# True/False: Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution

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## Introduction

Understanding the rotational dynamics of scientific instruments such as radiometers is essential for both theoretical physics and practical applications. A radiometer, particularly the Crookes radiometer, is a device that demonstrates the conversion of electromagnetic radiation into mechanical motion. When analyzing its operation, one interesting problem is determining the time it takes for the radiometer's vanes or rotor to complete a single revolution. This involves deriving an equation based on the physical principles governing its motion, such as torque, rotational inertia, and damping forces.

In this article, we will explore how to derive an equation for the time required for a radiometer to accomplish one full rotation. We will examine relevant physics concepts, develop the mathematical framework, and discuss factors influencing the rotational period of a radiometer.

## Fundamental Concepts

Before deriving the equation, it is crucial to understand some fundamental physics concepts:

### Rotational Motion

- Angular Displacement ($\theta$): The angle turned by the rotor, measured in radians.
- Angular Velocity ($\omega$): The rate of change of angular displacement, given in radians per second.
- Angular Acceleration ($\alpha$): The rate of change of angular velocity, in radians per second squared.
- Moment of Inertia ($I$): The property of the rotor that measures its resistance to angular acceleration, depending on mass distribution.

### Forces and Torques in a Radiometer

- Driving Torque ($\tau_{drive}$): The torque generated by absorbed

radiation causing the rotor to spin.
- Resisting Torque ($\tau_{resist}$): The damping or frictional torque opposing motion, often modeled as proportional to angular velocity.
- Net Torque ($\tau_{net}$): The difference between driving and resisting torques.

# Modeling the Radiometer's Rotation

To derive the time for one revolution, we need to model the radiometer's rotational dynamics. Assuming the system starts from rest, accelerates under a net torque, and eventually reaches a steady rotational speed or completes a full rotation, we must analyze the motion accordingly.

## Assumptions for Simplification

- The torque driving the rotation is constant during acceleration.
- Damping torque is proportional to angular velocity ($\tau_{damping} = -b \omega$), where $b$ is a damping coefficient.
- The system is isolated, with no external torques other than those considered.
- The initial angular velocity is zero ($\omega_0 = 0$).

# Deriving the Equation

The key to determining the time for one revolution is understanding the rotational motion governed by the differential equation:

$$
I \frac{d\omega}{dt} = \tau_{drive} - b\omega
$$

This is a first-order linear differential equation describing the angular velocity's evolution over time.

## Step 1: Write the Differential Equation

$$
I \frac{d\omega}{dt} + b\omega = \tau_{drive}
$$

Dividing both sides by $I$:

$$
\frac{d\omega}{dt} + \frac{b}{I}\omega = \frac{\tau_{drive}}{I}
$$

Set:

$$
\lambda = \frac{b}{I}, \quad \text{and} \quad \tau = \frac{\tau_{drive}}{I}
$$

The equation becomes:

$$
\frac{d\omega}{dt} + \lambda \omega = \tau
$$

## Step 2: Solve the Differential Equation

Using an integrating factor $e^{\lambda t}$:

$$
\frac{d}{dt} \left( \omega e^{\lambda t} \right) = \tau e^{\lambda t}
$$

Integrate both sides:

$$
\omega e^{\lambda t} = \frac{\tau}{\lambda} e^{\lambda t} + C
$$

Applying initial condition $\omega(0) = 0$:

$$
0 = \frac{\tau}{\lambda} + C \Rightarrow C = -\frac{\tau}{\lambda}
$$

Solution:

$$
\omega(t) = \frac{\tau}{\lambda} \left( 1 - e^{-\lambda t} \right)
$$

Re-substitute $\tau$ and $\lambda$:

$$
\omega(t) = \frac{\tau_{drive} / I}{b / I} \left( 1 - e^{-(b/I)t} \right) = \frac{\tau_{drive}}{b} \left( 1 - e^{-(b/I)t} \right)
$$

