Solving The Linear System Of Equations Using Elimination. 8x - 6y= -20-8x + 4y = 24

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Understanding how to solve systems of equations is a fundamental skill in algebra that helps in modeling real-world problems. Among the various methods available, the elimination method stands out for its efficiency and simplicity, especially when the equations are structured favorably. In this article, we will explore how to solve the specific system of linear equations:

- $8x - 6y = -20$
- $-8x + 4y = 24$

using the elimination method. We will walk through the step-by-step process, explain key concepts, and provide tips to master this technique.

Understanding the System of Equations

Before diving into the elimination method, it's important to understand what a system of equations is and how it can be solved. A system of equations consists of two or more equations with the same variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

In our case, the system is:

1. $8x - 6y = -20$
2. $-8x + 4y = 24$

Notice that these equations are linear, meaning each variable is raised to the first power, and their graphs would be straight lines.

What Is the Elimination Method?

The elimination method involves adding or subtracting the equations to eliminate one variable, thereby allowing you to solve for the remaining variable. This method is particularly effective when the coefficients of one variable are opposites or can be made opposites through multiplication.

Key steps in the elimination method include:

- Align the equations properly.
- Adjust the equations (if necessary) to create matching coefficients for one variable.
- Add or subtract the equations to eliminate that variable.
- Solve for the remaining variable.
- Substitute back to find the other variable.

Applying the Elimination Method to Our System

Let's apply the elimination method step-by-step to our system:

1. $8x - 6y = -20$
2. $-8x + 4y = 24$

Step 1: Observe the coefficients

Notice that the coefficients of x are 8 and -8, which are opposites. This is perfect for elimination of x.

Step 2: Add the equations

Adding both equations will eliminate x:

$(8x - 6y) + (-8x + 4y) = -20 + 24$

Simplify:

$(8x - 8x) + (-6y + 4y) = 4$

Which simplifies to:

$0x - 2y = 4$

or simply:

$-2y = 4$

Step 3: Solve for y

Divide both sides by -2:

$y = 4 / -2$

$y = -2$

Now we have the value of y.

Finding the Value of x

Having determined $y = -2$, substitute this value back into one of the original equations to find x.

Step 1: Use the first equation

$8x - 6y = -20$

Substitute $y = -2$:

$8x - 6(-2) = -20$

Simplify:

$8x + 12 = -20$

Step 2: Solve for x

Subtract 12 from both sides:

$8x = -20 - 12$

$8x = -32$

Divide both sides by 8:

$x = -32 / 8$

$x = -4$

Result:

The solution to the system is:

$x = -4, y = -2$

This means the point (-4, -2) is the intersection point of the two lines represented by the equations.

Verifying the Solution

It's always good practice to verify your solution by plugging the values back into both original equations.

Check in the first equation:

$8x - 6y = -20$

Plug in $x = -4$, $y = -2$:

$8(-4) - 6(-2) = -20$

$-32 + 12 = -20$

$-20 = -20$ ✓

Check in the second equation:

$-8x + 4y = 24$

Plug in $x = -4$, $y = -2$:

$-8(-4) + 4(-2) = 24$

$32 - 8 = 24$

$24 = 24$ ✓

The solution is verified.

Key Tips for Solving Systems Using Elimination

- Always look for coefficients that are opposites or can be made opposites to facilitate elimination.
- Multiply equations when necessary to align coefficients.
- Perform operations carefully, paying attention to signs.
- Check your solutions by substituting back into the original equations.
- Practice with different systems to recognize patterns and improve speed.

Common Mistakes to Avoid

- Not aligning equations properly, leading to errors in addition or subtraction.
- Overlooking signs, which can cause incorrect solutions.
- Failing to simplify equations after operations, making subsequent steps harder.
- Skipping verification, risking accepting incorrect solutions.

