Please Explain Why The X-value At Which The Two Functions' Values Are Equal Is 10/3.

Please Explain Why The X-value At Which The Two Functions' Values Are Equal Is 10/3.

Understanding the point at which two functions intersect is a fundamental concept in algebra and calculus. It involves determining the x-value where the output (or y-value) of both functions is identical. This intersection point provides insight into the behavior of the functions, their relative positions, and their relationships over different domains.

In this article, we will explore a specific example: why the x-value at which two particular functions are equal is 10/3. We will delve into the mathematical reasoning behind this, analyze the functions involved, and demonstrate how to find their intersection point step by step. By the end, you will have a comprehensive understanding of how to approach such problems and why the solution is exactly 10/3.

Understanding Functions and Their Intersections

What Are Functions?

Functions are mathematical relations that assign exactly one output (or y-value) for each input (or x-value). They are fundamental in describing relationships between quantities.

For example, consider the functions:

- $f(x) = 2x + 1$
- $g(x) = x^2$

Each function maps an x-value to a y-value based on its rule.

What Does It Mean for Functions to Be Equal?

When two functions are equal at a certain x-value, it means:

$$f(x) = g(x)$$

At that point, both functions produce the same output. The x-value where this occurs is called the point of intersection or solution to the equation $f(x) = g(x)$.

Why Is Finding the Intersection Important?

Identifying the intersection points helps in:

- Understanding the behavior and relationship between functions
- Solving real-world problems involving equality
- Graphically visualizing where two phenomena coincide
- Analyzing solutions in calculus, such as limits and integrals

---

Mathematical Foundations: Solving for the Intersection Point

General Approach

To find the x-value where two functions $f(x)$ and $g(x)$ are equal, follow these steps:

1. Set the two functions equal to each other:

$$f(x) = g(x)$$

2. Solve the resulting equation for x.

3. Verify the solutions by substituting back into the original functions.

Case Study: Demonstrating Why the Intersection Occurs at x = 10/3

Suppose we are given two functions:

- $f(x) = 3x - 2$
- $g(x) = 2x + 4$

Our goal is to find the x-value where these functions are equal and demonstrate why it is $\frac{10}{3}$.

Step 1: Set the functions equal to each other

$$
3x - 2 = 2x + 4
$$

Step 2: Solve for x

Subtract $2x$ from both sides:

$$
3x - 2x - 2 = 4
$$
$$
x - 2 = 4
$$

Add 2 to both sides:

$$
x = 6
$$

In this example, the intersection point occurs at $x = 6$.

Applying to the Specific Functions Leading to x = 10/3

Now, consider a more complex pair of functions where the solution is $\frac{10}{3}$. Suppose the functions are:

- $f(x) = 2x^2 - x$

- $g(x) = 4x - 6$

Our task: find the x-value where $f(x) = g(x)$ and understand why it is $\frac{10}{3}$.

Step 1: Set the functions equal

$$
2x^2 - x = 4x - 6
$$

Step 2: Rearrange to standard quadratic form

Bring all terms to one side:

$$
2x^2 - x - 4x + 6 = 0
$$

Simplify:

$$
2x^2 - 5x + 6 = 0
$$

Step 3: Solve the quadratic equation

Quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a = 2$, $b = -5$, and $c = 6$.

Calculate discriminant:

$$
\Delta = (-5)^2 - 4 \times 2 \times 6 = 25 - 48 = -23
$$

\]

Since the discriminant is negative, the solutions are complex (not real). So, in this case, the functions do not intersect on the real number line.

Designing a Scenario Where the Intersection Occurs at x = 10/3

To find functions that intersect at \(x = \frac{10}{3} \), we can construct functions with this property.

Suppose:

- \(f(x) = ax + b \)
- \(g(x) = cx + d \)

We want:

\[
f\left(\frac{10}{3}\right) = g\left(\frac{10}{3}\right)
\]

which implies:

\[
a \times \frac{10}{3} + b = c \times \frac{10}{3} + d
\]

Let's choose \(a = 2 \), \(c = 3 \), and find \(b \) and \(d \) such that they are equal at \(x = \frac{10}{3} \).

Calculate:

\[
f\left(\frac{10}{3}\right) = 2 \times \frac{10}{3} + b = \frac{20}{3} + b
\]
\[
g\left(\frac{10}{3}\right) = 3 \times \frac{10}{3} + d = 10 + d
\]

Set equal:

$$
\frac{20}{3} + b = 10 + d
$$

Assuming $b = 0$, then:

$$
\frac{20}{3} = 10 + d
$$

Express 10 as $\frac{30}{3}$:

$$
\frac{20}{3} = \frac{30}{3} + d
$$

Subtract $\frac{30}{3}$:

$$
\frac{20}{3} - \frac{30}{3} = d \Rightarrow -\frac{10}{3} = d
$$

Thus, the functions:

- $f(x) = 2x$
- $g(x) = 3x - \frac{10}{3}$

intersect at $x = \frac{10}{3}$.

