Verify The Identity. Show Your Worksin (theta)/3 Cos (theta)/3=1/2sin 2(theta)/3

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Introduction

Mathematics, especially trigonometry, is full of fascinating identities that help us simplify complex expressions and deepen our understanding of angles and their relationships. One common task encountered in trigonometry is verifying whether a given identity holds true for all values within its domain. In this article, we will thoroughly verify the identity:

\[
\frac{\sin (\theta)}{3} \times \frac{\cos (\theta)}{3} = \frac{1}{2} \times
\frac{\sin \left(\frac{2\theta}{3}\right)}{}
\]

by carefully showing all the steps involved in the process.

Understanding the Identity

Before diving into the verification process, it's essential to analyze the structure of the given identity:

\[
\frac{\sin (\theta)}{3} \times \frac{\cos (\theta)}{3} = \frac{1}{2} \times
\frac{\sin \left(\frac{2\theta}{3}\right)}{}
\]

At first glance, the left side involves the product of sine and cosine functions divided by 3, while the right side involves a sine function of a scaled angle, multiplied by 1/2. Recognizing standard trigonometric identities and formulas can significantly aid this process.

Step 1: Simplify the Left Side

Let's start by simplifying the left-hand side (LHS):

\[

\frac{\sin (\theta)}{3} \times \frac{\cos (\theta)}{3}
\]

which simplifies to:

\[
\frac{\sin (\theta) \cos (\theta)}{9}
\]

Now, recall the double-angle identity for sine:

\[
\sin (2\theta) = 2 \sin (\theta) \cos (\theta)
\]

Using this, we can express the numerator:

\[
\sin (\theta) \cos (\theta) = \frac{\sin (2\theta)}{2}
\]

Thus, the LHS becomes:

\[
\frac{\sin (2\theta)}{2 \times 9} = \frac{\sin (2\theta)}{18}
\]

Step 2: Simplify the Right Side

Now, examine the right-hand side (RHS):

\[
\frac{1}{2} \times \sin \left(\frac{2\theta}{3}\right)
\]

This remains as is for now, but to compare both sides, we need to relate
\(\sin (2\theta)\) and \(\sin \left(\frac{2\theta}{3}\right)\).

Step 3: Express \(\sin (2\theta)\) in terms of \(\sin
\left(\frac{2\theta}{3}\right)\)

This is the key step in verifying the identity. We need to see if \(\sin
(2\theta)\) can be expressed in terms of \(\sin
\left(\frac{2\theta}{3}\right)\).

Recall the sine multiple-angle formula:

$$
\sin (k x) = 2 \sin \left(\frac{k x}{2}\right) \cos \left(\frac{k x}{2}\right)
$$

Applying this to $\sin (2\theta)$:

$$
\sin (2\theta) = 2 \sin (\theta) \cos (\theta)
$$

But this is the same as the double-angle identity we used earlier.

Alternatively, we can consider the substitution:

$$
x = \frac{2\theta}{3}
$$
which implies:

$$
2\theta = 3x
$$

Therefore:

$$
\sin (2\theta) = \sin (3x)
$$

Now, use the triple-angle identity for sine:

$$
\sin (3x) = 3 \sin x - 4 \sin^3 x
$$

Expressed explicitly:

$$
\sin (2\theta) = 3 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3 \left(\frac{2\theta}{3}\right)
$$

Step 4: Rewrite the LHS in terms of $\sin \left(\frac{2\theta}{3}\right)$

Recall that the LHS simplified to:

$$
\frac{\sin (2\theta)}{18}

\]

Substituting the triple-angle identity:

\[
\frac{3 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3
\left(\frac{2\theta}{3}\right)}{18}
\]

We can split the fraction:

\[
\frac{3 \sin \left(\frac{2\theta}{3}\right)}{18} - \frac{4 \sin^3
\left(\frac{2\theta}{3}\right)}{18}
\]

which simplifies to:

\[
\frac{\sin \left(\frac{2\theta}{3}\right)}{6} - \frac{2 \sin^3
\left(\frac{2\theta}{3}\right)}{9}
\]

Step 5: Compare with the RHS

Recall the RHS:

\[
\frac{1}{2} \sin \left(\frac{2\theta}{3}\right)
\]

Our objective is to verify whether the LHS simplifies to this expression.

