Use The Zero Product Property To Find The Solutions To The Equation (x 2) (x 3) = 12.

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When working with algebraic equations, especially those involving products of expressions, the Zero Product Property is an essential tool. In this article, we will explore how to leverage the Zero Product Property to solve the equation (x 2) (x 3) = 12. Understanding this process not only helps in solving this specific problem but also builds foundational skills applicable to a wide range of algebraic equations.

Understanding The Zero Product Property

Before diving into solving the specific equation, it is crucial to understand what the Zero Product Property states and how it applies to algebra.

What Is The Zero Product Property?

The Zero Product Property asserts that if the product of two factors equals zero, then at least one of the factors must be zero. Mathematically, it is expressed as:

- If $AB = 0$, then either $A = 0$ or $B = 0$.

This property is fundamental when solving quadratic equations and equations that can be factored into simpler expressions.

Applying The Zero Product Property

While the property directly applies when the product equals zero, in equations where the product equals a non-zero value, the typical approach involves rearranging the equation into a form where the property can be used. This often includes:

- Expressing the equation as a product of factors.
- Setting each factor equal to the value that makes the product equal to the right side of the equation.
- Solving for the variable in each resulting equation.

In our specific case, since the equation equals 12 (not zero), the strategy involves algebraic manipulation to relate it to the Zero Product Property.

Solving the Equation (x 2) (x 3) = 12

Let's now focus on the specific problem: solving (x 2) (x 3) = 12 using the Zero Product Property.

Step 1: Recognize the Equation Structure

The given equation is:

- *(x 2)(x 3) = 12*

At first glance, it appears that (x 2) and (x 3) are factors multiplied together, but the key is to interpret these expressions correctly. Typically, in algebra, when parentheses contain a variable and a number, it suggests a product involving the variable.

However, the notation "x 2" and "x 3" could be interpreted as:

- **Option 1:** (x + 2)(x + 3) = 12
- **Option 2:** (x - 2)(x - 3) = 12
- **Option 3:** (x2)(x3) = 12

Given common algebraic notation, the most likely intended meaning is:

- (x + 2)(x + 3) = 12

which is a standard form, or possibly:

- (x - 2)(x - 3) = 12

For clarity, we will proceed with the assumption that the original expression is:
(x + 2)(x + 3) = 12

If your notation differs, the process remains similar, just adjusting the expressions accordingly.

Step 2: Expand the Expression

Next, expand the product:

- (x + 2)(x + 3) = x(x + 3) + 2(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6

So, the equation becomes:

- x^2 + 5x + 6 = 12

Step 3: Rewrite the Equation in Standard Quadratic Form

Subtract 12 from both sides:

- x^2 + 5x + 6 - 12 = 0

Simplify:

- x^2 + 5x - 6 = 0

Now, the equation is a quadratic in standard form, ready for factoring or applying the Zero Product Property.

Factoring the Quadratic Equation

Step 4: Factor the Quadratic

Look for two numbers that multiply to -6 and add to 5:

- Factors of -6: (1, -6), (-1, 6), (2, -3), (-2, 3)
- Sum of factors: $1 + (-6) = -5$, $-1 + 6 = 5$, $2 + (-3) = -1$, $-2 + 3 = 1$

The pair that adds to 5 is -1 and 6, but since their product is -6, and their sum is 5, these are the numbers we need.

Actually, check -1 and 6:

- $-1\ 6 = -6$
- $-1 + 6 = 5$

Perfect! So, factorization:

- $x^2 + 5x - 6 = (x - 1)(x + 6) = 0$

Step 5: Apply The Zero Product Property

Set each factor equal to zero:

- 1. $x - 1 = 0$ ⍰ $x = 1$
- 2. $x + 6 = 0$ ⍰ $x = -6$

These are the solutions to the original equation.

Summary of the Solution Process

Here's a quick recap:

1. Identify the structure of the equation: $(x + 2)(x + 3) = 12$.
2. Expand the factors to form a quadratic: $x^2 + 5x + 6 = 12$.
3. Rewrite the quadratic in standard form: $x^2 + 5x - 6 = 0$.
4. Factor the quadratic: $(x - 1)(x + 6) = 0$.

5. Use the Zero Product Property to find solutions: $x = 1$ and $x = -6$.

Other Variations and Tips

Handling Different Equations

If the original problem had a different structure, such as $(x - 2)(x - 3) = 12$, the steps would be similar:

- Expand the factors.
- Bring all terms to one side.
- Factor the quadratic if possible.
- Apply the Zero Product Property to find solutions.

