Solve Each Equation. Remember To Check For Extraneous Solutions. K+2/k-4-4k/k-4=1

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When faced with algebraic equations involving fractions, such as K + 2/(K - 4) - 4K/(K - 4) = 1, it's essential to approach the problem methodically. Solving such equations requires not only finding the potential solutions but also verifying their validity to avoid extraneous solutions—solutions that satisfy the algebraic manipulations but are not valid in the original equation due to restrictions like division by zero. In this article, we will explore the step-by-step process of solving the equation K + 2/(K - 4) - 4K/(K - 4) = 1, emphasizing the importance of checking for extraneous solutions at the end.

Understanding the Equation

Before diving into the solution process, it's crucial to understand the components of the equation:

- The variable involved is K.
- The equation contains fractions with denominators involving K - 4.
- The goal is to isolate K and find its value(s) that satisfy the entire equation.

The presence of fractions suggests that we should consider clearing denominators to simplify the equation, but we must also be cautious of restrictions such as K ≠ 4, because division by zero is undefined.

Step-by-Step Solution Process

1. Identify restrictions on the variable

Since the denominators are K - 4, we must exclude any solutions that make the denominator zero:

- K ≠ 4

This restriction will be checked after solving the equation.

2. Find a common denominator to clear fractions

The denominators are both K - 4. To eliminate the fractions, multiply both sides of the equation by K - 4:

- Original equation:

K + 2/(K - 4) - 4K/(K - 4) = 1

- Multiply both sides by $k - 4$:

$(k - 4) k + (k - 4) [2/(k - 4)] - (k - 4) [4k/(k - 4)] = (k - 4) 1$

Simplify each term:

- $(k - 4) k = k(k - 4) = k^2 - 4k$
- $(k - 4) [2/(k - 4)] = 2$ (since $k - 4$ cancels)
- $(k - 4) [4k/(k - 4)] = 4k$ (again, $k - 4$ cancels)
- $(k - 4) 1 = k - 4$

Putting it all together:

$k^2 - 4k + 2 - 4k = k - 4$

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3. Simplify the resulting equation

Combine like terms:

- Left side: $k^2 - 4k - 4k + 2 = k^2 - 8k + 2$
- Right side: $k - 4$

Now, the simplified equation is:

$k^2 - 8k + 2 = k - 4$

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4. Bring all terms to one side to form a quadratic equation

Subtract k and add 4 to both sides:

$k^2 - 8k + 2 - k + 4 = 0$

Combine like terms:

$k^2 - 9k + 6 = 0$

This quadratic equation is now ready for solving.

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5. Solve the quadratic equation

The quadratic equation:

$k^2 - 9k + 6 = 0$

Use the quadratic formula:

$k = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

Where:

- $a = 1$
- $b = -9$
- $c = 6$

Calculate discriminant:

$\Delta = b^2 - 4ac = (-9)^2 - 4\ 1\ 6 = 81 - 24 = 57$

Since $\Delta > 0$, there are two real solutions:

$k = [9 \pm \sqrt{57}] / 2$

Expressed explicitly:

$k = [9 + \sqrt{57}] / 2$ and $k = [9 - \sqrt{57}] / 2$

Checking for Extraneous Solutions

After finding potential solutions, it's critical to verify whether they satisfy the original equation and do not violate any restrictions, especially division by zero.

1. Verify solutions do not make denominators zero

Recall the restriction:

- $k \neq 4$

Calculate approximate decimal values:

- $\sqrt{57} \approx 7.55$

So,

- $k_1 \approx (9 + 7.55) / 2 \approx 16.55 / 2 \approx 8.275$
- $k_2 \approx (9 - 7.55) / 2 \approx 1.45 / 2 \approx 0.725$

Neither solution equals 4, so both are valid in terms of denominator restrictions.

2. Substitute back into the original equation

To confirm, substitute $k \approx 8.275$ and $k \approx 0.725$ into the original:

$k + 2/(k - 4) - 4k/(k - 4) = 1$

- For $k \approx 8.275$:

- $k - 4 \approx 4.275$

- Compute:

8.275 + 2 / 4.275 - 4 8.275 / 4.275

- 2 / 4.275 ≈ 0.468

- 4 8.275 ≈ 33.1

- 33.1 / 4.275 ≈ 7.74

- Sum:

8.275 + 0.468 - 7.74 ≈ 8.743 - 7.74 ≈ 1.003

- Slight discrepancy due to rounding, but very close to 1, confirming the solution.

- For K ≈ 0.725:

- K - 4 ≈ -3.275

- Compute:

0.725 + 2 / -3.275 - 4 0.725 / -3.275

- 2 / -3.275 ≈ -0.611

- 4 0.725 ≈ 2.9

- 2.9 / -3.275 ≈ -0.885

- Sum:

0.725 - 0.611 - (-0.885) = 0.725 - 0.611 + 0.885 ≈ 0.999

- Again, very close to 1, confirming the solutions are valid.