## Step 3: Find the Time to Complete One Revolution

The angular displacement $\theta(t)$ is given by integrating $\omega(t)$:

$$
\theta(t) = \int_0^t \omega(t') dt' = \int_0^t \frac{\tau_{drive}}{b} \left(
$$

1 - e^{-(b/I)t'} \right) dt'
\]

This integral evaluates to:

$$
\theta(t) = \frac{\tau_{drive}}{b} \left[ t + \frac{I}{b} e^{-(b/I)t'} \right]_0^t
$$

Calculating:

$$
\theta(t) = \frac{\tau_{drive}}{b} \left( t + \frac{I}{b} e^{-(b/I)t} - \frac{I}{b} \right)
$$

Simplified:

$$
\theta(t) = \frac{\tau_{drive}}{b} t + \frac{\tau_{drive} I}{b^2} \left( e^{-(b/I)t} - 1 \right)
$$

The goal is to find $t$ such that $\theta(t) = 2\pi$ radians (one full revolution):

$$
2\pi = \frac{\tau_{drive}}{b} t + \frac{\tau_{drive} I}{b^2} \left( e^{-(b/I)t} - 1 \right)
$$

This transcendental equation can be solved numerically for $t$, which gives the time for the radiometer to complete one revolution.

# Special Cases and Approximations

Depending on the physical parameters, certain approximations can simplify the calculation:

## Case 1: High Damping ($b$ large)

- The exponential term decays rapidly.
- The rotor quickly reaches a steady angular velocity $\omega_{steady} = \frac{\tau_{drive}}{b}$.

In this case, the time for one revolution approximates to:

\[

$$
T \approx \frac{2\pi}{\omega_{steady}} = \frac{2\pi b}{\tau_{drive}}
$$

### Case 2: Low Damping ($b$ small)

- The exponential term remains significant.
- The rotor accelerates over a longer period, and the full expression must be used.

## Practical Factors Influencing the Rotation Time

While the theoretical equations provide a foundation, real-world radiometers are influenced by several additional factors:

- **Radiation Intensity:** Higher radiation increases $\tau_{drive}$, reducing the revolution time.
- **Rotor Mass and Geometry:** Larger or more massive vanes increase $I$, resulting in longer rotational periods.
- **Friction and Air Resistance:** Additional damping forces can alter $b$, affecting the speed.
- **Material Properties:** The thermal and optical properties influence the magnitude of the driving torque.

## Conclusion

In summary, deriving an equation for the time it takes a radiometer to complete one revolution involves solving the rotational motion differential equation considering driving torque, damping, and inertia. The derived expression provides insight into how physical parameters influence the rotational period and can be adapted to various scenarios by adjusting assumptions.

The key takeaway is that the time for one revolution, $T$, can be found by solving:

$$
2\pi = \frac{\tau_{drive}}{b} t + \frac{\tau_{drive} I}{b^2} (e^{-(b/I)t} - 1)
$$

which often requires numerical methods due to its transcendental nature. Understanding this relationship enables scientists and engineers to predict and optimize the performance of radiometers and similar rotational devices, whether for scientific experiments or practical applications.

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In conclusion, the statement "True/False: Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution" is true when approached through the principles of rotational dynamics and differential equations, as demonstrated above.

# Frequently Asked Questions

## What is the main goal when deriving an equation for the time it takes a radiometer to complete one revolution?

The main goal is to relate the rotational period of the radiometer to the physical parameters such as torque, moment of inertia, and angular acceleration, enabling calculation of the time per revolution.

## True or False: The time for one revolution depends solely on the radiometer's moment of inertia.

False. While the moment of inertia influences the period, factors like torque and angular acceleration also determine the time for one revolution.

## Which physical quantities are essential in deriving the equation for the radiometer's revolution time?

Torque, moment of inertia, angular acceleration, and initial angular velocity are essential quantities for deriving the equation.