From Step 4, the LHS is:

\[
\frac{\sin \left(\frac{2\theta}{3}\right)}{6} - \frac{2 \sin^3
\left(\frac{2\theta}{3}\right)}{9}
\]

Let's write the RHS as:

\[
\frac{1}{2} \sin \left(\frac{2\theta}{3}\right)
\]

To check equality, bring all to a common denominator:

- The RHS can be expressed as:

\[
\frac{3}{6} \sin \left(\frac{2\theta}{3}\right)
\]

- The LHS is:

\[
\frac{\sin \left(\frac{2\theta}{3}\right)}{6} - \frac{2 \sin^3
\left(\frac{2\theta}{3}\right)}{9}
\]

Expressed with a common denominator of 18:

\[
\frac{3 \sin \left(\frac{2\theta}{3}\right)}{18} - \frac{4 \sin^3
\left(\frac{2\theta}{3}\right)}{18}
\]

Similarly, the RHS:

\[
\frac{9 \sin \left(\frac{2\theta}{3}\right)}{18}
\]

Now, compare:

\[
\frac{3 \sin \left(\frac{2\theta}{3}\right)}{18} - \frac{4 \sin^3
\left(\frac{2\theta}{3}\right)}{18} \stackrel{?}{=} \frac{9 \sin
\left(\frac{2\theta}{3}\right)}{18}
\]

Simplify the difference:

\[
\frac{3 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3
\left(\frac{2\theta}{3}\right)}{18} \stackrel{?}{=} \frac{9 \sin
\left(\frac{2\theta}{3}\right)}{18}
\]

Multiply both sides by 18:

\[
3 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3
\left(\frac{2\theta}{3}\right) = 9 \sin \left(\frac{2\theta}{3}\right)
\]

Bring all to one side:

\[
3 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3

\left(\frac{2\theta}{3}\right) - 9 \sin \left(\frac{2\theta}{3}\right) = 0
\]

Simplify:

\[
-6 \sin \left(\frac{2\theta}{3}\right) - 4 \sin^3
\left(\frac{2\theta}{3}\right) = 0
\]

Factor out \(\sin \left(\frac{2\theta}{3}\right)\):

\[
\sin \left(\frac{2\theta}{3}\right) \left(-6 - 4 \sin^2
\left(\frac{2\theta}{3}\right)\right) = 0
\]

Set each factor to zero:

1. \(\sin \left(\frac{2\theta}{3}\right) = 0\)
2. \(-6 - 4 \sin^2 \left(\frac{2\theta}{3}\right) = 0\)

The first gives:

\[
\sin \left(\frac{2\theta}{3}\right) = 0
\]

which holds for specific angles.

The second:

\[
-6 - 4 \sin^2 \left(\frac{2\theta}{3}\right) = 0 \Rightarrow 4 \sin^

Frequently Asked Questions

How can I verify the identity (cos θ/3) / (sin θ/3) = 1/2 sin 2θ/3?

Start by expressing both sides using standard identities. Rewrite the left side as (cos θ/3) / (sin θ/3) = cotangent θ/3. Recall that sin 2θ/3 = 2 sin θ/3 cos θ/3, so the right side becomes 1/2 2 sin θ/3 cos θ/3 = sin θ/3 cos θ/3. Then verify that cotangent θ/3 equals sin θ/3 / cos θ/3, confirming the identity.

What are the key trigonometric identities needed to verify (cos θ/3) / (sin θ/3) = 1/2 sin 2θ/3?

The main identities are cotangent $\theta/3 = \cos\theta/3 / \sin\theta/3$ and $\sin 2\theta/3 = 2 \sin\theta/3 \cos\theta/3$. These allow you to rewrite and simplify both sides to confirm the equality.

Can substitution help in verifying the identity involving (cos θ/3) / (sin θ/3)?

Yes, substituting specific values of θ (excluding values where denominators are zero) can help numerically verify the equality. For example, try $\theta/3 = 45°$, then check both sides to see if they are equal.

Is the identity (cos θ/3) / (sin θ/3) = 1/2 sin 2θ/3 valid for all θ?

It is valid where both sides are defined. Since the expression involves division by $\sin\theta/3$, θ should not be such that $\sin\theta/3 = 0$. Otherwise, the identity holds within the domain where these functions are defined.

How does the double-angle identity for sine assist in verifying (cos θ/3) / (sin θ/3) = 1/2 sin 2θ/3?

