times \frac{15}{17} = \frac{45}{51}
\]

Actually, to combine directly:

\[
\frac{1}{3} + \frac{15}{17} = \frac{17}{51} + \frac{45}{51} = \frac{62}{51}
\]

So, the value of the expression:

\[
\boxed{
\sin(x) + \cos y = \frac{62}{51}
}
\]

which is approximately 1.2157.

Additional Insights and Applications

Why Understanding These Calculations Matters

- Trigonometric identities are fundamental in fields such as engineering, physics, and computer science.
- They help solve real-world problems involving angles and distances, such as in navigation, astronomy, and signal processing.
- Being able to determine unknown angles or function values from given data is a key skill in problem-solving.

Extending the Problem to Other Expressions

- Similar steps can be used to evaluate other expressions such as tan(x) - csc(y), $\sin^2 x + \cos^2 y$, or more complex combinations.
- For example, to find tan(x):

$$
\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{3} \times \frac{3}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}
$$

- Similarly, csc(y):

$$
\csc y = \frac{1}{\sin y} = \frac{17}{8}
$$

Conclusion

In this comprehensive analysis, we examined the given trigonometric conditions:

- sin(x) = 1/3
- sec(y) = 17/15

and determined the corresponding values of cos(x), sin(y), and cos(y). Using these, we evaluated the sum sin(x) + cos(y), arriving at a final value of 62/51.

This process illustrates the importance of understanding fundamental identities and the careful consideration of the domain of angles. Mastery of such techniques enables solving a wide range of problems in trigonometry and related fields, emphasizing the importance of step-by-step reasoning and attention to detail in mathematical problem-solving.

If you encounter similar problems, remember to:

- Clarify the domain and quadrant considerations.
- Use Pythagorean identities to find unknown functions.
- Substitute known values carefully.
- Simplify expressions with common denominators for clarity.

By practicing these steps, you'll strengthen your trigonometric problem-solving skills and deepen your understanding of the relationships between different functions.

Frequently Asked Questions

Given that sin(x) = 1/3 and x is between 0 and 2π, what is the value of cos(x)?

$\cos(x) = \sqrt{1 - \sin^2(x)} = \sqrt{1 - (1/3)^2} = \sqrt{1 - 1/9} = \sqrt{8/9} = (2\sqrt{2})/3$.

If sec(y) = 17/15 and y is between 0 and 2π, what is sin(y)?

Since $\sec(y) = 1/\cos(y)$, $\cos(y) = 15/17$. Then, $\sin(y) = \sqrt{1 - \cos^2(y)} = \sqrt{1 - (15/17)^2} = \sqrt{1 - 225/289} = \sqrt{64/289} = 8/17$.

What is the value of tan(x) given sin(x) = 1/3 and x in the range [0, 2π]?

$\tan(x) = \sin(x)/\cos(x) = (1/3) / (2\sqrt{2}/3) = 1 / (2\sqrt{2}) = \sqrt{2}/4$.

Calculate the value of cot(y) given sec(y) = 17/15 and y between 0 and 2π.

$\cot(y) = 1 / \tan(y) = \cos(y) / \sin(y) = (15/17) / (8/17) = 15/8$.

Evaluate the expression: sin(x) + cos(y), with the given values.

$\sin(x) + \cos(y) = 1/3 + 15/17 = (17/51) + (45/51) = 62/51$.

Find the value of the expression: tan(x) cot(y) with the given data.

$\tan(x) \cot(y) = (\sqrt{2}/4) (15/8) = (\sqrt{2}\ 15) / (4\ 8) = (15\sqrt{2}) / 32$.

Additional Resources

Ifsin(x) = 1/3 and sec(y) = 17/15, where x and y are between 0 and 2, evaluate the expression is a challenging trigonometric problem that combines the application of fundamental identities with strategic reasoning. This type of problem is particularly valuable in understanding how to manipulate and interpret trigonometric functions within specified domains, especially when given certain values. It requires a solid grasp of sine, secant, and their interrelated identities, as well as how to work with the constraints of the angle ranges to derive precise values. This article aims to comprehensively analyze this problem, breaking down the key concepts, solving step-by-step, and providing insights into the underlying mathematical principles.

Understanding the Given Information

The problem provides two key pieces of information:
- sin(x) = 1/3
- sec(y) = 17/15

Additionally, both x and y are constrained to be within the interval [0, 2], which, in the context of angles, typically refers to radians. The goal is to evaluate a certain expression involving x and y, although the expression itself appears to be missing in the initial prompt. For the purpose of this review, let's assume we are tasked with evaluating an expression such as sin(x) + sec(y) or a similar combination, since such problems commonly involve these functions.

Note: If the exact expression differs, the approach discussed will still be applicable, focusing on how to find the necessary trigonometric values from the given information.

Fundamental Trigonometric Concepts Involved

Before diving into calculations, it’s essential to review the core concepts at play:

1. Sine Function (sin)

- Defines the ratio of the opposite side to the hypotenuse in a right triangle.
- Range: [-1, 1].
- In this problem, sin(x) = 1/3, which indicates that x corresponds to an angle where the sine value is positive and less than 1, implying x is in the first or second quadrant (since sine is positive in both).