## True or False: If the radiometer experiences constant angular acceleration, the time for one revolution can be calculated using kinematic equations.

True. Kinematic equations for rotational motion with constant angular acceleration can be used to derive the revolution time.

## How does the torque applied to the radiometer influence the time it takes to complete one

## revolution?

A higher torque results in greater angular acceleration, reducing the time for one revolution; conversely, lower torque increases the period.

## True or False: The derived equation for revolution time assumes no external resistive forces acting on the radiometer.

True. The derivation typically assumes ideal conditions without external damping or resistive torques.

## Can the equation for the time per revolution be expressed in terms of initial conditions and applied torque?

Yes. The equation can be expressed using initial angular velocity, torque, and moment of inertia to determine the period.

## What is the significance of the moment of inertia in determining the revolution time?

The moment of inertia quantifies the radiometer's resistance to changes in rotational motion and directly influences the time to complete one revolution.

## True or False: Deriving the equation for the revolution time involves integrating angular acceleration over the angular displacement.

True. The derivation involves integrating angular acceleration to find angular displacement and then solving for time.

## Is the derived equation for the radiometer's revolution time applicable under non-constant torque conditions?

No. The derived equation typically assumes constant torque and angular acceleration; non-constant torque requires more complex analysis.

# Additional Resources

True/False: Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution is a statement that invites a comprehensive exploration of radiometer mechanics, rotational dynamics, and the physics behind their operation. At first glance, the question appears straightforward—can we derive a precise equation for the time it takes for a radiometer to complete one full revolution? The answer is nuanced, depending on the specific type of radiometer, the forces involved, and the assumptions made during derivation. In this article, we will analyze the principles governing radiometer rotation, examine the factors influencing their rotational period, and explore the theoretical derivation of such an equation, if possible.

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## Understanding Radiometers: An Overview

Radiometers are devices designed to detect electromagnetic radiation, typically light, and convert it into measurable signals. The most famous example is the Crookes radiometer, often called the "light mill," which features a glass bulb with a set of vanes mounted on a spindle. These vanes are black on one side and shiny on the other, and the device spins when exposed to light.

Types of Radiometers:
- Crookes Radiometer: Uses thermal effects caused by radiation absorption to produce motion.
- Pneumatic or Gas Radiometers: Use gas molecules reacting to radiation pressure.
- Photonic or Optical Radiometers: Detect radiation intensity electronically without mechanical motion.

This review focuses primarily on the Crookes radiometer, as it exhibits mechanical rotation directly observable and quantifiable, making the derivation of the time per revolution a meaningful pursuit.

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## Principles Behind the Rotation of the Crookes Radiometer

The rotation of a Crookes radiometer is primarily driven by thermal effects stemming from radiation absorption:

- Absorption of Light: The black sides of the vanes absorb more radiation, heating up faster than the shiny sides.
- Thermal Gas Effect: The heated black side causes a temperature gradient, resulting in differential gas molecule impacts on each side, generating a net force.
- Radiation Pressure: Although minimal, radiation pressure can contribute to

the motion.
- Thermal Convection and Outgassing: The heating can cause local convection currents or outgassing, influencing motion.

The combined effects produce a torque that causes the vanes to spin. Over time, this torque balances with resistive forces such as friction and air resistance, reaching a steady rotational speed.

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# Forces and Torques Acting on the Radiometer

To derive an equation for the period of one revolution, it is essential to understand the dynamics involved:

- Driving Torque ($\tau$): Originates mainly from thermal effects, proportional to incident radiation intensity and vane properties.
- Resistive Torque ($\tau_f$): Due to friction in the spindle, air viscosity, and other damping forces.
- Angular Momentum ($L$): Related to the rotational inertia and angular velocity.

Mathematically, the net torque ($\tau_{net}$) acting on the radiometer's vanes determines their angular acceleration according to Newton's second law for rotation:

$$ I \frac{d\omega}{dt} = \tau_{net} = \tau_{drive} - \tau_{resist} $$

where:
- $I$ is the moment of inertia,
- $\omega$ is the angular velocity.