Using $\sin 2\theta/3 = 2 \sin\theta/3 \cos\theta/3$, we can rewrite the right side as $\sin\theta/3 \cos\theta/3$. Recognizing that the left side is cotangent θ/3 (which is $\cos\theta/3 / \sin\theta/3$), helps confirm the relationship between both sides.

What steps are involved in algebraically verifying the identity (cos θ/3) / (sin θ/3) = 1/2 sin 2θ/3?

Step 1: Express the right side as $1/2\ 2 \sin\theta/3 \cos\theta/3 = \sin\theta/3 \cos\theta/3$. Step 2: Recognize the left side as $\cot\theta/3$. Step 3: Show that $\cot\theta/3 = \sin\theta/3 / \cos\theta/3$. Step 4: Confirm that both sides are equivalent by substitution or algebraic manipulation.

Are there specific values of θ where this identity is easier to verify?

Yes, choosing θ such that θ/3 is a common angle (like 30°, 45°, or 60°) makes the verification straightforward since their sine and cosine values are well-known.

How can I graph both sides to visually verify the

identity $(\cos \theta/3) / (\sin \theta/3) = 1/2 \sin 2\theta/3$?

Plot $y = (\cos \theta/3) / (\sin \theta/3)$ and $y = 1/2 \sin 2\theta/3$ for various θ values within the domain. If the graphs coincide, it confirms the identity visually. Use graphing tools or software for accurate visualization.

What common mistakes should I avoid when verifying this trigonometric identity?

Avoid dividing by zero when $\sin \theta/3 = 0$, and ensure you correctly apply identities like $\sin 2\theta/3 = 2 \sin \theta/3 \cos \theta/3$. Also, be cautious with angle domain restrictions and simplify step-by-step to prevent algebraic errors.

Additional Resources

Verify The Identity. Show Your Work in $(\theta)/3 \cos (\theta)/3 = 1/2 \sin 2(\theta)/3$

In the realm of trigonometry, verifying identities is a fundamental skill that enhances understanding of the relationships between angles and their functions. The process involves demonstrating that two expressions are equivalent through algebraic manipulations and known identities. The given identity:

$$
\frac{\theta}{3} \cos \frac{\theta}{3} = \frac{1}{2} \sin \frac{2\theta}{3}
$$

presents an intriguing challenge that offers insight into the interplay between angle transformations and fundamental trigonometric identities. This article aims to meticulously verify this identity, explaining each step in detail, contextualizing the significance of such identities, and exploring their applications.

Understanding the Components of the Identity

Before diving into the verification process, it’s essential to understand each component of the given expression:

The Left-Hand Side (LHS): $\frac{\theta}{3} \cos \frac{\theta}{3}$

- $\frac{\theta}{3}$: Represents a scaled version of the angle θ. It suggests that the expression involves an angle division, a common technique in trigonometry to simplify or relate different expressions.
- $\cos \frac{\theta}{3}$: The cosine of the scaled angle, representing the ratio of the adjacent side to the hypotenuse in a right-angled triangle with

angle $\frac{\theta}{3}$.

The Right-Hand Side (RHS): $\frac{1}{2} \sin \frac{2\theta}{3}$

- $\sin \frac{2\theta}{3}$: The sine of twice the scaled angle, which invokes the double-angle identity.
- Multiplication by $\frac{1}{2}$: A scalar factor that adjusts the magnitude of the sine function.

Key Trigonometric Identities Relevant to Verification

To verify the identity, certain fundamental identities are instrumental:

1. Double-Angle Identity for Sine

$$
\sin 2x = 2 \sin x \cos x
$$

This identity allows expressing the sine of a double angle as a product of sine and cosine functions of the original angle.

2. Product-to-Sum Identities

Although not directly necessary here, understanding these identities can offer alternative approaches.

3. Basic Algebraic Manipulations

Simplification often involves factoring, expanding, or substituting equivalent expressions to reveal the equality.

Step-by-Step Verification Process

The goal is to demonstrate that:

$$
\frac{\theta}{3} \cos \frac{\theta}{3} = \frac{1}{2} \sin \frac{2\theta}{3}
$$

holds true for all relevant values of θ.

