Question 5. Find F'(x)Solution. (a) F(x) = In Arc Tan (2x) (b) F(x) = F(x)= Ex Sechx

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Introduction

In calculus, differentiation is a fundamental concept that allows us to analyze the rate of change of functions. Determining the derivatives of various functions is crucial in fields such as physics, engineering, economics, and many more. This article focuses on solving two specific differentiation problems involving composite and hyperbolic functions.

The problems are as follows:
- Part (a): Find the derivative of $F(x) = \ln(\arctan(2x))$.
- Part (b): Find the derivative of $F(x) = e^x \operatorname{sech} x$.

Understanding how to differentiate these functions requires a good grasp of chain rule, product rule, and properties of hyperbolic functions. We will delve into each problem step-by-step, providing detailed explanations and relevant formulas to enhance comprehension.

Part (a): Differentiating $F(x) = \ln(\arctan(2x))$

Understanding the function

The function $F(x) = \ln(\arctan(2x))$ involves a composition of functions:
- The natural logarithm function $\ln u$,
- The inverse tangent (arctangent) function $\arctan v$,
- The linear function $2x$.

Hence, differentiating this function involves applying the chain rule multiple times.

Step-by-step differentiation

1. Recognize the composite structure

Let:
- $u = \arctan(2x)$,
- So, $F(x) = \ln u$.

The derivative of $\ln u$ with respect to u is $\frac{1}{u}$.

2. Differentiate $u = \arctan(2x)$

Using the chain rule:
- $\frac{d}{dx} \arctan(2x) = \frac{1}{1 + (2x)^2} \times \frac{d}{dx}(2x) = \frac{1}{1 + 4x^2} \times 2 = \frac{2}{1 + 4x^2}$.

3. Combine the derivatives

Applying the chain rule:
$$
F'(x) = \frac{1}{u} \times \frac{du}{dx} = \frac{1}{\arctan(2x)} \times \frac{2}{1 + 4x^2}.
$$

Final derivative expression:

$$
\boxed{
F'(x) = \frac{2}{(1 + 4x^2) \cdot \arctan(2x)}.
}
$$

Additional notes

- The derivative is valid for all x where $\arctan(2x) \neq 0$ (i.e., $x \neq 0$), as the logarithm is undefined for zero or negative arguments.
- Understanding the domain is crucial when applying derivatives involving logarithmic functions.

Part (b): Differentiating $F(x) = e^x \operatorname{sech} x$

Understanding the function

The function $F(x) = e^x \operatorname{sech} x$ is a product of exponential and hyperbolic secant functions. To differentiate it, we need to apply the product rule, which states:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) v(x) + u(x) v'(x).
$$

Step-by-step differentiation

1. Identify $u(x)$ and $v(x)$

Let:

- $u(x) = e^x$,
- $v(x) = \operatorname{sech} x$.

2. Differentiate each component

- $u'(x) = e^x$,
- $v(x) = \operatorname{sech} x$.

Recall that:

$$
\operatorname{sech} x = \frac{1}{\cosh x}.
$$

The derivative of $\operatorname{sech} x$ is:

$$
\frac{d}{dx} \operatorname{sech} x = - \operatorname{sech} x \tanh x,
$$

which can be derived from the quotient rule or known hyperbolic derivatives.

3. Apply the product rule

$$
F'(x) = u'(x) v(x) + u(x) v'(x) = e^x \operatorname{sech} x + e^x \times \left(- \operatorname{sech} x \tanh x \right).
$$

Simplify:

$$
F'(x) = e^x \operatorname{sech} x - e^x \operatorname{sech} x \tanh x.
$$

Factor out common terms:

$$
F'(x) = e^x \operatorname{sech} x (1 - \tanh x).
$$

Final derivative expression:

$$
\boxed{
F'(x) = e^x \operatorname{sech} x (1 - \tanh x).
}
$$

Additional notes

- The hyperbolic tangent $\tanh x$ is related to $\sinh x$ and \(

$\cosh x$ as:

$$
\tanh x = \frac{\sinh x}{\cosh x}.
$$

- The hyperbolic secant $\operatorname{sech} x$ is always positive and decreases as $|x|$ increases.
- The expression $1 - \tanh x$ influences the sign and magnitude of the derivative across different x values.

Summary and Key Takeaways

Differentiation Techniques Used

- Chain Rule: Essential for differentiating compositions like $\ln(\arctan(2x))$.
- Product Rule: Necessary for differentiating products such as $e^x \operatorname{sech} x$.
- Hyperbolic Function Derivatives: Understanding derivatives of hyperbolic functions like $\operatorname{sech} x$ and $\tanh x$ is crucial.