Additional Practice Problems

To strengthen your understanding, try solving these systems using elimination:

1. $3x + 2y = 7$ and $5x - 2y = 11$
2. $4x + y = 9$ and $2x - y = 3$
3. $6x - 4y = 8$ and $-3x + 2y = -4$

Remember to look for coefficients that can be eliminated and follow the step-by-step process outlined above.

Conclusion

The elimination method is a powerful tool for solving systems of linear equations efficiently. By carefully manipulating equations to eliminate one variable, you can quickly find the solution set. In the specific example of solving:

- $8x - 6y = -20$
- $-8x + 4y = 24$

we saw how the coefficients of x directly cancel out when added, leading to a straightforward calculation of y, followed by substitution to find x. Mastering this technique enhances your algebraic problem-solving skills and prepares you for more complex mathematical challenges.

Whether you're tackling homework problems or real-world applications, understanding and practicing the elimination method will significantly improve your ability to solve systems of equations confidently and accurately.

Frequently Asked Questions

What is the first step to solve the system of equations $8x - 6y = -20$ and $-8x + 4y = 24$ using elimination?

The first step is to align the equations and determine a way to eliminate one variable, often by adding or subtracting the equations to cancel out either x or y.

How do you eliminate the variable x in the system $8x - 6y = -20$ and $-8x + 4y = 24$?

Add the two equations together, which cancels out x because $8x + (-8x) = 0$, leaving an equation with only y to solve.

What is the result after adding the equations $8x - 6y = -20$ and $-8x + 4y = 24$?

Adding them gives $(8x - 8x) + (-6y + 4y) = -20 + 24$, which simplifies to $0x - 2y = 4$, or simply $-2y = 4$.

How do you find the value of y from the equation $-2y = 4$?

Divide both sides by -2 to get $y = 4 / -2$, which simplifies to $y = -2$.

Once y is found, how do you determine the value of x?

Substitute $y = -2$ into either original equation (for example, $8x - 6y = -20$) and solve for x.

What is the value of x when $y = -2$ in the equation $8x - 6(-2) = -20$?

Substitute $y = -2$: $8x - 6(-2) = -20$ becomes $8x + 12 = -20$. Subtract 12 from both sides: $8x = -32$. Divide both sides by 8: $x = -4$.

Additional Resources

Solving the linear system of equations using the elimination method is a fundamental skill in algebra that enables students and mathematicians alike to find solutions to simultaneous equations efficiently. This technique is particularly advantageous when the coefficients of one variable are opposites or easily manipulated to cancel out, simplifying the process of solving for the remaining variable. In this article, we will explore the step-by-step process of applying the elimination method to the given system of equations:

$$8x - 6y = -20$$
$$-8x + 4y = 24$$

By analyzing this system, we will demonstrate how to eliminate variables systematically, interpret the results, and understand the broader implications of the solutions obtained through this approach.

Understanding the System of Equations

Before diving into the elimination process, it is essential to understand the structure and components of the system:

Equation 1: $8x - 6y = -20$

Equation 2: $-8x + 4y = 24$

This system contains two equations with two variables, x and y. The goal is to determine the values of these variables that satisfy both equations simultaneously.

Key characteristics:

- The coefficients of x in the two equations are opposites (8 and -8), which immediately suggests potential for straightforward elimination.
- The coefficients of y are -6 and 4, which may require scaling for effective elimination.

Understanding these coefficients guides the choice of which variable to eliminate and how to manipulate the equations effectively.

Principles of the Elimination Method

The elimination method involves manipulating the equations—through addition, subtraction, or scaling—to cancel out one variable, leaving a single-variable equation that is easier to solve. The general principles are:

1. Align coefficients: Adjust the equations so that the coefficients of one variable are equal in magnitude but opposite in sign.
2. Add or subtract equations: Combine the equations to eliminate the targeted variable.
3. Solve for the remaining variable: Once one variable is eliminated, solve the resulting equation.
4. Back-substitute: Substitute the known value into one of the original equations to find the other variable.