Final Answer and Summary

The solutions to the equation K + 2/(K - 4) - 4K/(K - 4) = 1 are:

1. K ≈ (9 + ⍰ 57) / 2 ≈ 8.275
2. K ≈ (9 - ⍰ 57) / 2 ≈ 0.725

Both solutions are valid because they do not violate the restriction K ≠ 4 and satisfy the original equation upon substitution.

Key Takeaways for Solving Equations with Fractions

- Always identify restrictions on the variable to avoid extraneous solutions.
- Clear denominators carefully, multiplying through by the least common denominator (LCD).
- Simplify the resulting algebraic equations systematically, often reducing to quadratics.
- Use the quadratic formula when necessary, and evaluate the discriminant to determine the nature of solutions.
- Always check potential solutions by substituting back into the original equation to confirm their validity and ensure no division by zero occurs.

By following these steps, you can confidently solve complex rational equations and ensure your solutions are accurate and meaningful. Remember, checking for extraneous solutions is an essential part of the process to maintain mathematical integrity.

Frequently Asked Questions

How do I approach solving the equation (k + 2)/(k - 4) - 4k/(k - 4) = 1?

Start by recognizing the common denominator (k - 4), combine the fractions on the left, and then solve the resulting equation for k while checking for any extraneous solutions caused by restrictions on the denominator.

What should I do first when solving (k + 2)/(k - 4) - 4k/(k - 4) = 1?

Combine the fractions since they share the same denominator: (k + 2 - 4k)/(k - 4) = 1, simplifying the numerator to (-3k + 2).

How do I handle the equation (-3k + 2)/(k - 4) = 1?

Cross-multiply to eliminate the denominator: -3k + 2 = 1 (k - 4), then expand and solve for k.

What is the next step after cross-multiplying in the equation (-3k + 2) = (k - 4)?

Distribute and then combine like terms: -3k + 2 = k - 4, and then move all terms to one side to solve for k.

How do I solve the equation -3k + 2 = k - 4?

Subtract k from both sides: -4k + 2 = -4, then subtract 2: -4k = -6, and finally divide both sides by -4 to find k.

What is the solution for k after solving -4k = -6?

k = (-6)/(-4) = 3/2 or 1.5.

How do I check for extraneous solutions in this problem?

Substitute k = 3/2 back into the original equation to ensure the denominators are not zero and that the equation holds true.

Are there any restrictions on K in the original equation?

Yes, K cannot be 4 because it makes the denominator zero in the original equation.

Is K = 3/2 a valid solution for the original equation?

Yes, since plugging K = 3/2 into the original equation does not make any denominator zero and the equation holds true, making it a valid solution.

Additional Resources

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Introduction

Mathematics, particularly algebra, forms a foundational pillar for logical reasoning and problem-solving in numerous fields. Among the essential algebraic skills is solving rational equations — equations involving fractions with variables in the numerator, denominator, or both. These equations often appear in academic settings, standardized tests, and real-world applications, making mastery vital for students and professionals alike.

In this comprehensive review, we will examine a specific rational equation:

$$\frac{K+2}{k-4} - \frac{4k}{k-4} = 1$$

Our goal is to solve for K, identify any extraneous solutions, and understand the underlying principles necessary for solving similar equations. This process involves techniques like combining fractions, simplifying expressions, solving linear equations, and verifying solutions against domain restrictions.

Understanding the Equation

Before jumping into the solution process, it is crucial to analyze the structure of the given equation:

$$\frac{K+2}{k-4} - \frac{4k}{k-4} = 1$$

Recognizing the Variables and Constants

- The variable is K (or k), which appears in both fractions.
- The denominators involve k-4, which imposes restrictions on the domain; specifically, $k \neq 4$ to avoid division by zero.
- The equation involves a linear combination of rational expressions equaling 1.

Initial Observations

- Both fractions share the same denominator ($k-4$), making it a candidate for combining into a single fraction.
- The numerator of the combined fraction would be $(K+2) - 4k$.
- Since the goal is to solve for K, and the variables appear in the numerator as well, careful algebraic manipulation is required.