Understanding Why the Intersection Is Exactly at x = 10/3

This example illustrates the principle: the point where two linear functions intersect depends on their slopes and intercepts. By setting their expressions equal and solving for x, we can find the precise point of intersection.

In the constructed example, the key steps are:

1. Set the functions equal: $2x = 3x - \frac{10}{3}$
2. Solve for x:

$$
2x = 3x - \frac{10}{3}
$$
$$
- x = - \frac{10}{3}
$$
$$
x = \frac{10}{3}
$$

This confirms that the functions intersect at $x = \frac{10}{3}$.

Real-World Applications of Finding Intersection Points

Understanding why two functions intersect at a specific x-value has practical implications across various fields:

- Economics: Determining equilibrium points where supply and demand curves meet.
- Physics: Finding points where two moving objects occupy the same position.
- Biology: Analyzing overlapping growth rates of populations.
- Engineering: Identifying operating points where different system components perform equally.

In each case, accurately solving for the intersection point allows for better decision-making and system optimization.

Summary: Why Is the Intersection at x = 10/3?

In essence, the x-value where two functions are equal is determined by solving the algebraic equation obtained by setting those functions equal to each other. When the functions are linear, quadratic, or of higher degree, the process involves algebraic manipulation and solving equations, often with the quadratic formula.

Specifically, in cases where the intersection occurs at $x = \frac{10}{3}$, this results from the particular coefficients and constants chosen in the functions. The process involves:

- Setting the functions equal.
- Rearranging to standard form.
- Solving for x.
- Confirming that the solution matches \(\frac{10}{3} \).

This algebraic process explains precisely why the intersection point occurs at that x-value, providing clarity and confidence in understanding function behavior.

Conclusion

Determining the x-value

Frequently Asked Questions

What does it mean when two functions have equal values at a specific x-value?

It means that at that particular x-value, both functions produce the same output or y-value, indicating they intersect at that point.

How do you find the x-value where two functions are equal?

By setting the two functions equal to each other and solving the resulting equation for x.

Why is the x-value at which the functions are equal equal to 10/3?

Because solving the equation derived from setting the functions equal yields $x = 10/3$, which is the point of their intersection.

Can you give an example of two functions that intersect at x = 10/3?

Yes, for example, if $f(x) = 2x + 4$ and $g(x) = 3x - 2$, solving $2x + 4 = 3x - 2$ results in $x = 10/3$.

Is the x-value 10/3 always the point of intersection for these functions?

Only if it satisfies both functions' equations; in this context, it is the solution to their equality, so yes.

How does understanding where functions are equal help in graphing?

Finding the intersection points helps accurately sketch where the functions cross paths, revealing important features of their graphs.

What steps are involved in verifying that x = 10/3 is the solution?

Substitute x = 10/3 into both functions and check if both yield the same y-value; if they do, then x = 10/3 is the point of equality.

Why is solving for x important in understanding the relationship between two functions?

It helps identify points of intersection, which are crucial for analyzing their behavior, comparative growth, and solutions to equations involving both functions.

Additional Resources

Please Explain Why The X-value At Which The Two Functions' Values Are Equal Is 10/3.

In the realm of mathematics, understanding how and why certain points of intersection occur is fundamental to grasping the behavior and relationships of functions. Among these, the point where two functions have the same value at a specific x-coordinate often reveals deeper insights into their nature. Today, we explore why the x-value at which two particular functions are equal is precisely $\frac{10}{3}$. This explanation not only clarifies the reasoning behind this specific value but also illuminates key concepts in algebra and function analysis that are widely applicable across various mathematical contexts.

The Core Problem: Identifying the Intersection Point of Two Functions

Before delving into the specifics of why the x-value is $\frac{10}{3}$, it is essential to understand the general setup. Consider two functions:

- $f(x)$
- $g(x)$

The question at hand is: At which x-value do these functions have the same output? Mathematically, this translates to solving the equation:

\[

f(x) = g(x)
\]

The solution(s) to this equation give the x-coordinate(s) where the two functions intersect.

Example: Suppose the functions are:

\[
f(x) = 2x + 4
\]
\[
g(x) = -x + 8
\]

To find the intersection point, we set:

\[
2x + 4 = -x + 8
\]

Solving for \(x\):

\[
2x + x = 8 - 4
\]
\[
3x = 4
\]
\[
x = \frac{4}{3}
\]

This is a straightforward example that illustrates the process: setting the two functions equal and solving for \(x\).

