The Value 5pi/4 Is A Solution For The Equation 3 Sqrt Sin Theta +2=-1true Or False

The Value 5pi/4 Is A Solution For The Equation 3 Sqrt Sin Theta +2=-1true Or False

When exploring solutions to trigonometric equations, it's essential to verify whether specific angle values satisfy the given equations. One such question involves determining if the value $\frac{5\pi}{4}$ is a solution to the equation $3 \sqrt{\sin \theta} + 2 = -1$. At first glance, this may seem straightforward, but a closer look reveals the importance of understanding the properties of sine, square roots, and the structure of the equation itself. This article delves into evaluating whether $\frac{5\pi}{4}$ is a solution for the equation and clarifies the reasoning behind the answer, offering insights into solving similar trigonometric equations.

Understanding the Equation: $3 \sqrt{\sin \theta} + 2 = -1$

Before assessing whether $\frac{5\pi}{4}$ is a solution, it’s crucial to analyze the components and structure of the equation:

1. The Role of the Square Root Function

The equation involves $\sqrt{\sin \theta}$, which introduces a key consideration: the domain of the square root function. Since the square root of a negative number is not real, $\sqrt{\sin \theta}$ is only defined for $\sin \theta \geq 0$. This restriction is fundamental in determining possible solutions, as the expression $3 \sqrt{\sin \theta} + 2$ must be real-valued.

2. Range of the Expression

Given that $\sqrt{\sin \theta} \geq 0$ for $\sin \theta \geq 0$, the expression $3 \sqrt{\sin \theta} + 2$ will always be greater than or equal to 2 because:

- The minimum value of $\sqrt{\sin \theta}$ is 0.
- The maximum value of $\sqrt{\sin \theta}$ is 1 (since $\sin \theta \leq 1$).

Therefore, the range of $3 \sqrt{\sin \theta} + 2$ is from 2 to 5.

3. The Right-Hand Side (-1)

The equation states that the expression equals -1, which raises an immediate issue: since $3 \sqrt{\sin \theta} + 2 \geq 2$, it can never be equal to -1, which is less than 2.

Evaluating Whether $\frac{5\pi}{4}$ Is a Solution

Now, with the understanding of the equation's structure, let's analyze whether substituting $\theta = \frac{5\pi}{4}$ satisfies the equation.

1. Find $\sin \left(\frac{5\pi}{4} \right)$

The angle $\frac{5\pi}{4}$ radians corresponds to 225°, a known point on the unit circle.

- $\sin \left(\frac{5\pi}{4} \right) = -\frac{\sqrt{2}}{2}$

Since this sine value is negative, it immediately conflicts with the domain requirement for $\sqrt{\sin \theta}$, which mandates $\sin \theta \geq 0$.

2. Implication for the Square Root

Because $\sin \left(\frac{5\pi}{4} \right)$ is negative, $\sqrt{\sin \left(\frac{5\pi}{4} \right)}$ is undefined in the real numbers.

This means that:

- The expression $3 \sqrt{\sin \theta} + 2$ cannot be evaluated at $\theta = \frac{5\pi}{4}$ in the real domain.

3. Conclusion on the Solution

Given that the expression is undefined at $\theta = \frac{5\pi}{4}$, it cannot satisfy the equation $3 \sqrt{\sin \theta} + 2 = -1$. Therefore, $\frac{5\pi}{4}$ is not a solution.

Is $\frac{5\pi}{4}$ a Solution? True or False?

Based on the analysis above, the answer is straightforward:

- Since $\sin \left(\frac{5\pi}{4} \right)$ is negative, $\sqrt{\sin \theta}$ is undefined for $\theta = \frac{5\pi}{4}$ in real numbers.
- The left side of the equation cannot be evaluated at this angle, making it impossible to satisfy the equation.
- The right side of the equation is a negative number (-1), but the left side's minimum value is 2, which is always positive.