Final Results Recap

- Part (a):

$$
\boxed{
F'(x) = \frac{2}{(1 + 4x^2) \cdot \arctan(2x)}.
}
$$

- Part (b):

$$
\boxed{
F'(x) = e^x \operatorname{sech} x (1 - \tanh x).
}
$$

Applications of These Derivatives

Understanding these derivatives has broad applications:
- In physics, for calculating rates of change involving inverse trigonometric functions and hyperbolic functions.
- In engineering, for analyzing signal processing involving hyperbolic functions.
- In mathematics, for solving differential equations and modeling phenomena with these functions.

Conclusion

Differentiating complex functions like \(\ln(\arctan(2x)) \) and \(e^x \operatorname{sech} x \) showcases the power of calculus tools such as the chain rule, product rule, and properties of hyperbolic functions. Mastering these techniques enables one to analyze a wide range of mathematical models and real-world problems involving advanced functions.

By carefully applying the rules and understanding the properties of inverse and hyperbolic functions, students and professionals can confidently tackle similar differentiation problems, enhancing their analytical skills and mathematical proficiency.

Frequently Asked Questions

How do you find the derivative of F(x) = ln(Arc Tan(2x))?

To differentiate F(x) = ln(Arc Tan(2x)), apply the chain rule: F'(x) = (1 / Arc Tan(2x)) d/dx [Arc Tan(2x)]. The derivative of Arc Tan(2x) is 2 / (1 + (2x)^2). So, F'(x) = (1 / Arc Tan(2x)) [2 / (1 + 4x^2)].

What is the derivative of F(x) = ln(Arc Tan(2x))?

F'(x) = [2 / (1 + 4x^2)] divided by Arc Tan(2x), so F'(x) = (2 / (1 + 4x^2)) (1 / Arc Tan(2x)).

How do you differentiate F(x) = Ex Sechx'?

Assuming F(x) = e^x sech(x), use the product rule: F'(x) = e^x sech(x) + e^x d/dx [sech(x)]. Since d/dx [sech(x)] = -sech(x) tanh(x), the derivative is F'(x) = e^x sech(x) - e^x sech(x) tanh(x).

What is the derivative of F(x) = e^x sech(x)?

F'(x) = e^x sech(x) - e^x sech(x) tanh(x).

Can you simplify the derivative of F(x) = e^x sech(x)?

Yes, F'(x) = e^x sech(x) [1 - tanh(x)].

What is the derivative of the natural log of an inverse tangent function?

For F(x) = ln(Arc Tan(2x)), the derivative is F'(x) = (2 / (1 + 4x^2)) / Arc Tan(2x).

Are there common applications for derivatives of inverse tangent and hyperbolic functions?

Yes, derivatives of inverse tangent and hyperbolic functions are used in calculus problems involving rates, optimization, and modeling phenomena like hyperbolic motion or signal processing.

What is the importance of understanding derivatives of composite functions like these?

Understanding derivatives of composite functions such as ln(Arc Tan(2x)) and e^x sech(x) is crucial for solving complex calculus problems involving implicit functions, optimization, and modeling real-world phenomena.

How do you interpret the derivative F'(x) in these cases?

F'(x) represents the rate of change of the function F(x) with respect to x, providing insight into how the function behaves locally, such as increasing or decreasing tendencies and concavity.

Additional Resources

Differentiation: Unlocking the Derivative of Complex Functions

When delving into the intricate world of calculus, understanding how to find derivatives — especially for composite and hyperbolic functions — is fundamental. This article explores the detailed process of differentiating two intriguing functions, offering a comprehensive guide that will empower students, educators, and enthusiasts alike to master these derivative calculations with confidence. Let's embark on this analytical journey, examining each function step-by-step and uncovering the mathematical elegance behind their derivatives.

Understanding the Derivative of F(x) =

ln(Arctan(2x))

This function combines a natural logarithm with an inverse tangent (arctangent) function, encapsulating a layered structure that demands the application of both the chain rule and knowledge of derivatives of inverse functions.