This approach is favored for its clarity and efficiency, especially when coefficients are conveniently set for elimination.

Step-by-Step Solution of the Given System

Let's proceed with applying the elimination method to the specific system:

$$ 8x - 6y = -20 \quad (1) $$
$$ -8x + 4y = 24 \quad (2) $$

Step 1: Align coefficients for elimination

Notice that the coefficients of x are already opposites: 8 and -8. This makes elimination of x straightforward by adding the equations directly.

Step 2: Add the equations to eliminate x

Adding (1) and (2):

$$
(8x - 6y) + (-8x + 4y) = -20 + 24
$$

Simplify:

$$
8x - 8x - 6y + 4y = 4
$$
$$
0x - 2y = 4
$$

which simplifies to:

$$
-2y = 4
$$

Step 3: Solve for y

Divide both sides by -2:

$$
y = \frac{4}{-2} = -2
$$

Result: $ y = -2 $

Step 4: Substitute $ y = -2 $ into one of the original equations to solve for x

Choose Equation (1):

$$
8x - 6(-2) = -20
$$

Simplify:

$$
8x + 12 = -20
$$

Subtract 12 from both sides:

$$
8x = -20 - 12 = -32
$$

Divide both sides by 8:

$$
x = \frac{-32}{8} = -4
$$

Final solution:

$$
x = -4, \quad y = -2
$$

Verifying the Solution

Verification ensures the solution satisfies both equations:

- Substitute $x = -4$ and $y = -2$ into Equation (2):

$$
-8(-4) + 4(-2) = 24
$$

Calculate:

$$
32 - 8 = 24
$$

which confirms the solution is correct.

Broader Implications and Analytical Insights

The successful application of elimination in this case underscores several important points:

Efficiency of the Elimination Method

- When coefficients are opposites or easily scaled, elimination can be the quickest route to a solution.
- It requires minimal algebraic manipulation compared to substitution, especially with systems where variables are tightly coupled.

Insights into System Types

- Systems with coefficients that are multiples or negatives of each other tend to be more straightforward to solve using elimination.
- The method also reveals whether the system is consistent (has solutions), inconsistent (no solutions), or dependent (infinitely many solutions). In this case, a unique solution was found, indicating a consistent and independent system.

Educational Significance

- Mastering elimination enhances problem-solving skills, enabling students to handle more complex systems involving multiple variables.
- It provides a foundation for understanding matrix methods and linear algebra concepts, such as Gaussian elimination.

Practical Applications

- Engineering, physics, economics, and computer science frequently involve solving systems of equations.
- The elimination method, due to its systematic nature, can be easily programmed into algorithms for computational solutions.

Extensions and Variations of the Elimination Method

While the example provided was straightforward, real-world systems often involve more complex coefficients or additional variables. Variations include:

- Scaling equations: Multiplying equations to align coefficients.
- Using matrices: Representing systems as matrices and applying row operations.
- Combining methods: Employing substitution after elimination to confirm solutions or

handle more variables.

These techniques build upon the fundamental principle demonstrated here, broadening the scope and applicability of linear algebra.

Conclusion: The Power of Elimination in Systematic Problem Solving

The elimination method remains a cornerstone technique in algebra for solving systems of linear equations. Its strength lies in simplicity and efficiency, especially when coefficients are conducive to straightforward cancellation. The example of solving \(8x - 6y = -20\) and \(-8x + 4y = 24\) illustrates how strategic manipulation and understanding of coefficients can lead to rapid solution discovery.

As systems grow in complexity, the fundamental principles of elimination continue to underpin more advanced methods, including matrix algebra and computational algorithms. Mastery of this technique not only empowers students to solve equations confidently but also lays the groundwork for exploring the more intricate landscapes of linear systems in higher mathematics and applied sciences.

In essence, solving linear systems through elimination exemplifies the elegance of algebraic manipulation and systematic reasoning, vital skills for anyone delving into quantitative problem-solving across disciplines.

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