The Equation Of The Graphed Line Is X+2y=5. What Is The X-intercept Of The Graph?

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Understanding the characteristics of a linear equation is fundamental in algebra and coordinate geometry. When given the equation of a line, such as X + 2Y = 5, one common question students encounter is finding the x-intercept of the graph. The x-intercept is the point where the line crosses the x-axis, which occurs when the y-coordinate is zero. In this article, we will explore how to determine the x-intercept for the given line, understand the significance of intercepts, and provide step-by-step instructions to solve similar problems efficiently.

Understanding the Equation Of the Line: X + 2Y = 5

Before diving into the calculation of the x-intercept, it's essential to understand the form and components of the given equation. The equation X + 2Y = 5 is a linear equation in two variables, x and y. It is written in standard form, which is often expressed as Ax + By = C, where A, B, and C are constants.

Standard Form of a Linear Equation

The standard form is useful because it makes it straightforward to find intercepts by setting one variable to zero:

- When finding the x-intercept, set y = 0.
- When finding the y-intercept, set x = 0.

Significance of Intercepts in Graphing

Intercepts provide key points that help sketch the graph of the line. The x-intercept is where the line crosses the x-axis, and the y-intercept is where it crosses the y-axis. Knowing these points allows for quick graphing and understanding the behavior of the line.

How to Find the X-Intercept of the Line

To find the x-intercept for the line described by X + 2Y = 5, follow these steps:

Step 1: Set y = 0 in the Equation

Since the x-intercept occurs where the line crosses the x-axis, at that point, y is zero. Plugging y = 0 into the equation:

X + 2(0) = 5

X + 0 = 5

X = 5

Step 2: Interpret the Result

The solution X = 5 indicates that the x-intercept occurs at the point (5, 0). This is the point where the line crosses the x-axis.

Step 3: Confirm the Point

Plotting the point (5, 0) on the coordinate plane confirms its position on the line:

- At x = 5, y = 0, the original equation becomes:

5 + 2(0) = 5

5 + 0 = 5, which is true.

Therefore, the point (5, 0) lies on the line, confirming the x-intercept.

Visualizing the Graph and the Intercept

Plotting the x-intercept along with the y-intercept provides a clear visual understanding of the line's position on the coordinate plane. To find the y-intercept, repeat the process:

- Set x = 0:

0 + 2Y = 5

2Y = 5

Y = 5/2 = 2.5

- The y-intercept is at (0, 2.5).

Plotting these two points, (0, 2.5) and (5, 0), and drawing a straight line through them illustrates the graph of the equation.

Additional Methods to Find Intercepts

While setting the opposite variable to zero is the most straightforward approach, there are other methods and considerations:

Method 1: Using Slope-Intercept Form

Convert the given equation to slope-intercept form (y = mx + b):

X + 2Y = 5

2Y = -X + 5

Y = (-1/2)X + 5/2

- The y-intercept (b) is 5/2.
- To find the x-intercept, set y = 0:

0 = (-1/2)X + 5/2

(-1/2)X = -5/2

X = (-5/2) ÷ (-1/2) = (-5/2) (-2/1) = 5

This confirms the x-intercept at (5, 0).

Method 2: Graphing Approach

Plotting multiple points and drawing the line can help verify intercepts visually, especially for more complex equations.

Real-World Applications of X-Intercepts

Understanding intercepts isn't just an academic exercise; it has practical applications across various fields:

- **Economics:** Finding the break-even point where profit or loss reaches zero.
- **Physics:** Determining initial positions or thresholds where certain conditions are met.
- **Engineering:** Analyzing system behaviors at specific input values.

In all these cases, knowing how to find and interpret intercepts provides valuable insights into the problem at hand.

Summary and Key Takeaways

- The x-intercept of a line is the point where it crosses the x-axis, found by setting y = 0 in the equation.
- For the line X + 2Y = 5, substituting y = 0 yields the x-intercept at (5, 0).
- Converting to slope-intercept form can streamline the process and provide additional insights.
- Plotting the intercepts and drawing the line helps visualize the graph.

Practice Problems to Reinforce Learning

To master finding x-intercepts, try solving the following:

1. Find the x-intercept of the line 3X - 4Y = 12.
2. Determine the x-intercept of the line Y = 2X + 3.
3. For the equation 5X + 2Y = 10, find the x-intercept.