Therefore, the statement: "The value $\frac{5\pi}{4}$ is a solution for the equation $3 \sqrt{\sin \theta} + 2 = -1$" is false.

General Tips for Solving Similar Trigonometric Equations

When working with equations involving square roots and sine functions, keep these tips in mind:

1. Always Check the Domain

- For $\sqrt{\sin \theta}$, ensure $\sin \theta \geq 0$.
- For other functions, verify their domains to avoid extraneous solutions or undefined expressions.

2. Understand the Range of the Expression

- Determine the minimum and maximum values the expression can take.
- Use this to assess whether the equation is solvable for given values.

3. Use Unit Circle Values

- Remember the sine values for common angles (e.g., $0, \pi/2, \pi, 3\pi/2, 2\pi$) to evaluate potential solutions.

4. Be Aware of Extraneous Solutions

- Squaring both sides or manipulating equations can introduce solutions outside the original domain.
- Always verify solutions within the original context.

Conclusion

In summary, the equation $3 \sqrt{\sin \theta} + 2 = -1$ cannot be satisfied at $\theta = \frac{5\pi}{4}$ because the sine of $\frac{5\pi}{4}$ is negative, rendering $\sqrt{\sin \theta}$ undefined in the real numbers, and the overall left side cannot be negative to match -1. This reasoning confirms that the claim "The value $5\pi/4$ is a solution for the equation" is false.

Understanding the properties of trigonometric functions, their domains, and ranges is crucial for solving equations accurately. Always verify whether the given angle values fall within the permissible domain before attempting to substitute them into the equation. This approach prevents misconceptions and ensures precise solutions in trigonometry.

In conclusion:

The statement is false: $\frac{5\pi}{4}$ is not a solution for the equation $3 \sqrt{\sin \theta} + 2 = -1$.

Frequently Asked Questions

Is $5\pi/4$ a solution to the equation $3\sqrt{\sin \theta} + 2 = -1$?

False. When $\theta = 5\pi/4$, $\sin \theta = -\sqrt{2}/2$. Substituting into the equation results in a value greater than -1, so it does not satisfy the equation.

How do I verify if $5\pi/4$ satisfies the equation $3\sqrt{\sin \theta} + 2 = -1$?

Calculate $\sin(5\pi/4)$, then compute $3\sqrt{\sin(5\pi/4)} + 2$ and see if it equals -1. Since $\sin(5\pi/4) = -\sqrt{2}/2$, the expression does not equal -1, so $5\pi/4$ is not a solution.

What is the value of $\sin(5\pi/4)$?

$\sin(5\pi/4) = -\sqrt{2}/2$.

Can the expression $3\sqrt{\sin \theta} + 2$ equal -1 for any real θ?

Yes, but only if the value inside the square root makes the entire expression equal to -1. Since $\sqrt{\sin \theta}$ is only real when $\sin \theta \geq 0$, and for negative $\sin \theta$, the square root is imaginary, so the equation does not hold for negative $\sin \theta$.

Is the equation $3\sqrt{\sin\theta} + 2 = -1$ valid for $\theta = 5\pi/4$?

No, because at $\theta = 5\pi/4$, $\sin\theta = -\sqrt{2}/2$, which makes $\sqrt{\sin\theta}$ invalid in the real numbers, and the left side does not equal -1.

What is the domain concern when evaluating $\sqrt{\sin\theta}$ in this equation?

Since the square root function requires non-negative inputs in the real numbers, $\sin\theta$ must be ≥ 0 for $\sqrt{\sin\theta}$ to be real. For negative $\sin\theta$, the expression becomes imaginary.

Does the equation $3\sqrt{\sin\theta} + 2 = -1$ have real solutions?

It depends on the value of $\sin\theta$. For real solutions, $\sqrt{\sin\theta}$ must be real, and the equation must satisfy the equality. Typically, this equation has solutions only when $\sin\theta \geq 0$.

What is the correct way to test if a specific angle, like $5\pi/4$, satisfies the equation?