When the Equation Equals Zero

If the equation was directly set to zero, such as $(x + 2)(x + 3) = 0$, the Zero Product Property could be applied immediately:

- Set each factor equal to zero:
- $x + 2 = 0$ □ $x = -2$
- $x + 3 = 0$ □ $x = -3$

Conclusion

Using the Zero Product Property is a powerful method for solving equations involving products of expressions. By recognizing the structure of the equation, expanding factors, rewriting in standard quadratic form, and factoring, you can efficiently find all solutions. Remember, the key is to manipulate the equation into a form where the Zero Product Property applies directly—either by setting a product equal to zero or by transforming the equation into a zero-product form.

Applying these techniques to the equation $(x\ 2)\ (x\ 3) = 12$ (interpreted as $(x + 2)(x + 3) = 12$) demonstrates the process clearly. With practice, solving such equations becomes straightforward, making algebraic problem-solving a more manageable and rewarding task.

Frequently Asked Questions

What is the Zero Product Property?

The Zero Product Property states that if the product of two factors is zero, then at least one of the factors must be zero.

How do you apply the Zero Product Property to solve the equation (x - 2)(x - 3) = 12?

First, set each factor equal to zero after bringing all terms to one side, or rearranged, then solve for x. Alternatively, rewrite the equation to set each factor equal to zero if possible, or isolate the factors to solve.

Is the equation (x - 2)(x - 3) = 12 suitable for the Zero Product Property?

Yes, but first you need to set the equation equal to zero. You can do this by subtracting 12 from both sides to get (x - 2)(x - 3) - 12 = 0 and then proceed accordingly.

What are the steps to solve (x - 2)(x - 3) = 12 using the Zero Product Property?

First, rewrite the equation as (x - 2)(x - 3) = 12. Then, move 12 to the other side to get (x - 2)(x - 3) - 12 = 0. Alternatively, expand the left side, set equal to zero, and solve the resulting quadratic equation.

Can you solve (x - 2)(x - 3) = 12 by factoring directly?

No, because the product equals 12, not zero. To use the Zero Product Property, set the equation equal to zero by subtracting 12 from both sides, then factor and solve.

What is the expanded form of (x - 2)(x - 3)?

The expanded form is x^2 - 5x + 6.

How do you solve the quadratic equation x^2 - 5x + 6 = 0?

Factor the quadratic as (x - 2)(x - 3) = 0, then set each factor equal to zero: x - 2 = 0 and x - 3 = 0, giving solutions x = 2 and x = 3.

What are the solutions to the original equation (x - 2)(x - 3) = 12?

The solutions are x = 2 and x = 3, obtained by solving the quadratic equation x^2 - 5x + 6 = 0.

Why is it important to set the equation equal to zero before applying the Zero Product Property?

Because the Zero Product Property applies only when the product equals zero. Therefore, you must manipulate the equation to have zero on one side before factoring and solving.

Can the Zero Product Property be used directly on (x - 2)(x - 3) = 12?

No, because the property applies when the product equals zero. To use it directly, you need to rewrite the equation as (x - 2)(x - 3) - 12 = 0, then factor and solve.

Additional Resources

Use The Zero Product Property To Find The Solutions To The Equation (x 2) (x 3) = 12

When approaching algebraic equations, one of the fundamental techniques that students and mathematicians often utilize is the Zero Product Property. This property states that if the product of two factors equals zero, then at least one of the factors must be zero. While this principle is straightforward, its application to more complex equations—such as those involving products set equal to non-zero numbers—requires strategic manipulation and understanding. In this article, we will explore how to leverage the Zero Product Property to solve the equation $(x^2)(x^3) = 12$, breaking down each step to ensure clarity and mastery of the concept.

Understanding the Zero Product Property

What Is the Zero Product Property?

The Zero Product Property is a key principle in algebra which states:

> If $ab = 0$, then either $a = 0$ or $b = 0$.

This property simplifies solving equations where the product of two factors equals zero. It's particularly useful because it allows us to break down complex equations into simpler linear equations.

Key features:

- Only applies directly when the entire expression equals zero.
- Serves as a shortcut to factor-based equations.
- Essential for solving quadratic and higher-degree polynomial equations.

Limitations:

- Cannot be directly applied if the product equals a number other than zero.
- Requires that the equation be factored into a product of factors.

Rearranging the Equation for Application

Original Equation

Our starting point is the equation:

$$
(x^2)(x^3) = 12
$$

At first glance, this is a multiplication of two powers of x, set equal to 12. To apply the Zero Product Property effectively, we need to convert this equation into a form where the property's constraints are satisfied.

