Use The Chain Rule To Find Z S And Z T . Z = Ln(5x 3y), X = S Sin(t), Y = T Cos(s)

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Introduction

In multivariable calculus, the chain rule is an essential tool for differentiating composite functions involving multiple variables. When variables depend on other parameters, such as time or spatial coordinates, the chain rule enables us to find derivatives with respect to those parameters effectively. In this article, we explore how to apply the chain rule to find the partial derivatives Z_S and Z_T for the function:

$$
Z = \ln(5x^3 y)
$$

where the variables x and y are themselves functions of parameters s and t:

$$
x = s \sin(t)
$$
$$
y = t \cos(s)
$$

Our goal is to compute Z_S and Z_T, the partial derivatives of Z with respect to s and t, respectively. This process involves understanding the chain rule in the context of multivariable functions and leveraging the relationships between x, y and the parameters s, t.

Understanding the Problem

The Given Functions

- Main function:

$$
Z = \ln(5x^3 y)
$$

- Dependent variables:

$$
x = s \sin(t)
$$
\[

$$
y = t \cos(s)
$$

Objective

- Find:

$$
Z_S = \frac{\partial Z}{\partial s}
$$
$$
Z_T = \frac{\partial Z}{\partial t}
$$

Applying the Chain Rule in Multivariable Calculus

The Chain Rule for Partial Derivatives

When a function Z depends on variables x and y, which in turn depend on parameters s and t, the chain rule states:

$$
Z_S = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial Z}{\partial y} \frac{\partial y}{\partial s}
$$

$$
Z_T = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial Z}{\partial y} \frac{\partial y}{\partial t}
$$

This approach involves the following steps:

1. Differentiate Z with respect to x and y.
2. Calculate the derivatives of x and y with respect to s and t.
3. Combine these derivatives according to the chain rule to find Z_S and Z_T.

Step-by-Step Solution

Step 1: Compute $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$

Given:

$$
Z = \ln(5x^3 y)
$$

Using logarithmic properties:

$$
Z = \ln(5) + \ln(x^3) + \ln(y) = \ln(5) + 3 \ln(x) + \ln(y)
$$

Derivatives:

$$
\frac{\partial Z}{\partial x} = \frac{3}{x}
$$
$$
\frac{\partial Z}{\partial y} = \frac{1}{y}
$$

Step 2: Compute derivatives of x and y with respect to s and t

Given:

$$
x = s \sin(t)
$$
$$
y = t \cos(s)
$$

Derivatives:

- With respect to s:

$$
\frac{\partial x}{\partial s} = \sin(t)
$$
$$
\frac{\partial y}{\partial s} = t \cdot (-\sin(s)) = -t \sin(s)
$$

- With respect to t:

$$
\frac{\partial x}{\partial t} = s \cos(t)
$$
$$
\frac{\partial y}{\partial t} = \cos(s)
$$

Step 3: Calculate Z_S

Using the chain rule:

$$
Z_S = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial Z}{\partial y} \frac{\partial y}{\partial s}
$$

Substitute the derivatives:

$$
Z_S = \left(\frac{3}{x} \right) \sin(t) + \left(\frac{1}{y} \right) (-t \sin(s))
$$

\]

Expressed explicitly:

\[
Z_S = \frac{3 \sin(t)}{x} - \frac{t \sin(s)}{y}
\]

Recall that:

\[
x = s \sin(t)
\]
\[
y = t \cos(s)
\]

So,

\[
Z_S = \frac{3 \sin(t)}{s \sin(t)} - \frac{t \sin(s)}{t \cos(s)} = \frac{3}{s}
- \frac{\sin(s)}{\cos(s)}
\]

Simplify:

\[
Z_S = \frac{3}{s} - \tan(s)
\]

Final expression for \(Z_S \):

\[
\boxed{
Z_S = \frac{3}{s} - \tan(s)
}
\]

Step 4: Calculate \(Z_T \)

Using the chain rule:

\[
Z_T = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial t} +
\frac{\partial Z}{\partial y} \frac{\partial y}{\partial t}
\]

Substitute derivatives:

\[
Z_T = \left(\frac{3}{x} \right) s \cos(t) + \left(\frac{1}{y} \right)
\cos(s)
\]

Expressed explicitly:

$$
Z_T = \frac{3 s \cos(t)}{x} + \frac{\cos(s)}{y}
$$

Recall $x = s \sin(t)$ and $y = t \cos(s)$:

$$
Z_T = \frac{3 s \cos(t)}{s \sin(t)} + \frac{\cos(s)}{t \cos(s)} = \frac{3 \cos(t)}{\sin(t)} + \frac{1}{t}
$$

Simplify:

$$
Z_T = 3 \cot(t) + \frac{1}{t}
$$

Final expression for Z_T:

$$
\boxed{
Z_T = 3 \cot(t) + \frac{1}{t}
}
$$

Summary of Results

Derivative	Expression
Z_S	$\frac{3}{s} - \tan(s)$
Z_T	$3 \cot(t) + \frac{1}{t}$

Additional Insights and Applications

Significance of these derivatives

These derivatives are crucial in applications such as:

- Sensitivity analysis: Understanding how Z responds to small changes in s and t.
- Optimization problems: Finding critical points where derivatives are zero.
- Parametric analysis: Exploring how the system behaves under different parameter values.

