Use The Chain Rule To Find Dw/dt.w = Xey/z, X = T9, Y = 2 T, Z = 7 + 8tdw/dt=____

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Understanding how to differentiate complex functions involving multiple variables is a fundamental skill in calculus, particularly when dealing with composite functions. The chain rule provides a systematic way to handle the differentiation of such functions, especially when variables depend on a common parameter, often time t. In this article, we will explore the application of the chain rule to evaluate the derivative $\frac{dw}{dt}$ of a given function involving multiple variables, and we will interpret the problem step-by-step, ensuring clarity in each part of the process.

Introduction to the Chain Rule

The chain rule is a core concept in differential calculus used to differentiate composite functions. It states that if a variable w depends on y, which in turn depends on z, and z depends on t, then the derivative of w with respect to t can be expressed as:

$$
\frac{dw}{dt} = \frac{dw}{dy} \times \frac{dy}{dz} \times \frac{dz}{dt}
$$

This rule extends naturally to functions with multiple variables that are functions of a common parameter, such as time t. The key is to identify all the dependencies and differentiate accordingly.

Understanding the Given Function and Variables

The problem presents a function:

$$
w = X e^{y/z}
$$

with the following variable definitions:

- $X = T^9$
- $Y = 2T$
- $Z = 7 + 8 t \frac{dw}{dt}$

and the goal is to compute:

$$
\frac{dw}{dt}
$$

at a certain point, with the relation:

$$
\frac{dw}{dt} = ?
$$

Notice that the problem involves multiple variables X, Y, Z, some of which depend on T and t. The key is to understand the interdependencies and apply the chain rule accordingly.

Step-by-Step Solution Approach

To solve for $\frac{dw}{dt}$, we need to:

1. Express w explicitly in terms of T, y, and z.
2. Identify how each variable depends on t.
3. Compute derivatives of each component with respect to t.
4. Combine these derivatives following the chain rule.

Let's analyze each part carefully.

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1. Expressing w in terms of variables

Given:

$$
w = X e^{y/z}
$$

where:

- \(X = T^9 \)
- \(y = 2T \)
- \(z = 7 + 8 t \frac{dw}{dt} \)

Here, \(w \) appears on both sides because \(z \) involves \(\frac{dw}{dt}\). This indicates the derivative is defined implicitly, and we need to resolve this relationship carefully.

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2. Understanding the dependencies

- \(X = T^9 \): depends on \(T \).
- \(y = 2T \): depends on \(T \).
- \(z = 7 + 8 t \frac{dw}{dt} \): depends on \(t \) and \(\frac{dw}{dt}\).

Since \(T \) is a function of \(t \) (though not explicitly given), we may assume \(T \) is either constant or a known function of \(t \). For the sake of this problem, let's assume \(T \) is constant unless specified otherwise, simplifying the derivatives.

Calculating the Derivative \(\frac{dw}{dt}\)

Applying the chain rule to \(w = X e^{y/z} \), we recognize that \(w \) is a composite function of \(T \), \(y \), and \(z \), with \(z \) depending on \(t \) and \(\frac{dw}{dt}\). To resolve the implicit nature, it's helpful to differentiate both sides of the equation with respect to \(t \):

\[
\frac{d}{dt} w = \frac{d}{dt} \left(X e^{y/z} \right)
\]

Using the product rule:

\[
\frac{dw}{dt} = \frac{dX}{dt} e^{y/z} + X \frac{d}{dt} \left(e^{y/z} \right)
\]

Since \(X = T^9 \), and assuming \(T \) is constant, then:

\[
\frac{dX}{dt} = 0
\]

Thus:

$$
\frac{dw}{dt} = X \frac{d}{dt} \left(e^{y/z} \right)
$$

Next, differentiate $e^{y/z}$:

$$
\frac{d}{dt} \left(e^{y/z} \right) = e^{y/z} \times \frac{d}{dt} \left(y/z \right)
$$

Now, focus on differentiating y/z:

$$
\frac{d}{dt} \left(\frac{y}{z} \right) = \frac{z \frac{dy}{dt} - y \frac{dz}{dt}}{z^2}
$$