The Specific Functions and Derivation of \(x = \frac{10}{3}\)

Now, let’s consider the specific pair of functions that lead to the intersection at \(x = \frac{10}{3}\). Assume the functions are:

\[

f(x) = 3x + 2
\]
\[
g(x) = 2x + 4
\]

Our goal is to find the \(x\) where \(f(x) = g(x)\):

\[
3x + 2 = 2x + 4
\]

Subtract \(2x\) from both sides:

\[
3x - 2x + 2 = 4
\]
\[
x + 2 = 4
\]

Subtract 2 from both sides:

\[
x = 2
\]

In this simplified case, the intersection occurs at \(x = 2\). But to arrive at \(x = \frac{10}{3}\), the functions are more complex. Suppose the functions are:

\[
f(x) = 4x - 2
\]
\[
g(x) = 2x + c
\]

Let's determine the value of \(c\) that makes the intersection at \(x = \frac{10}{3}\). Set \(f(x) = g(x)\):

\[
4x - 2 = 2x + c
\]

Substitute $x = \frac{10}{3}$:

$$
4 \times \frac{10}{3} - 2 = 2 \times \frac{10}{3} + c
$$

Calculate each side:

$$
\frac{40}{3} - 2 = \frac{20}{3} + c
$$

Express -2 as $\frac{-6}{3}$:

$$
\frac{40}{3} - \frac{6}{3} = \frac{20}{3} + c
$$

Simplify numerator:

$$
\frac{34}{3} = \frac{20}{3} + c
$$

Subtract $\frac{20}{3}$ from both sides:

$$
\frac{34}{3} - \frac{20}{3} = c
$$

$$
\frac{14}{3} = c
$$

Therefore, if $c = \frac{14}{3}$, the functions intersect at $x = \frac{10}{3}$.

This example illustrates that the specific x-value of $\frac{10}{3}$ (or approximately 3.333) is obtained through algebraic manipulation, aligning the functions at that point.

Why Is $x = \frac{10}{3}$? Analyzing the Algebraic Roots

The core of understanding why the intersection occurs at $x = \frac{10}{3}$ involves examining the equations that led us there. Typically, this results from solving a quadratic or higher-degree polynomial, where the roots directly indicate the intersection points.

Suppose the difference between the functions is represented as:

$$
h(x) = f(x) - g(x)
$$

Our goal is to find the roots of $h(x) = 0$. If $h(x)$ is quadratic, say:

$$
h(x) = ax^2 + bx + c
$$

then solving $h(x) = 0$ using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

might yield roots involving fractions, one of which could be $\frac{10}{3}$.

Example:

Let:

$$
h(x) = 3x^2 - 10x + 10
$$

Set $h(x) = 0$:

$$
3x^2 - 10x + 10 = 0
$$

Applying quadratic formula:

$$
x = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 3 \times 10}}{2 \times 3}
$$

Calculate discriminant:

$$
100 - 120 = -20
$$

Since the discriminant is negative, no real roots exist here. But suppose the quadratic is:

$$
h(x) = 3x^2 - 10x
$$

Set to zero:

$$
3x^2 - 10x = 0
$$
$$
x(3x - 10) = 0
$$

Solutions:

$$
x = 0 \quad \text{or} \quad 3x - 10 = 0 \Rightarrow x = \frac{10}{3}
$$

Here, the root $x = \frac{10}{3}$ naturally emerges from solving the quadratic. It signifies the precise point where the functions, represented by $h(x)$, intersect.

Geometric Interpretation: Why the Fractional Value?

Understanding why the intersection point is at $x = \frac{10}{3}$ also benefits from a geometric perspective. Graphically, each function corresponds to a curve or line. The intersection point is where these curves cross, which translates to a specific x-coordinate that satisfies both equations simultaneously.

When the solution involves fractions like $\frac{10}{3}$, it often indicates that the intersection occurs at a point that is not a simple whole number but rather a rational fraction, reflecting the precise balance point between the two functions.

Broader Significance: The Importance of Exact Solutions

Knowing the exact x-coordinate where functions intersect — such as $\frac{10}{3}$ — has practical significance beyond pure mathematics:

- Precision in Engineering: Exact intersection points are critical when designing systems that depend on crossing thresholds.
- Data Analysis: Precise solutions help identify critical transition points in datasets modeled by functions.
- Mathematical Modeling: Exact roots inform the stability and behavior of models in physics, economics, and other sciences.

Moreover, expressing solutions as fractions emphasizes the importance of algebraic exactness over decimal approximations, which can introduce rounding errors and misinterpretations.

Conclusion: The Mathematical Rationale Behind $x = \frac{10}{3}$

In summary, the reason why the x-value at which two functions are equal is exactly $\frac{10}{3}$ stems from algebraic solving of their equations. Whether it involves solving linear equations or roots of quadratic functions, the fractional form arises naturally from the algebraic process, ensuring an exact solution. Recognizing this process enhances our understanding of how functions interact and intersect, offering insights that extend across various branches of mathematics and applied sciences.

Understanding why the intersection occurs at this specific point not only clarifies a particular problem but also reinforces fundamental concepts in solving equations, analyzing functions, and interpreting the geometric meaning of solutions. As mathematics continues to underpin technological and scientific progress, mastering these foundational ideas remains crucial for innovation and discovery.

Please Explain Why The X Value At Which The Two Functions Values Are Equal Is 10 3

Please Explain Why The X Value At Which The Two Functions Values Are Equal Is 10 3

Related Articles

- The Car Is Moving At 10 M/s Relative To Bill. How Fast Does Amy See The Car As Moving?
- A Data Value Is Considered _______ If Its Z-score Is Less Than 2 Or Greater Than 2.
- Solve The Following Simultaneous EquationUsing Any Suitable Method X+6y=2, 3x +2y =10

Back to Home