What Happens To The Graph Of F(x) When It Is Multipliedby A Number Between 0 And 1?

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Understanding how the graph of a function transforms under various operations is fundamental in mathematics, especially in the study of functions and their behaviors. When the function $f(x)$ is multiplied by a number between 0 and 1, the resulting graph undergoes a specific type of transformation known as vertical compression or vertical scaling. This article explores in detail what happens to the graph of $f(x)$ when it is multiplied by a scalar k such that $0 < k < 1$, including the effects on the shape, size, and key features of the graph.

Understanding the Concept of Scalar Multiplication in Functions

What Does Multiplying a Function by a Scalar Mean?

Multiplying a function $f(x)$ by a scalar k involves creating a new function:

$$g(x) = k \times f(x)$$

This operation scales the output values (the y-values) of the original function by the factor k. When k is greater than 1, the graph is stretched vertically; when $0 < k < 1$, the graph is compressed vertically.

Visualizing the Transformation

To understand the impact, think of the original graph as a set of points (x, y). Multiplying by k transforms each point into (x, ky). Since each y-coordinate is scaled by k, the entire graph is "shrunk" towards the x-axis when $0 < k < 1$.

Effects of Multiplying by a Number Between 0 and 1

Vertical Compression Explained

When $f(x)$ is multiplied by a scalar k where $0 < k < 1$, the graph experiences a vertical compression:

- Definition: The graph is compressed toward the x-axis, reducing the height of all points without altering the x-positions.
- Result: The overall shape of the graph remains the same, but the peaks and troughs are less pronounced.

Mathematical Illustration

- For a point (x, y) on $f(x)$, the corresponding point on $g(x) = kf(x)$ is (x, ky).
- Since $0 < k < 1$, ky has a smaller magnitude than y, pulling the point closer to the x-axis.

Graphical Examples

- Consider $f(x) = \sin(x)$. Multiplying by 0.5 results in $g(x) = 0.5 \sin(x)$, which appears as a sine wave with half the amplitude.
- For a quadratic $f(x) = x^2$, multiplying by 0.3 results in $g(x) = 0.3 x^2$, making the parabola narrower vertically.

Key Features Affected by the Multiplication

Amplitude and Range

- Amplitude: The maximum and minimum values of $f(x)$ are scaled by k.
- Range: If $f(x)$ has a range $[a, b]$, then $g(x) = kf(x)$ has a range $[k a, k b]$.

Intercepts

- Y-intercept: Since $g(0) = kf(0)$, the y-intercept is scaled by k.
- X-intercepts: The roots of $f(x)$ (where $f(x) = 0$) are unaffected because multiplying by $k \neq 0$ does not change zeros.

Critical Points and Shape

- Critical points, where the derivative $f'(x)$ is zero, are preserved in

$g(x)$, but the height of the peaks and troughs are scaled.
- The overall shape and curvature remain the same, only scaled vertically.

Impact on Specific Types of Functions

Polynomial Functions

- For polynomial functions, multiplying by k compresses the graph vertically.
- The roots of the polynomial remain unchanged, but the graph's maxima and minima are scaled down.

Trigonometric Functions

- For functions like $\sin(x)$ or $\cos(x)$, the amplitude is multiplied by k.
- The period remains unchanged, but the peaks and valleys are closer to the x-axis.

Exponential and Logarithmic Functions

- Exponential functions e^{x} scaled by k maintain their exponential growth but with reduced values.
- Logarithmic functions $\log(x)$ are scaled vertically, affecting their range but not their domain.

Practical Applications of Vertical Compression

Modeling Real-World Phenomena

- Vertical scaling helps in adjusting models to better fit data.
- For example, in physics, scaling the velocity or acceleration functions to match observed data involves similar transformations.

Graph Transformations in Engineering

- Engineers often scale graphs to analyze specific features or to normalize data.

Educational Use

- Demonstrating the effects of scalar multiplication aids students in understanding function transformations and graph behaviors.

Summary of Effects When $0 < k < 1$

Aspect	Effect of Multiplying by k (where $0 < k < 1$)
Overall Graph	Vertically compressed toward the x-axis
Amplitude	Reduced proportionally to k
Range	Scaled down by k
Y-intercept	Multiplied by k
Critical Points	Same x-values; y-values scaled
Shape of Graph	Preserved; only size changes

Conclusion

Multiplying a function $f(x)$ by a scalar k between 0 and 1 results in a vertical compression of its graph. This transformation reduces the height of all points on the graph proportionally, bringing peaks closer to the x-axis and diminishing the amplitude of oscillations or the height of the parabola's vertex. The key features such as roots, critical points, and general shape remain unchanged in their x-positions, but their y-values are scaled.