Recall that:

- $y = 2T$, with T constant, so:

$$
\frac{dy}{dt} = 0
$$

- $z = 7 + 8 t \frac{dw}{dt}$

This is an implicit relationship involving $\frac{dw}{dt}$. Let's differentiate z with respect to t:

$$
\frac{dz}{dt} = 8 \frac{dw}{dt} + 8 t \frac{d^2w}{dt^2}
$$

However, since the problem asks for $\frac{dw}{dt}$, and z depends on it, we encounter a differential equation involving $\frac{dw}{dt}$.

Deriving the Differential Equation for $\frac{dw}{dt}$

Putting everything together:

$$
\frac{dw}{dt} = X e^{y/z} \times \frac{z \times 0 - y \times \frac{dz}{dt}}{z^2} = - X e^{y/z} \times \frac{y \times \frac{dz}{dt}}{z^2}
$$

\]

Substitute \(y = 2T \):

\[
\frac{dw}{dt} = - X e^{2T/z} \times \frac{2T \times \frac{dz}{dt}}{z^2}
\]

Now, recall that:

\[
z = 7 + 8 t \frac{dw}{dt}
\]

and

\[
\frac{dz}{dt} = 8 \frac{dw}{dt} + 8 t \frac{d^2w}{dt^2}
\]

Therefore, the differential equation becomes:

\[
\frac{dw}{dt} = - X e^{2T/z} \times \frac{2T}{z^2} \left(8 \frac{dw}{dt} + 8 t \frac{d^2w}{dt^2} \right)
\]

This is a second-order differential equation involving \(\frac{dw}{dt}\) and \(\frac{d^2w}{dt^2}\). Solving this explicitly requires initial conditions or further assumptions.

Special Cases and Simplifications

Given the complex dependencies, the problem simplifies significantly if we assume:

- \(T \) is constant.
- The term involving \(\frac{d^2w}{dt^2}\) is negligible or the system reaches a steady state where derivatives are zero.

Under these assumptions, the equation reduces to:

\[
\frac{dw}{dt} = - X e^{2T/z} \times \frac{2T}{z^2} \times 8 \frac{dw}{dt}
\]

which simplifies to:

$$
\frac{dw}{dt} = - 8 \times X e^{2T/z} \times \frac{2T}{z^2} \times \frac{dw}{dt}
$$

or:

$$
\frac{dw}{dt} \left(1 + 16 T X e^{2T/z} / z^2 \right) = 0
$$

This leads to the trivial solution:

$$
\frac{dw}{dt} = 0
$$

which suggests that, at equilibrium or under certain conditions, the derivative is zero.

Conclusion and Final Expression

The key insight is that, without specific initial conditions or additional information about the dependencies, the derivative $\frac{dw}{dt}$ is implicitly defined via a differential equation involving itself.

However, based on the initial assumptions and the structure of the problem, the most straightforward answer—assuming steady state or no change—is:

Frequently Asked Questions

What is the main purpose of using the chain rule in this problem?

The chain rule is used to find the derivative of a composite function, specifically dw/dt, based on the given functions of X, Y, and Z.

Given the functions $X = T^9$, $Y = 2T$, and $Z = 7 + 8t$, how do you express w in terms of X, Y, and Z?

w is given as $w = X e^{Y/Z}$.

How do you compute dw/dt using the chain rule in this context?

Compute partial derivatives of w with respect to X, Y, Z, then multiply each by their respective derivatives with respect to t, and sum the results: $dw/dt = (\partial w/\partial X)(dX/dt) + (\partial w/\partial Y)(dY/dt) + (\partial w/\partial Z)(dZ/dt)$.

What is the partial derivative of w with respect to X?

Since $w = X e^{Y/Z}$, $\partial w/\partial X = e^{Y/Z}$.

What is the partial derivative of w with respect to Y?

$\partial w/\partial Y = X e^{Y/Z} (1/Z)$.

What is the partial derivative of w with respect to Z?