Calculate $\sin(\theta)$, then compute $3\sqrt{\sin\theta} + 2$; if the result equals -1, then θ is a solution. For $5\pi/4$, since $\sin\theta$ is negative, it does not satisfy the equation in real numbers.

Is the statement '$5\pi/4$ is a solution for the equation $3\sqrt{\sin\theta} + 2 = -1$' true or false?

False. Because $\sin(5\pi/4)$ is negative, and the square root of a negative number is not real, so $5\pi/4$ does not satisfy the equation in real numbers.

Additional Resources

Understanding the Value $5\pi/4$ as a Solution to the Equation $3\sqrt{\sin\theta} + 2 = -1$: A Detailed Review

---

Introduction

Mathematics, especially trigonometry, often involves solving equations that include sine functions, square roots, and constant coefficients. One common challenge is determining whether a specific angle value satisfies a given equation. In this review, we explore whether $5\pi/4$ (which is 225° in degrees) is a solution to the equation:

$$3 \sqrt{\sin \theta} + 2 = -1$$

This problem combines several important concepts: the properties of the sine function, the implications of square roots, and the conditions for solutions to trigonometric equations. We will analyze the equation step-by-step, examine the properties of the involved functions, and ultimately decide whether the statement is true or false.

Fundamental Concepts & Background

Before diving into the specific problem, it’s essential to understand some core principles:

Sine Function ($\sin \theta$)

- The sine of an angle θ, in the context of the unit circle, oscillates between -1 and 1:

$$-1 \leq \sin \theta \leq 1$$

- The sine function is positive in the first and second quadrants (0 to π), and negative in the third and fourth quadrants (π to 2π).

Square Root of Sin θ ($\sqrt{\sin \theta}$)

- The square root function, $\sqrt{x}$, is only defined for $x \geq 0$ in the set of real numbers.
- Therefore, for $\sqrt{\sin \theta}$ to be real-valued, we must have:

$$\sin \theta \geq 0$$

- If $\sin \theta$ is negative, $\sqrt{\sin \theta}$ is not real, which impacts the solvability of the equation over real numbers.

Implication for the Equation

- The original equation involves $\sqrt{\sin \theta}$, which constrains the possible solutions to those where $\sin \theta \geq 0$.
- The entire expression:

$$3 \sqrt{\sin \theta} + 2$$

will then be constrained to certain ranges, depending on the value of $\sin \theta$ in $[0,1]$.

Analyzing the Equation

Let’s restate the equation:

$$3 \sqrt{\sin \theta} + 2 = -1$$

The goal is to determine if $\theta = 5\pi/4$ satisfies this equation.

Step 1: Isolate $\sqrt{\sin \theta}$

Subtract 2 from both sides:

$$3 \sqrt{\sin \theta} = -1 - 2$$
$$3 \sqrt{\sin \theta} = -3$$

Divide both sides by 3:

$$\sqrt{\sin \theta} = -1$$

Step 2: Analyze the Result

The key point here is the value of $\sqrt{\sin \theta}$:

- Since $\sqrt{\sin \theta} \geq 0$ (for real solutions), the equation:

$$\sqrt{\sin \theta} = -1$$

cannot be true because the square root of any real number ≥ 0 cannot be negative.

- Therefore, the only way for the equation to hold is if the right side is non-negative, which it isn't.

Conclusion from Step 2

- The equation has no real solutions because it implies that $\sqrt{\sin \theta}$ must be negative, which is impossible.

Evaluating θ = 5π/4

Given the above, we now check whether $\theta = 5\pi/4$ (225°) satisfies the original equation.

Step 1: Find sin(5π/4)

- The sine of 5π/4:

$$\sin \left(\frac{5\pi}{4} \right) = -\frac{\sqrt{2}}{2} \approx -0.7071$$

- This is a negative value, less than zero.

Step 2: Can √sin θ be evaluated for θ = 5π/4?

- Since $\sin(5\pi/4)$ is negative, $\sqrt{\sin(5\pi/4)}$ involves $\sqrt{-0.7071}$, which is not a real number.