Understanding this transformation is essential for analyzing how functions behave under scaling, which has practical applications across mathematics, physics, engineering, and data analysis. Recognizing the effects of multiplying by a number less than 1 enables better interpretation of scaled models and graphical data, providing deeper insight into the relationships and behaviors of various functions.

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Keywords: function transformation, vertical compression, scalar multiplication, graph of $f(x)$, scaling effects, amplitude reduction, graph features

Frequently Asked Questions

How does multiplying a function f(x) by a number between 0 and 1 affect its graph?

It causes the graph of f(x) to be vertically compressed or scaled down, making it appear closer to the x-axis without changing its shape.

Does multiplying f(x) by a number less than 1 change the x-intercepts of the graph?

No, the x-intercepts remain the same because multiplying by a scalar only affects the y-values, not where the function crosses the x-axis.

What is the visual effect on the graph of f(x) when it is multiplied by a scalar between 0 and 1?

The graph becomes shorter vertically, with all points closer to the x-axis, effectively compressing it vertically.

If f(x) is scaled by a factor of 0.5, how does the amplitude of the graph change?

The amplitude is halved, meaning the peaks and troughs are reduced to half their original height.

Does multiplying by a number between 0 and 1 affect the domain of f(x)?

No, the domain remains unchanged; only the y-values are scaled.

Can multiplying by a number between 0 and 1 turn a positive function into a negative one?

No, multiplying by a positive number less than 1 will not change the sign of the function's output; negative or positive values are preserved.

How does this multiplication impact the overall shape of the graph for functions like sine or cosine?

It compresses the oscillations vertically, reducing the height of the waves but not altering their frequency or period.

Is the point (x, 0) affected when multiplying f(x) by a number between 0 and 1?

Yes, if f(x) passes through (x, 0), that point remains unchanged because scaling by a positive scalar preserves zeros of the function.

Additional Resources

What Happens To The Graph Of F(x) When It Is Multiplied By A Number Between 0 And 1?

Understanding how the graph of a function transforms under various manipulations is fundamental in mathematics, especially in the study of functions and their properties. A common transformation involves multiplying the output of a function by a scalar value. When this scalar is a number between 0 and 1, it produces a distinctive effect on the graph—primarily, a vertical compression or "shrinking." This article provides a comprehensive analysis of this transformation, exploring its geometric interpretation, algebraic implications, effects on key function features, and practical applications.

Introduction to Function Transformations

Before delving into the specific case of multiplying a function by a number between 0 and 1, it’s essential to establish a foundational understanding of function transformations.

A function transformation involves changing the graph of a function to produce a new graph. Common transformations include shifts, stretches, compressions, and reflections. These transformations alter the original graph's position, size, or orientation without changing the fundamental nature of the function.

Multiplying the output $f(x)$ by a scalar a (where a is a real number) is one such transformation, often called a vertical scaling or vertical dilation. The effect depends heavily on the value of a.

Understanding the Scalar Multiplier: Values Between 0 and 1

When the scalar a satisfies $0 < a < 1$, the transformation can be summarized as:

$$
g(x) = a \times f(x)
$$

This operation scales the original function's output values by a factor less than 1, leading to a vertical compression. Unlike a scalar greater than 1, which stretches the graph vertically, a scalar between 0 and 1 compresses the graph toward the x-axis.

Key characteristics of the scalar a in (0,1):

- Vertical Compression: The graph becomes "squished" vertically, reducing the amplitude of the function.

- Preservation of Domain: The domain remains unchanged since the transformation only affects y-values.
- Alteration of Range: The range is scaled by a, leading to smaller maximum and minimum values if the original function is bounded.

Geometric Interpretation of Vertical Compression

Visualizing the effect of multiplying by a scalar between 0 and 1 can be approached geometrically:

- Original Graph: Consider the graph of $f(x)$, with points $(x, f(x))$.
- Transformed Graph: After multiplying by a, each point becomes $(x, a \times f(x))$.
- Effect: For every point on the original graph, the y-coordinate is scaled down proportionally. The larger the original y-value, the more it shrinks toward the x-axis.