$\partial w/\partial Z = X e^{Y/Z} (-Y/Z^2)$.

How do you find dX/dt, dY/dt, and dZ/dt given the functions T and t?

Since $X = T^9$ and $Y = 2T$, need to know T in terms of t; assuming T is a function of t, then $dX/dt = 9T^8 \, dT/dt$, $dY/dt = 2 \, dT/dt$, and $dZ/dt = 8 \, dt/dt = 8$.

What is the value of dZ/dt in this problem?

Given $Z = 7 + 8t$, so $dZ/dt = 8$.

How do you substitute all derivatives into the chain rule formula to find dw/dt?

Plug in the partial derivatives of w with respect to X, Y, Z, and the derivatives dX/dt, dY/dt, dZ/dt into the formula: $dw/dt = (\partial w/\partial X)(dX/dt) + (\partial w/\partial Y)(dY/dt) + (\partial w/\partial Z)(dZ/dt)$.

What is the final expression for dw/dt in terms of T and t?

$dw/dt = e^{Y/Z} [(9T^8)(dT/dt) + (2T)(1/Z)(dT/dt) - (Y/Z^2)(8) T]$.

Additional Resources

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Introduction: Deciphering a Complex Derivative Problem

In the realm of calculus, the chain rule stands as a fundamental technique for differentiating composite functions—functions nested within other functions. Today, we explore a nuanced problem involving derivatives with respect to time, t, where the goal is to find $\frac{dw}{dt}$ given a complex relationship involving multiple functions and variables. The problem statement is:

$$
\text{Use the chain rule to find } \frac{dw}{dt} \text{ in the expression } w = X e^{y/z},
$$

where the variables are defined as:

- $X = T^9$,
- $Y = 2T$,
- $Z = 7 + 8t \frac{dw}{dt}$.

The challenge lies in differentiating w with respect to t, knowing that w depends on X, Y, Z through the exponential function, and that Z itself involves $\frac{dw}{dt}$. This recursive nature introduces an implicit derivative problem, requiring careful application of the chain rule, substitution, and algebraic manipulation.

This article will systematically dissect the problem, clarify the steps involved, and guide you through the process of solving for $\frac{dw}{dt}$ in a clear, detailed manner suitable for students and professionals alike.

Understanding the Problem Structure

Before jumping into calculations, it's essential to interpret the problem components and how they interrelate.

The Main Function: $w = X e^{y/z}$

- w is expressed as the product of X and an exponential function involving y and z.

- $X = T^9$ indicates X is a function of T, but since T is not explicitly time-dependent in the problem statement, we assume T

is either a constant or a known function of t. For simplicity, and typical in such problems, T is considered a function of t.

- $Y = 2 T$, which is a linear function of T.

- $Z = 7 + 8 t \frac{dw}{dt}$, which explicitly depends on t and on $\frac{dw}{dt}$ itself, making the problem implicit and recursive.

Given the above, our goal is to find $\frac{dw}{dt}$. Because w depends on X, Y, Z, which in turn depend on T and t, we will need to apply multivariable chain rule techniques and implicit differentiation.

Step 1: Expressing w Clearly

Given:

$$
w = X e^{y/z}.
$$

Since y is not explicitly defined but is related to $Y = 2T$, we need to clarify whether y is simply Y. Typically, in such notation, y corresponds to Y, so:

$$
w = X e^{Y/Z}.
$$

Thus, the variables are:

$$
X = T^9,
$$
$$
Y = 2T,
$$
$$
Z = 7 + 8 t \frac{dw}{dt}.
$$

Note that T is a function of t, so we may write:

$$
X = T(t)^9,
$$
$$
Y = 2 T(t),
$$

$$
Z = 7 + 8 t \frac{dw}{dt}.
$$

Step 2: Differentiating w with respect to t

Applying the multivariable chain rule:

$$
\frac{dw}{dt} = \frac{\partial w}{\partial X} \frac{dX}{dt} + \frac{\partial w}{\partial Y} \frac{dY}{dt} + \frac{\partial w}{\partial Z} \frac{dZ}{dt}.
$$

Let's compute each partial derivative, then find derivatives of the variables, and finally substitute everything back, considering the recursive nature of Z.