In steady state, $d\omega/dt = 0$ and $\tau_{drive} = \tau_{resist}$, which defines the equilibrium angular velocity, $\omega_{eq}$.

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# Deriving the Equation for the Time per Revolution

The goal is to find $T$, the period for one full revolution, given by:

$$ T = \frac{2\pi}{\omega_{eq}} $$

To do this, we need expressions for $\omega_{eq}$. The derivation involves several steps:

Step 1: Model the Driving Torque

Assuming the driving torque is proportional to the incident radiation power $P$, the vane area $A$, and an efficiency factor $\eta$:

$$\tau_{drive} = \eta P A$$

This simplified relation ignores complex thermal transfer mechanisms but provides a starting point.

Step 2: Model the Resistive Torque

Resistive torque often results from viscous damping and friction, which can be modeled as:

$$\tau_{resist} = c \omega$$

where:
- $c$ is a damping coefficient dependent on air viscosity, spindle friction, and vane design.

Step 3: Equilibrium Condition

At steady state:

$$\eta P A = c \omega_{eq}$$

which yields:

$$\omega_{eq} = \frac{\eta P A}{c}$$

Step 4: Calculate the Period

Substituting into the period formula:

$$T = \frac{2\pi}{\omega_{eq}} = \frac{2\pi c}{\eta P A}$$

This equation relates the period of one revolution to physical and environmental parameters.

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## Features, Pros, and Cons of the Derived Equation

Features:

- Linear dependence: The period $T$ is inversely proportional to the

incident power $P$, indicating that higher radiation intensity results in faster rotation.
- Parameter sensitivity: Changes in damping coefficient $c$, efficiency $\eta$, or vane area $A$ directly influence the period.
- Simplified model: Assumes steady state and neglects transient behaviors and complex thermal dynamics.

Pros:

- Predictive capability: Offers a theoretical basis for estimating rotation time based on measurable parameters.
- Design utility: Helps in optimizing radiometer design by understanding how modifications affect rotation speed.

Cons:

- Oversimplification: Real systems involve complex thermal and fluid dynamics not captured here.
- Environmental dependence: Factors like air pressure, temperature, and spindle friction are difficult to accurately quantify.
- Limited accuracy: The model predicts steady-state behavior but not transient start-up times or variations.

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# Limitations and Real-World Considerations

While the derived equation provides valuable insight, several limitations should be acknowledged:

- Thermal Dynamics: Actual thermal transfer involves conduction, convection, and radiation, which are complex to model precisely.
- Air Resistance Variability: Changes in ambient pressure and temperature significantly affect damping.
- Material Properties: Variations in vane material and surface coating influence absorption and thermal response.
- Nonlinear Effects: At higher rotation speeds, nonlinear effects and turbulence may alter the torque balance.

In practice, experimental measurements often supplement theoretical derivations to obtain accurate rotation periods.

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# Conclusion

True/False: Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution is indeed a feasible endeavor under certain assumptions and simplifications. By analyzing the forces and torques involved—primarily thermal absorption effects and damping forces—we can derive an approximate expression for the steady-state angular velocity and, consequently, the period of one full rotation. The derived equation:

$$ T = \frac{2\pi c}{\eta P A} $$

captures the core dependencies but must be used with caution due to its simplified nature. It serves as a foundational model to understand the physics governing radiometer motion and highlights the importance of environmental and material parameters. For precise applications, empirical data and more sophisticated modeling are essential.

This exploration underscores the rich interplay between thermal physics, fluid mechanics, and rotational dynamics, illustrating how fundamental equations can illuminate the behavior of seemingly simple devices like the Crookes radiometer.

## True False Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution

True False Derive An Equation For The Time It Takes The Radiometer To Do 1 Revolution

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