Step-by-Step Solution Process

Step 1: Combine the Fractions

Given the common denominator $k-4$, rewrite the equation as:

$$ \frac{(K+2) - 4k}{k-4} = 1 $$

This simplifies the problem to solving:

$$ \frac{K+2 - 4k}{k-4} = 1 $$

Step 2: Eliminate the Denominator

To eliminate the fraction, multiply both sides of the equation by the denominator $k-4$, considering the domain restriction $k \neq 4$:

$$ K + 2 - 4k = 1 \times (k - 4) $$

which simplifies to:

$$ K + 2 - 4k = k - 4 $$

Step 3: Isolate K

Rearranging the equation to solve for K:

$$ K + 2 - 4k = k - 4 $$

Subtract k from both sides:

$$ K + 2 - 4k - k = -4 $$

which simplifies to:

$$ K + 2 - 5k = -4 $$

Subtract 2 from both sides:

$$ K - 5k = -4 - 2 $$

$$ K - 5k = -6 $$

Now, combine like terms:

$$ (K) - 5k = -6 $$

Since K is the variable, and k is also a variable, we need to clarify whether K and k are the same or different variables.

Important clarification: In the original equation, the variable appears as both K and k. Typically, in algebraic equations, a single variable is used, but here, the notation suggests possibly different variables.

Assumption: The problem is intended to be solved for K in terms of k, or perhaps K and k are the same variable.

If K and k are the same variable, then the equation simplifies to:

$$ \frac{K+2}{K-4} - \frac{4K}{K-4} = 1 $$

which is more consistent with standard algebra practice.

Clarification and correction:

Given the potential confusion, let's assume that the variable is K, and the notation with lowercase k was a typographical inconsistency.

Therefore, the corrected equation is:

$$ \frac{K+2}{K-4} - \frac{4K}{K-4} = 1 $$

Solving the Corrected Equation

Step 1: Combine the fractions

$$ \frac{(K+2) - 4K}{K-4} = 1 $$

which simplifies to:

$$ \frac{K + 2 - 4K}{K-4} = 1 $$

$$ \frac{-3K + 2}{K-4} = 1 $$

Step 2: Cross-multiplied to remove the denominator

$$ -3K + 2 = 1 \times (K - 4) $$

$$ -3K + 2 = k - 4 $$

Step 3: Solve for K

Bring all K terms to one side:

$$ -3K - k = -4 - 2 $$

$$ -4K = -6 $$

Divide both sides by -4:

$$ K = \frac{-6}{-4} = \frac{3}{2} $$

Step 4: Check for extraneous solutions

Recall that the original denominators involve $K-4$. The solution $K = \frac{3}{2}$ does not make the denominator zero, since:

$$ K - 4 = \frac{3}{2} - 4 = \frac{3}{2} - \frac{8}{2} = -\frac{5}{2} \neq 0 $$

Thus, the solution is valid within the domain.

Verifying the Solution

Plugging $K = \frac{3}{2}$ back into the original equation:

$$ \frac{\frac{3}{2} + 2}{\frac{3}{2} - 4} - \frac{4 \times \frac{3}{2}}{\frac{3}{2} - 4} = 1 $$

Simplify numerator and denominator:

$$ \frac{\frac{3}{2} + \frac{4}{2}}{\frac{3}{2} - \frac{8}{2}} - \frac{4 \times \frac{3}{2}}{\frac{3}{2} - 4} = 1 $$

$$ \frac{\frac{7}{2}}{-\frac{5}{2}} - \frac{6}{-\frac{5}{2}} = 1 $$

Calculate each term:

$$ \frac{7/2}{-5/2} = \frac{7/2} \times \frac{-2/5} = \frac{7 \times -2}{2 \times 5} = \frac{-14}{10} = -\frac{7}{5} $$

Similarly:

$$ \frac{6}{-\frac{5}{2}} = 6 \times \frac{-2}{5} = \frac{-12}{5} $$

Now, sum the two:

$$ -\frac{7}{5} - \frac{12}{5} = -\frac{19}{5} $$

This does not equal 1, indicating a discrepancy.

Because of this inconsistency, it suggests that either:

- The algebraic manipulations need reconsideration, or
- The initial assumption about the variables being the same was incorrect.

Revisiting the Original Equation

Given the initial confusion, perhaps the original equation was:

$$ \frac{K+2}{k-4} - \frac{4k}{k-4} = 1 $$

with both K and k as variables, and the goal is to solve for K in terms of k.

In this case:

$$ \frac{K+2}{k-4} - \frac{4k}{k-4} = 1 $$

Combine the fractions:

$$ \frac{K + 2 - 4k}{k-4} = 1 $$

Multiply both sides by $k-4$:

$$ K + 2 - 4k = k - 4 $$

Now, solve for K:

$$ K = k - 4 - 2 + 4k $$

$$ K = (k + 4k) - 6 $$

$$ K = 5k - 6 $$

Thus, the solution for K in terms of k is:

$$ K = 5k - 6 $$

Domain Restrictions and Extraneous Solutions

Since the denominators involve $k - 4$, we must exclude $k = 4$ from the domain, as division by zero is undefined.

- Domain restriction: $k \neq 4$

The solution for K depends on k, and as long as $k \neq 4$, the expression is valid.

Summary and Final Remarks

- The original

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