2. Secant Function (sec)

- Defined as the reciprocal of cosine: $\sec(\theta) = 1 / \cos(\theta)$.
- Range: $(-\infty, -1] \cup [1, \infty)$.
- Given sec(y) = 17/15, which is greater than 1, indicating y is in the first or fourth quadrant where cosine is positive or negative respectively.

3. Domain Constraints

- The angles x and y are within [0, 2], which is interpreted as radians. Since radians are standard in calculus, this restriction helps determine the possible quadrants and signs of the sine and cosine functions.

Step-by-Step Solution Approach

The solution involves several key steps:

Step 1: Find cos(x) from sin(x) = 1/3

Using the Pythagorean identity:
$$ \sin^2(x) + \cos^2(x) = 1 $$

Plugging in $\sin(x) = 1/3$:
$$ \left(\frac{1}{3}\right)^2 + \cos^2(x) = 1 $$
$$ \frac{1}{9} + \cos^2(x) = 1 $$
$$ \cos^2(x) = 1 - \frac{1}{9} = \frac{8}{9} $$

Thus:
$$ \cos(x) = \pm \sqrt{\frac{8}{9}} = \pm \frac{\sqrt{8}}{3} = \pm \frac{2\sqrt{2}}{3} $$

Determining the Sign of cos(x):
- Since sin(x) = 1/3 > 0, x is in the first or second quadrant.
- In the first quadrant, cos(x) > 0.
- In the second quadrant, cos(x) < 0.

Given the domain [0, 2], which covers the first and second quadrants for angles in radians, we need to specify the quadrant to determine the sign. If the problem indicates no further restriction, both are possible.

Step 2: Find cos(y) from sec(y) = 17/15

Since sec(y) = 1 / cos(y):
$$ \cos(y) = \frac{1}{\sec(y)} = \frac{15}{17} $$

Because sec(y) > 1, y is in the first or fourth quadrant:

- In the first quadrant, cos(y) > 0.
- In the fourth quadrant, cos(y) < 0.

Again, without explicit information, both are possible, but for simplicity, assume y is in the first quadrant where cos(y) > 0.

Step 3: Find sin(y) from cos(y)

Using the Pythagorean identity:
$$ \sin^2(y) + \cos^2(y) = 1 $$
$$ \sin^2(y) = 1 - \left(\frac{15}{17}\right)^2 $$
$$ \sin^2(y) = 1 - \frac{225}{289} = \frac{289 - 225}{289} = \frac{64}{289} $$
$$ \sin(y) = \pm \frac{8}{17} $$

Assuming y is in the first quadrant, $\sin(y) > 0$:
$$ \sin(y) = \frac{8}{17} $$

Evaluating the Expression

Suppose the expression to evaluate is:
$$ \text{Expression} = \sin(x) + \sec(y) $$

Using the values derived:
- $\sin(x) = 1/3$
- $\sec(y) = 17/15$

Thus:
$$ \text{Expression} = \frac{1}{3} + \frac{17}{15} $$

Find common denominator (15):
$$ \frac{5}{15} + \frac{17}{15} = \frac{22}{15} $$

Result:
$$ \boxed{\frac{22}{15}} $$

Additional Insights and Variations

Depending on the original expression, similar steps can be employed:

- If the expression involves other functions such as tangent, cotangent, or their combinations, the same approach applies: derive sine and cosine values first.
- The sign ambiguities in $\cos(x)$ and $\sin(y)$ depend on the quadrants, which should

be clarified based on the problem context.
- For angles outside the first quadrant, the signs change appropriately, affecting the final answer.

Pros and Cons of the Approach

Pros:
- Systematic method based on fundamental identities.
- Clear step-by-step process that can be adapted for different trigonometric expressions.
- Reinforces understanding of the relationships among trigonometric functions.

Cons:
- Assumes specific quadrants without explicit information, which can lead to multiple possible answers.
- Neglects potential domain restrictions if not specified.
- Slightly cumbersome if the expression involves more complex functions.

Features and Recommendations

Features of this approach:
- Uses basic identities and algebraic manipulation.
- Facilitates understanding of how to handle inverse functions and reciprocal identities.
- Applies domain considerations to determine signs of sine and cosine.

Recommendations:
- Always verify the quadrants for angles based on given conditions.
- Be cautious with signs when deriving sine or cosine from their reciprocal.
- Consider expressing all functions in terms of sine and cosine for consistency.

Conclusion

The problem of evaluating an expression given $\sin(x) = 1/3$ and $\sec(y) = 17/15$ is a classic example of applying fundamental trigonometric identities and understanding the geometric significance of these functions. By carefully determining the corresponding cosine and sine values, respecting the domain restrictions, and performing straightforward algebra, we can confidently evaluate the expression. This process not only yields the numerical answer but also deepens the understanding of the interplay between different trigonometric functions and their identities.

Final Note: If the original expression differs, the same principles apply: find the necessary sine and cosine values from the given information, determine signs based on the domain, and then evaluate the expression accordingly. This approach ensures a thorough and accurate solution to a wide range of trigonometric problems.

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