Step 3: Computing Partial Derivatives of w

Given:

$$
w = X e^{Y/Z}.
$$

- Partial derivative with respect to X:

$$
\frac{\partial w}{\partial X} = e^{Y/Z}.
$$

- Partial derivative with respect to Y:

$$
\frac{\partial w}{\partial Y} = X \cdot e^{Y/Z} \cdot \frac{\partial}{\partial Y}(Y/Z) = X e^{Y/Z} \cdot \frac{1}{Z},
$$

since $\partial (Y/Z)/\partial Y = 1/Z$.

- Partial derivative with respect to Z:

$$
\frac{\partial w}{\partial Z} = X e^{Y/Z} \cdot \frac{\partial}{\partial Z}(Y/Z) = X e^{Y/Z} \cdot \left(- \frac{Y}{Z^2} \right),
$$

because $\partial (Y/Z)/\partial Z = -Y/Z^2$.

Step 4: Derivatives of X, Y, Z with respect to t

- For $X = T^9$:

$$
\frac{dX}{dt} = 9 T^8 \frac{dT}{dt}.
$$

- For $Y = 2T$:

$$
\frac{dY}{dt} = 2 \frac{dT}{dt}.
$$

- For $Z = 7 + 8 t \frac{dw}{dt}$:

$$
\frac{dZ}{dt} = 8 \frac{dw}{dt} + 8 t \frac{d^2 w}{dt^2}.
$$

Note that Z involves $\frac{dw}{dt}$, which is the derivative we seek. Consequently, $\frac{dZ}{dt}$ involves $\frac{d^2 w}{dt^2}$. This indicates the problem's implicit nature and suggests that solving directly for $\frac{dw}{dt}$ will involve implicit algebraic steps.

Step 5: Assembling the Derivative Expression

Putting all components together, we have:

$$
\frac{dw}{dt} = e^{Y/Z} \cdot 9 T^8 \frac{dT}{dt} + X e^{Y/Z} \cdot \frac{1}{Z} \cdot 2 \frac{dT}{dt} + X e^{Y/Z} \cdot \left(- \frac{Y}{Z^2} \right) \cdot \left(8 \frac{dw}{dt} + 8 t \frac{d^2 w}{dt^2} \right).
$$

This complex expression contains $\frac{dw}{dt}$ and $\frac{d^2 w}{dt^2}$, indicating a second-order differential equation. To proceed, assumptions or approximations are needed, or the problem must be simplified to isolate $\frac{dw}{dt}$.

Step 6: Simplification and Solving Strategy

Given the complexity, a typical approach involves:

- Assuming T is independent of t or known function, simplifying derivatives of T.

- Recognizing that the recursive term involving $\frac{dw}{dt}$ within Z suggests substituting Z back into the expression and solving algebraically for $\frac{dw}{dt}$.

- Noticing that the presence of $\frac{d^2 w}{dt^2}$ indicates the need for differential equation techniques, or considering steady-state or particular solutions.

In many practical problems, the recursive dependence is handled by substituting Z explicitly and solving algebraically for $\frac{dw}{dt}$, often assuming T is constant or known.

Step 7: Final Expression and Interpretation

Assuming T is constant for simplicity (i.e., $\frac{dT}{dt} = 0$), the derivatives of X and Y vanish:

$$
\frac{dX}{dt} = 0, \quad \frac{dY}{dt} = 0,
$$

and the derivative reduces to:

$$
\frac{dw}{dt} = - X e^{Y/Z} \frac{Y}{Z^2} \left(8 \frac{dw}{dt} + 8 t \frac{d^2 w}{dt^2} \right).
$$

Rearranged:

$$
\frac{dw}{dt} + X e^{Y/Z} \frac{Y}{Z^2} \left(8 \frac{dw}{dt} + 8 t \frac{d^2 w}{dt^2} \right) = 0
$$

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