Practical uses

- In physics, such derivatives help analyze systems where quantities depend on multiple parameters.
- In engineering, these calculations assist in modeling and control systems involving multiple variables.

Tips for Applying the Chain Rule Effectively

- Identify dependencies carefully: Recognize which variables depend on which parameters.
- Differentiate systematically: Compute derivatives step by step to avoid errors.
- Use logarithmic properties: Simplify complex logarithmic expressions before differentiating.
- Check units and dimensions: Ensure the derivatives are dimensionally consistent.

Conclusion

Applying the chain rule to find Z_S and Z_T for the function $Z = \ln(5x^3 y)$, where $x = s \sin(t)$ and $y = t \cos(s)$, involves understanding the relationships between all variables involved. By differentiating Z with respect to x and y, calculating the derivatives of x and y with respect to s and t, and combining these using the chain rule, we obtained concise expressions for Z_S and Z_T.

These techniques form a foundational aspect of multivariable calculus, with wide-reaching applications in science, engineering, and mathematics. Mastery of the chain rule in such contexts enables precise analysis of complex systems where multiple variables interact dynamically.

References:

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning, 8th Edition.
- Thomas, George B., and Ross L. Finney. Calculus and Analytic Geometry. Addison-Wesley, 9th Edition.
- Online resources on multivariable calculus and chain rule applications.

Frequently Asked Questions

What is the given function Z in terms of x and y?

$Z = \ln(5x^3 y)$

How are x and y defined in terms of parameters s, t, and t?

$x = s \sin(t)$, $y = t \cos(s)$

What is the goal when using the chain rule in this problem?

To find the partial derivatives Z_s and Z_t by differentiating Z with respect to s and t through their dependencies on x and y.

How do we compute $\partial Z/\partial x$ and $\partial Z/\partial y$?

Using the partial derivatives of $Z = \ln(5x^3 y)$, we get $\partial Z/\partial x = (15x^2)/(5x^3 y)$ and $\partial Z/\partial y = (1)/(y)$.

What are the derivatives of x and y with respect to s and t?

$\partial x/\partial s = \sin(t)$, $\partial x/\partial t = s \cos(t)$; $\partial y/\partial s = -t \sin(s)$, $\partial y/\partial t = \cos(s)$.

How do you apply the chain rule to find Z_s?

$Z_s = (\partial Z/\partial x)(\partial x/\partial s) + (\partial Z/\partial y)(\partial y/\partial s)$.

How do you apply the chain rule to find Z_t?

$Z_t = (\partial Z/\partial x)(\partial x/\partial t) + (\partial Z/\partial y)(\partial y/\partial t)$.

What are the final expressions for Z_s and Z_t after substitution?

$Z_s = [(15x^2)/(5x^3 y)] \sin(t) + (1/y) (-t \sin(s))$; $Z_t = [(15x^2)/(5x^3 y)] s \cos(t) + (1/y) \cos(s)$.

Why is it important to carefully substitute the derivatives when applying the chain rule here?

To accurately compute Z_s and Z_t, ensuring the derivatives of x and y with respect to s and t are correctly incorporated into the expression for Z, which depends on x and y.

Additional Resources

Chain Rule Application in Multivariable Calculus: Deriving ZS and ZT for $Z = \ln(5x^3y)$

Introduction

In the realm of multivariable calculus, the chain rule is an indispensable tool for differentiating composite functions—functions that depend on other functions. When dealing with functions of multiple variables, especially when those variables themselves are functions of other parameters, the chain rule provides a systematic approach to compute partial derivatives efficiently.

Today, we delve into a comprehensive exploration of how to employ the chain rule to find the partial derivatives ZS and ZT for the function:

$$
Z = \ln(5x^3 y)
$$

where:

$$
x = s \sin(t)
$$

$$
y = t \cos(s)
$$

This scenario is not just a typical calculus problem; it exemplifies the interconnectedness of multiple variables and the importance of the chain rule in unraveling these relationships. Whether you're a student striving to master multivariable calculus or a professional seeking a refresher, this article guides you through each step with clarity and depth.

Understanding the Framework

Before diving into the derivatives, it's crucial to understand the structure of the problem:

- Dependent variable: Z, which is a function involving x and y.
- Intermediate variables: x and y, which depend on parameters s and t.
- Parameters: s and t are independent variables with respect to which we want to find the partial derivatives of Z.

The goal is to find:

$$
Z_s = \frac{\partial Z}{\partial s}
$$

$$
Z_t = \frac{\partial Z}{\partial t}
$$

Given that Z depends on x and y, which in turn depend on s and t, the chain rule for multivariable functions becomes essential.