- In the real number system, $\sqrt{\sin \theta}$ is undefined for negative $\sin \theta$.

- Therefore, the expression $3 \sqrt{\sin \theta} + 2$ is not real-valued at $\theta = 5\pi/4$.

Implication

- Because $\sqrt{\sin \theta}$ is undefined for $\sin \theta < 0$ in real numbers, $\theta = 5\pi/4$ cannot be a solution to the original equation over the real numbers.

Is the Equation True or False for $\theta = 5\pi/4$?

Based on the above analysis:

- The calculation of $\sin(5\pi/4)$ yields a negative value.
- The square root of a negative number is not defined over the real numbers, meaning the expression $3 \sqrt{\sin \theta} + 2$ is undefined at $\theta = 5\pi/4$.
- Since the question asks whether $5\pi/4$ is a solution to the equation, the answer must consider the domain of the functions involved.

Conclusion:

- In the real number system, $\theta = 5\pi/4$ does not satisfy the equation because $\sqrt{\sin \theta}$ is undefined.
- Therefore, the statement "$5\pi/4$ is a solution for the equation $3\sqrt{\sin \theta} + 2 = -1$" is false.

Additional Considerations

While the previous conclusion is based on the real numbers, it's worth noting some additional points:

Complex Numbers Perspective

- If we extend to complex numbers, then $\sqrt{\sin\theta}$ for negative $\sin\theta$ can be defined via complex roots.
- However, such an extension goes beyond the scope of standard trigonometric equations and typical problem contexts.

Domain Restrictions & Practical Implications

- Most standard trigonometric equations are considered over the domain where $\sqrt{\sin\theta}$ is real-valued.
- The restriction $\sin\theta \geq 0$ ensures the square root is well-defined and real.

Summary of Key Points

- The equation involves $\sqrt{\sin\theta}$, which requires $\sin\theta \geq 0$ for real solutions.
- At $\theta = 5\pi/4$, $\sin\theta$ is negative, making the square root undefined over the reals.
- The right side of the equation is negative, but $\sqrt{\sin\theta}$ cannot be negative, eliminating the possibility of a real solution at $\theta = 5\pi/4$.
- Hence, the statement that "$5\pi/4$ is a solution" is False in the real number context.

Final Verdict

The statement: "The value $5\pi/4$ is a solution for the equation $3\sqrt{\sin \theta} + 2 = -1$" is False.

This conclusion is grounded in the fundamental properties of square roots in real numbers and the behavior of the sine function at $\theta = 5\pi/4$. The analysis confirms that the equation has no solution at this particular angle within the real domain, primarily due to the negative sine value, which renders $\sqrt{\sin \theta}$ undefined.

Summary & Takeaways

- **When dealing with equations involving $\sqrt{\sin \theta}$, always check the domain restrictions.**
- **For real solutions, $\sin \theta$ must be ≥ 0.**
- **At $\theta = 5\pi/4$, $\sin \theta$ is negative, so $\sqrt{\sin \theta}$ is undefined.**
- **The algebraic manipulation shows the equation cannot be satisfied for any θ where $\sqrt{\sin \theta}$ is real, including $5\pi/4$.**
- **Therefore, the assertion that $5\pi/4$ is a solution is false.**

In essence, understanding the domain constraints and properties of the functions involved is crucial when solving and evaluating trigonometric equations.

[The Value 5pi 4 Is A Solution For The Equation 3 Sqrt Sin Theta 2 1true Or False]

The Value 5pi 4 Is A Solution For The Equation 3 Sqrt Sin Theta 2 1true Or False

Related Articles

- A Vehicle Start Moving At 15m/s. How Long Will It Take To Stop At A Distance Of 15m?
- The Tape Diagram Represents An Equation. Write An Equation To Represent The Image.
- Frida Owed$75onher Credit Card. Then She Charged$21more. What Was Her Newbalance?

Back to Home