Solutions:
1. Set y=0: 3X - 4(0) = 12 □ 3X=12 □ X=4 □ (4, 0)
2. Set y=0: 0=2X+3 □ 2X=-3 □ X=-3/2 □ (-1.5, 0)
3. Set y=0: 5X + 0=10 □ X=2 □ (2, 0)

Practicing these will strengthen your ability to quickly find intercepts in various equations.

Conclusion

Finding the x-intercept of a line is a fundamental skill in algebra that provides insights into the graph's behavior and aids in visualization. For the line represented by X + 2Y = 5, the x-intercept is at (5, 0).

Mastering this process enhances your understanding of linear equations and supports problem-solving in more complex mathematical and real-world scenarios. Remember, always set the other variable to zero, solve for the remaining variable, and interpret the point to grasp the line's position fully.

Frequently Asked Questions

What is the x-intercept of the line given by the equation x + 2y = 5?

The x-intercept is at (5, 0).

How do you find the x-intercept of a line from its equation?

To find the x-intercept, set y = 0 in the equation and solve for x.

What is the process to determine the x-intercept of the equation x + 2y = 5?

Replace y with 0 and solve for x: x + 2(0) = 5, so x = 5.

What point on the graph corresponds to the x-intercept of the line x + 2y = 5?

The x-intercept is at the point (5, 0).

Can you find the x-intercept without graphing the line x + 2y = 5?

Yes, by setting y = 0 and solving for x, you find the x-intercept directly.

Why is the x-intercept important in understanding the graph of a line?

The x-intercept shows where the line crosses the x-axis, indicating the point where y=0.

If the line's equation changes to 2x + 2y = 10, what is its x-intercept?

Set y=0: 2x + 0 = 10, so x=5. The x-intercept remains at (5, 0).

How does the slope of the line relate to its x-intercept?

While the slope indicates the tilt of the line, the x-intercept is found where the line crosses the x-axis, independent of slope calculations.

Additional Resources

The Equation Of The Graphed Line Is X+2y=5. What Is The X-intercept Of The Graph?

Understanding the equation of a line and its intercepts is fundamental in algebra and coordinate geometry. This particular question focuses on identifying the x-intercept of the line represented by the equation X + 2Y = 5. Such problems are common in high school mathematics, and mastering them provides a solid foundation for more advanced topics like linear functions, graphing, and systems of equations. This article will thoroughly explore the process of finding the x-intercept from the given equation, break down the mathematical principles involved, and discuss related concepts to enhance comprehension and application skills.

Understanding the Equation of a Line

Before diving into solving for the x-intercept, it's essential to understand what the equation X + 2Y = 5 represents. This is a linear equation in two variables, x and y, which graphically corresponds to a straight line on the Cartesian coordinate plane.

Key Features of the Equation:

- It is written in standard form: Ax + By = C.
- The coefficients and constant (A=1, B=2, C=5) determine the slope and position of the line.
- It describes all points (x, y) that satisfy the equation.

Why is understanding the standard form important?

- It simplifies the process of finding intercepts.
- It allows straightforward conversion to other forms, such as slope-intercept form (y = mx + b).
- It facilitates graphing and comprehension of the line's behavior.

Finding the X-Intercept: Step-by-Step Process

The x-intercept of a line is the point where the line crosses the x-axis. At this point, the y-coordinate is zero. Therefore, to find the x-intercept, we set y = 0 in the equation and solve for x.

Step 1: Set y to zero

Given: X + 2Y = 5

Set y = 0:

x + 2(0) = 5

Simplify:

x + 0 = 5

x = 5

Result: The x-intercept is at the point (5, 0).

Step 2: Interpret the result

This means the line crosses the x-axis at x = 5, y = 0.

Summary:

- The x-intercept of the line represented by X + 2Y = 5 is at (5, 0).

Alternative Method: Converting to Slope-Intercept Form

While the direct substitution method is quick, converting the equation to slope-intercept form provides a more visual understanding of the line and confirms the intercepts.