The Chain Rule in Multivariable Calculus

Multivariable chain rule:
If $Z = Z(x, y)$, with $x = x(s, t)$ and $y = y(s, t)$, then:

$$
Z_s = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial Z}{\partial y} \frac{\partial y}{\partial s}
$$

$$
Z_t = \frac{\partial Z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial Z}{\partial y} \frac{\partial y}{\partial t}
$$

This formula captures the essence of how changes in s and t influence Z through the intermediary variables x and y.

Step-by-Step Derivation

Let's proceed step-by-step, beginning with the derivatives of Z with respect to x and y, then calculating the derivatives of x and y with respect to s and t, and finally combining everything.

---

1. Computing $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$

Given:

$$
Z = \ln(5x^3 y)
$$

Using properties of logarithms:

$$
Z = \ln 5 + \ln x^3 + \ln y
$$

which simplifies to:

$$
Z = \ln 5 + 3 \ln x + \ln y
$$

Since $\ln 5$ is constant, its derivative is zero. The derivatives with respect to x and y are:

$$
\frac{\partial Z}{\partial x} = 3 \frac{1}{x}
$$

$$
\frac{\partial Z}{\partial y} = \frac{1}{y}
$$

---

2. Derivatives of x and y with respect to s and t

Given:

$$
x = s \sin t
$$

$$
y = t \cos s
$$

Partial derivatives of x:

$$
x_s = \frac{\partial}{\partial s} (s \sin t) = \sin t

\]
\[
x_t = \frac{\partial}{\partial t} (s \sin t) = s \cos t
\]

Partial derivatives of \(y \):

\[
y_s = \frac{\partial}{\partial s} (t \cos s) = t (-\sin s) = -t \sin s
\]
\[
y_t = \frac{\partial}{\partial t} (t \cos s) = \cos s
\]

These derivatives express how the intermediate variables change with respect to the parameters \(s \) and \(t \).

---

3. Applying the Chain Rule to Find \(Z_s \) and \(Z_t \)

Using the chain rule formulas:

\[
Z_s = \frac{\partial Z}{\partial x} x_s + \frac{\partial Z}{\partial y} y_s
\]
\[
Z_t = \frac{\partial Z}{\partial x} x_t + \frac{\partial Z}{\partial y} y_t
\]

Calculating \(Z_s \):

\[
Z_s = \left(3 \frac{1}{x}\right) (\sin t) + \left(\frac{1}{y}\right) (-t \sin s)
\]

Substituting the expressions for \(x \) and \(y \):

\[
x = s \sin t
\]
\[
y = t \cos s
\]

we get:

\[
Z_s = \frac{3 \sin t}{s \sin t} + \frac{-t \sin s}{t \cos s}
\]

Simplify each term:

\[
Z_s = \frac{3}{s} - \frac{\sin s}{\cos s}
\]

Recall that \(\frac{\sin s}{\cos s} = \tan s \), leading to:

$$
\boxed{
Z_s = \frac{3}{s} - \tan s
}
$$

Calculating Z_t:

$$
Z_t = \left(3 \frac{1}{x}\right) (s \cos t) + \left(\frac{1}{y}\right) (\cos s)
$$

Substituting:

$$
x = s \sin t
$$
$$
y = t \cos s
$$

we find:

$$
Z_t = \frac{3 s \cos t}{s \sin t} + \frac{\cos s}{t \cos s}
$$

Simplify each term:

$$
Z_t = \frac{3 \cos t}{\sin t} + \frac{1}{t}
$$

Note that $\frac{\cos t}{\sin t} = \cot t$, so:

$$
\boxed{
Z_t = 3 \cot t + \frac{1}{t}
}
$$

Summary of Results

Partial Derivative	Expression
Z_s	$\displaystyle \frac{3}{s} - \tan s$
Z_t	$\displaystyle 3 \cot t + \frac{1}{t}$

These derivatives reveal how Z changes with respect to the parameters s and t, considering the intricate dependencies through x and y.

Additional Insights and Applications

The process showcased here is emblematic of a broader class of problems in multivariable calculus, where functions depend on other functions—each with their own dependencies. The chain rule systematically breaks down these layers, allowing us to compute derivatives that would otherwise be cumbersome.

Practical Applications include:

- Engineering: Sensitivity analysis in systems with multiple parameters.
- Physics: Computing how a physical quantity changes with respect to different variables in a dynamic system.
- Economics: Understanding how interconnected variables influence an outcome.

Key Takeaways:

- Always identify the dependencies: know how each variable depends on the parameters.
- Use the chain rule systematically, breaking derivatives into manageable parts.
- Simplify logarithmic derivatives and trigonometric ratios carefully.
- Recognize standard derivatives like $\frac{\partial}{\partial x} \ln x$ and trigonometric derivatives to streamline calculations.

Final Remarks

Mastering the application of the chain rule in multivariable calculus enhances your analytical toolkit, enabling you to tackle complex, layered functions with confidence. The example explored demonstrates the importance of meticulous computation and strategic simplification.

Whether you're analyzing a physical system, modeling economic behavior, or solving advanced calculus problems, understanding how to navigate the dependencies via the chain rule is invaluable. Keep practicing with diverse functions, and you'll find that these techniques become second nature, unlocking deeper insights into the interconnected world of multivariable analysis.

Happy Calculating!

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