This process is akin to pressing or squeezing the graph vertically toward the x-axis, without affecting its horizontal positioning.

Algebraic and Analytical Effects

Multiplying a function by a scalar between 0 and 1 influences various algebraic features of the function:

1. Amplitude Changes in Periodic Functions

For functions like sine or cosine, which have a defined amplitude, the transformation reduces the amplitude:

$$
f(x) = A \sin(Bx + C) \implies g(x) = a \times A \sin(Bx + C)
$$

The new amplitude becomes $|a| \times A$, which falls within $[0, A)$. This results in a graph oscillating between $\pm aA$ instead of $\pm A$.

2. Vertical Distance from the x-axis

In general, the maximum and minimum values of the original function are scaled:

- Original maximum: f_{max}
- Transformed maximum: $a \times f_{max}$

Similarly for minimum values, the entire range contracts toward zero.

3. Effect on Critical Points and Intervals

Critical points (where the derivative is zero or undefined) are preserved in the x-coordinate but their y-values are scaled:

$$
g'(x) = a \times f'(x)
$$

Thus, the location of maxima, minima, or inflection points remains the same, but the function's height at these points diminishes.

Impact on Key Features of the Graph

The transformation modifies various important aspects of the graph:

1. Intercepts

- Y-intercept: The point at $(0, f(0))$ becomes $(0, a \times f(0))$. The y-intercept moves closer to the x-axis unless $f(0) = 0$.
- X-intercepts: These occur where $f(x) = 0$. Since multiplying by a leaves zeros unchanged, the x-intercepts remain the same.

2. Range and Domain

- Domain: Unaffected. The set of all x-values where the function is defined remains the same.
- Range: Scaled by a. If the original range is $[f_{min}, f_{max}]$, the new range is $[a \times f_{min}, a \times f_{max}]$.

3. Symmetry and Shape

- Symmetries such as even or odd properties are preserved because the transformation is a scalar multiplication, not a reflection.
- The overall shape remains similar but compressed vertically.

Special Considerations for Different Types of Functions

Different classes of functions respond uniquely to this transformation:

1. Polynomial Functions

Vertical compression reduces the y-values uniformly, but the degree and roots remain unchanged.

2. Trigonometric Functions

Amplitude diminishes proportionally, but the period, phase shift, and frequency stay intact.

3. Exponential and Logarithmic Functions

- For exponential functions like $f(x) = e^x$, the shape remains exponential, but the y-values are scaled.
- For logarithmic functions, the transformation compresses the graph vertically, affecting the y-values but not the domain.

4. Piecewise and Discontinuous Functions

The transformation applies pointwise, affecting only the y-values, so the domain and points of discontinuity are unaffected.

Practical Implications and Applications

Understanding the effect of multiplying a function by a scalar between 0 and 1 is crucial in various fields:

- Physics: Damping oscillations in mechanics or electrical engineering, where amplitude reduces over time.
- Economics: Modeling scenarios where quantities diminish proportionally.
- Data Visualization: Adjusting the scale of data representations to improve clarity.
- Signal Processing: Attenuation of signals by scalar factors.

In real-world modeling, choosing a scalar less than 1 can represent decay, attenuation, or "shrinking" effects, providing insights into system behaviors.

Summary and Conclusions

Multiplying a function $f(x)$ by a number a such that $0 < a < 1$ produces a clear and consistent transformation: a vertical compression of the graph towards the x-axis.

Key takeaways:

- The graph is scaled vertically by a factor of a.
- The domain remains unchanged.
- The x-intercepts are unaffected, but y-values and the overall height are reduced.
- The shape and symmetry of the graph are preserved.
- The transformation impacts key features like amplitude, maximum/minimum values, and range, but not the critical points' locations.

This transformation is fundamental in understanding how functions can be manipulated to model real-world phenomena involving proportional reductions, damping, or attenuation. Recognizing the geometric and algebraic implications of such scalar multiplications enhances mathematical intuition and analytical capabilities.

In conclusion, multiplying the graph of a function by a scalar between 0 and 1 compresses the graph vertically, bringing the curve closer to the x-axis without altering its domain or the x-coordinates of key points. This simple yet powerful transformation forms a cornerstone of function analysis, with wide-ranging applications across science, engineering, and mathematics.

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