Step 1: Express the RHS Using Double-Angle Identity

Recall the double-angle identity:

$$
\sin 2x = 2 \sin x \cos x
$$

Set $x = \frac{\theta}{3}$, then:

$$
\sin \frac{2\theta}{3} = 2 \sin \frac{\theta}{3} \cos \frac{\theta}{3}
$$

Substitute this into RHS:

$$
\frac{1}{2} \sin \frac{2\theta}{3} = \frac{1}{2} \times 2 \sin \frac{\theta}{3} \cos \frac{\theta}{3} = \sin \frac{\theta}{3} \cos \frac{\theta}{3}
$$

Step 2: Simplify RHS to Match LHS

Now, the RHS simplifies to:

$$
\sin \frac{\theta}{3} \cos \frac{\theta}{3}
$$

Compare this with the LHS:

$$
\frac{\theta}{3} \cos \frac{\theta}{3}
$$

At this point, notice that the two expressions are similar, but the LHS involves $\frac{\theta}{3}$ (multiplied by cosine), while the RHS is $\sin \frac{\theta}{3} \times \cos \frac{\theta}{3}$.

Step 3: Recognize the Discrepancy

The key difference is:

- LHS: $\frac{\theta}{3} \times \cos \frac{\theta}{3}$
- RHS: $\sin \frac{\theta}{3} \times \cos \frac{\theta}{3}$

For these to be equal, the following must hold:

$$
\frac{\theta}{3} \times \cos \frac{\theta}{3} = \sin \frac{\theta}{3} \times
$$

$$
\cos \frac{\theta}{3}
$$

Dividing both sides by $\cos \frac{\theta}{3}$ (assuming $\cos \frac{\theta}{3} \neq 0$):

$$
\frac{\theta}{3} = \sin \frac{\theta}{3}
$$

Step 4: Verify the Condition $\frac{\theta}{3} = \sin \frac{\theta}{3}$

The equation:

$$
x = \sin x
$$

has solutions at:

- $x = 0$
- $x \approx 0.88$ (within the interval $[- \pi/2, \pi/2]$), but more generally, the only real solution where $x = \sin x$ is at $x=0$.

Therefore, the original identity holds only at specific values of θ where:

$$
\frac{\theta}{3} = 0 \Rightarrow \theta = 0
$$

and potentially at points where $\cos \frac{\theta}{3} = 0$, which would make the division invalid.

Implications and Conclusions

The detailed analysis reveals that the initial identity:

$$
\frac{\theta}{3} \cos \frac{\theta}{3} = \frac{1}{2} \sin \frac{2\theta}{3}
$$

does not hold universally for all θ. Instead, it appears to be valid only at specific points, particularly when $\theta = 0$, since then:

- $\frac{\theta}{3} = 0$
- $\cos 0 = 1$

- $\sin 0 = 0$

Plugging in $\theta=0$:

$$
0 \times 1 = \frac{1}{2} \times 0 \Rightarrow 0=0
$$

which confirms the identity holds trivially at $\theta=0$.

However, for the identity to hold for all θ, the derivation must satisfy the condition:

$$
\frac{\theta}{3} = \sin \frac{\theta}{3}
$$

which is only true at $\theta=0$. Consequently, the initial expression is more accurately described as an identity valid only at specific points, not over the entire domain.

Broader Context and Practical Applications

Understanding the limitations and specific conditions under which trig identities hold is crucial in both theoretical mathematics and applied sciences.

1. Signal Processing and Fourier Analysis

Trigonometric identities underpin Fourier transforms, which decompose signals into sinusoidal components. Recognizing when identities hold allows engineers to simplify complex expressions and optimize algorithms.

2. Engineering and Physics

In wave mechanics, optics, and electromagnetism, precise manipulation of trigonometric functions facilitates problem-solving involving oscillations, interference, and wave propagation.

3. Mathematical Education

Verifying identities enhances critical thinking, algebraic skills, and conceptual understanding of the properties of functions.

Final Remarks and Recommendations

- When working with trigonometric identities, always verify the domain and the conditions under which the identities are valid.
- Use fundamental identities as tools for transformation and simplification, but be mindful of their limitations.
- Recognize that many identities are valid only at particular points or under specific constraints, which is vital in both pure and applied mathematics.

In conclusion, the identity:

$$
\frac{\theta}{3} \cos \frac{\theta}{3} = \frac{1}{2} \sin \frac{2\theta}{3}
$$

is valid only at particular values of θ, notably at $\theta=0$. The detailed verification process underscores the importance of understanding the foundational identities and their domains, fostering a deeper appreciation for the elegance and nuance inherent in trigonometry.

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