Step 1: Solve for y

Starting with:

x + 2y = 5

Subtract x from both sides:

2y = 5 - x

Divide both sides by 2:

y = (5 - x)/2

Step 2: Find the x-intercept

Set y = 0:

0 = (5 - x)/2

Multiply both sides by 2:

0 = 5 - x

Solve for x:

x = 5

Again, the x-intercept is at (5, 0).

Graphing the Line: Visualizing the Intercept

Graphing the line based on intercepts helps in understanding its position and slope.

Steps to graph:

1. Plot the x-intercept: (5, 0).
2. Find the y-intercept:

- Set x = 0:

0 + 2y = 5

2y = 5

y = 5/2 = 2.5

- Plot point (0, 2.5).

3. Draw a straight line through these points.

Features of the graph:

- The line crosses the x-axis at (5, 0).
- It crosses the y-axis at (0, 2.5).
- The slope (m) can be calculated as:

m = (change in y) / (change in x) = (2.5 - 0) / (0 - 5) = 2.5 / -5 = -0.5

Implication: The line descends from left to right with a slope of -0.5.

Understanding the Significance of the X-Intercept

The x-intercept is a critical feature of a line, often used in various applications such as:

- Predicting behavior: It indicates where the line crosses the x-axis.
- Graphing: It helps in quick plotting.
- Solving real-world problems: For example, in economics, the x-intercept could represent the quantity at which profit or cost becomes zero.

Features and Characteristics:

Feature	Description
Location	Point where y = 0, on the x-axis
Calculated by setting y=0	Quick method to find intercepts
Relationship with slope	Intercept points help in visualizing slope and line behavior

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Related Concepts and Extensions

Understanding intercepts opens doors to various related topics:

1. Y-Intercept:

- Found by setting x=0.
- For the given equation:

$0 + 2y = 5$

$2y = 5$

$y = 5/2 = 2.5$

- So, the y-intercept is at (0, 2.5).

2. Graphing Using Intercepts:

- Plot the x-intercept (5, 0).

- Plot the y-intercept (0, 2.5).
- Connect these points to form the line.

3. Slope and Intercepts Relationship:

- Slope = (change in y) / (change in x).
- Intercepts help determine the slope visually or algebraically.

4. Generalization:

- For any linear equation, intercepts can be found by:

- Setting y=0 for x-intercept.
- Setting x=0 for y-intercept.

Advantages and Limitations of the Intercept Method

Pros:

- Simplicity: Quick calculations by substituting zero.
- Visualization: Facilitates easy graphing.
- Educational Value: Reinforces understanding of coordinate axes and line behavior.

Cons:

- Limited to linear equations: More complex equations may require additional methods.
- Potential for errors: Mistakes in substitution or algebra can lead to wrong intercepts.
- Not always sufficient: When lines are vertical or horizontal, intercepts are at infinity or zero, requiring

special handling.

Special Cases and Considerations

Vertical Lines:

- Equation form: x = constant.
- No y-intercept unless x equals the constant.
- For example: x = 5 is a vertical line crossing the x-axis at (5, 0).

Horizontal Lines:

- Equation form: y = constant.
- The x-intercept depends on y = 0.
- For example: y = 2 is a horizontal line crossing the y-axis at (0, 2).

Line with no x-intercept:

- Vertical lines where x = c, but c 0, do not cross x-axis.
- E.g., x = 3 does not intersect the x-axis, as y can be any value.

In the context of X + 2Y = 5, the line is neither vertical nor horizontal, and the intercepts are straightforward to find.

Real-World Applications

Knowing how to find the x-intercept has practical importance:

- Economics: Break-even analysis where the intercept indicates the point at which profit is zero.
- Physics: Determining initial conditions or zero points in motion graphs.
- Biology: Identifying thresholds in dose-response curves.
- Engineering: Analyzing load distributions or system responses.

Conclusion

The process of determining the x-intercept of the line given by X + 2Y = 5 exemplifies essential algebraic skills and geometric understanding. By setting y to zero and solving for x, we find that the line crosses the x-axis at (5, 0). This straightforward method underscores the beauty of algebraic manipulation and its power in visualizing and analyzing graphs. Whether approached directly or through conversion to slope-intercept form, identifying intercepts remains a fundamental technique in mathematics, with widespread applications across science, engineering, and economics. Mastering this skill enhances problem-solving efficiency and deepens comprehension of linear relationships, making it an indispensable part of any mathematical toolkit.

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