Expressing the Equation Using Exponents

Recall that when multiplying powers with the same base, exponents add:

$$
x^2 \times x^3 = x^{2 + 3} = x^5
$$

Therefore, the original equation simplifies to:

$$
x^5 = 12
$$

Now, the equation is a simple algebraic expression: $x^5 = 12$.

Applying the Zero Product Property

Why the Zero Product Property Doesn't Directly Apply

At this stage, it might seem tempting to apply the Zero Product Property directly to solve for x. However, since the equation is now $x^5 = 12$, which equals a non-zero constant, the property isn't directly applicable because it requires the product to equal zero.

Key insight: The Zero Product Property is only valid when the entire expression equals zero, not any other number. Therefore, to leverage the property, we need to manipulate the equation further.

Using the Property with Factoring

One way to incorporate the Zero Product Property here is to set the equation equal to zero by bringing all terms to one side:

$$
x^5 - 12 = 0
$$

This is a polynomial equation, but not factored into a product of simpler factors yet. To use the Zero Product Property, we need to factor the polynomial.

Factoring the Equation

Factoring a Fifth-Degree Polynomial

Factoring $x^5 - 12$ directly can be challenging because it's not a standard difference of squares or sum/difference of cubes. However, we can consider using substitution or numerical methods. Alternatively, for illustrative purposes, if the polynomial were factorable into lower-degree polynomials, the Zero Product Property could be applied.

In this specific case, since $x^5 - 12$ is not easily factorable over the rationals, we recognize that the solution involves taking roots rather than factoring into linear factors.

Solving the Equation Using Roots

Isolating x

From the simplified form:

$$
x^5 = 12
$$

We can solve for x by taking the 5th root of both sides:

$$
x = \sqrt[5]{12}
$$

This solution is real and unique because the 5th root of a positive number is positive.

Features:

- The 5th root is well-defined for all real numbers.
- The solution provides the real root directly.

Complex Solutions (Optional Considerations)

While the real solution is straightforward, complex solutions also exist if we consider the roots of complex numbers.

Complex roots of $x^5 = 12$:

$$
x = \sqrt[5]{12} \times e^{i \frac{2\pi k}{5}}, \quad k=0,1,2,3,4
$$

This involves De Moivre's theorem and complex roots of unity. For most algebraic problem-solving purposes, focusing on the real root suffices unless the problem specifies complex solutions.

Summary of the Solution Process

1. Start with the original equation:

$$
(x^2)(x^3) = 12
$$

2. Simplify using exponent rules:

$$
x^{2+3} = x^5 = 12
$$

3. Rewrite as a zero-equal equation:

$$
x^5 - 12 = 0
$$

4. Solve for x:

$$
x = \sqrt[5]{12}
$$

5. Optional: Consider complex roots if context requires.

Advantages and Disadvantages of This Approach

Pros:

- Simplifies the original problem efficiently through exponent rules.
- Avoids unnecessary complex factorizations.
- Provides an exact real solution easily accessible via roots.

Cons:

- Does not directly demonstrate the Zero Product Property since the polynomial is not factored into a product of zero-equal factors.
- Might be confusing for beginners who expect the property to be applied directly to the original equation.
- Does not explore complex solutions unless explicitly addressed.

Alternative Methods and Additional Tips

- Graphical approach: Plotting $y = x^5$ and $y=12$ to find the intersection point.
- Numerical approximation: Using calculator or software to approximate $\sqrt[5]{12}$.
- Factoring over complex numbers: For deeper understanding, explore roots of unity and complex solutions.

Conclusion

While the direct application of the Zero Product Property isn't straightforward for the equation \((x^2)(x^3) = 12\), understanding how to manipulate the equation to utilize the property is invaluable. By recognizing that the product of powers of \(x\) simplifies to a single power, and then rewriting the equation as \(x^5 = 12\), we can straightforwardly solve for \(x\) using roots. This approach underscores the importance of algebraic manipulation and the strategic use of fundamental properties like the Zero Product Property. Mastering these techniques equips learners with versatile tools for tackling a wide range of algebraic equations efficiently and confidently.

Key Takeaways:

- The Zero Product Property applies directly only when an expression equals zero.
- Simplify equations to the form \(x^n = a\) to find solutions via roots.
- Recognize when to manipulate equations to leverage fundamental properties effectively.
- Be aware of the difference between real and complex solutions and approach accordingly.

By practicing these methods, students can build a strong foundation in algebraic problem-solving and develop confidence in handling more complex equations with elegance and precision.

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