Breaking Down the Function

Given:

$$ F(x) = \ln(\arctan(2x)) $$

This can be viewed as a composition of two functions:

- Outer function: $ \ln(u) $, where $ u = \arctan(2x) $
- Inner function: $ u = \arctan(2x) $

To differentiate $ F(x) $, we need to apply the chain rule, which states:

$$ \frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x) $$

Step 1: Differentiate the Outer Function

The derivative of $ \ln(u) $ with respect to $ u $ is:

$$ \frac{d}{du} \ln(u) = \frac{1}{u} $$

Applying the chain rule, this becomes:

$$ \frac{1}{\arctan(2x)} $$

Step 2: Differentiate the Inner Function $ \arctan(2x) $

Recall that:

$$ \frac{d}{dx} \arctan(kx) = \frac{k}{1 + (kx)^2} $$

for a constant $ k $.

Here, $k = 2$. Thus:

$$\frac{d}{dx} \arctan(2x) = \frac{2}{1 + (2x)^2} = \frac{2}{1 + 4x^2}$$

Step 3: Combine the Results

Putting it all together, we get:

$$F'(x) = \frac{1}{\arctan(2x)} \times \frac{2}{1 + 4x^2}$$

Thus, the derivative of $F(x) = \ln(\arctan(2x))$ is:

$$
\boxed{
F'(x) = \frac{2}{(1 + 4x^2) \cdot \arctan(2x)}
}
$$

Summary:

- The derivative involves the reciprocal of the arctangent function, multiplied by the derivative of the inside.
- The result emphasizes how nested functions require careful application of the chain rule.

Differentiating F(x) = E^x \operatorname{Sech} x

This function incorporates exponential and hyperbolic functions, specifically the hyperbolic secant, which is less common but equally important in advanced calculus contexts, especially in hyperbolic geometry and physics.

Understanding the Components

Given:

$$F(x) = e^x \operatorname{Sech} x$$

Here, the function is a product of two functions:

- $u(x) = e^x$
- $v(x) = \operatorname{Sech} x$

Since the function is a product, the product rule applies:

$$\frac{d}{dx}[u \cdot v] = u' \cdot v + u \cdot v'$$

Step 1: Differentiate $u(x) = e^x$

This is straightforward:

$$u' = e^x$$

Step 2: Differentiate $v(x) = \operatorname{Sech} x$

Recall the derivative of hyperbolic secant:

$$\frac{d}{dx} \operatorname{Sech} x = - \operatorname{Sech} x \tanh x$$

This stems from the definition:

$$\operatorname{Sech} x = \frac{1}{\cosh x}$$

and applying the quotient rule or chain rule.

Step 3: Apply the Product Rule

Putting it all together:

$$F'(x) = u' \cdot v + u \cdot v' = e^x \operatorname{Sech} x + e^x \left(- \operatorname{Sech} x \tanh x \right)$$

Factor out $e^x \operatorname{Sech} x$:

$$
F'(x) = e^x \operatorname{Sech} x \left(1 - \tanh x \right)
$$

Final Expression for $F'(x)$

$$
\boxed{
F'(x) = e^x \operatorname{Sech} x \left(1 - \tanh x \right)
}
$$

This concise form captures the essence of the derivative, combining exponential growth, hyperbolic secant decay, and the hyperbolic tangent's behavior.

Insights and Practical Applications

Understanding the derivatives of these functions isn't just an academic exercise; it has profound implications across multiple fields:

- Physics: Hyperbolic functions describe phenomena like special relativity and wave propagation.
- Engineering: Signal processing often involves inverse tangent functions and hyperbolic functions.
- Mathematics: Advanced calculus and differential equations frequently include nested functions requiring such derivatives.

Key Takeaways for Mastering These Derivatives

- Chain Rule is Fundamental: For compositions like $\ln(\arctan(2x))$, always identify inner and outer functions.
- Product Rule for Multiple Functions: When functions are multiplied, differentiate each separately and combine.
- Recall Standard Derivatives: Memorize derivatives of basic functions like e^x, $\arctan x$, $\operatorname{Sech} x$, and $\tanh x$.
- Simplify Carefully: Combining derivatives often leads to factorizations

that simplify the expression.

Conclusion

Both derivatives exemplify the beauty and complexity of calculus. The logarithmic and inverse tangent combination demonstrates the importance of the chain rule in nested functions, while the product of exponential and hyperbolic functions showcases the power of the product rule and hyperbolic identities. Mastering these derivatives enhances analytical skills and opens doors to solving more advanced problems in mathematics, physics, and engineering.

By practicing these techniques and understanding the underlying principles, students and professionals can confidently approach even the most challenging derivative problems with clarity and precision.

Question 5 Find F X Solution A F X In Arc Tan 2x B F X F X Ex Sechx

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