

What Is An Equation Of The Line That Passes Through The Points $(-5, -7)$ And $(5, 1)$

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Understanding how to find the equation of a line that passes through two specific points is a fundamental skill in algebra and coordinate geometry. In this article, we will explore the step-by-step process to determine the equation of the line passing through the points $(-5, -7)$ and $(5, 1)$. This involves calculating the slope, determining the line's equation in slope-intercept form, and understanding the significance of each component within the equation. Whether you're a student preparing for exams or a math enthusiast looking to deepen your understanding, mastering this concept is essential for analyzing linear relationships in various contexts.

Understanding the Basics of a Line Equation

What Is a Line Equation?

A line equation describes the relationship between the x and y coordinates of any point lying on a straight line. The most common form is the slope-intercept form:

- $y = mx + b$

where:

- m is the slope of the line, indicating its steepness.
- b is the y-intercept, the point where the line crosses the y-axis.

Knowing the equation of a line allows us to predict y-values for any given x-value, understand the line's direction, and analyze relationships between variables.

The Importance of Two Points in Determining a Line

A straight line in a plane is uniquely determined by any two distinct points. Given two points, you can:

- Calculate the slope.
- Use one of the points and the slope to find the line's equation.

This process applies universally, making it a foundational technique in coordinate geometry.

Calculating the Slope of the Line

Definition of Slope

The slope (m) quantifies the rate of change of y relative to x . It measures how much y increases or decreases as x increases.

Formula for Slope Between Two Points

Given two points $((x_1, y_1))$ and $((x_2, y_2))$, the slope is calculated as:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

This ratio represents the "rise over run," or how much y changes over the change in x.

Applying the Formula to the Given Points

Using the points:

- Point 1: $((-5, -7))$

- Point 2: $((5, 1))$

Calculate the slope:

$$m = (1 - (-7)) / (5 - (-5))$$

which simplifies to:

$$m = (1 + 7) / (5 + 5) = 8 / 10 = 4 / 5$$

Therefore, the slope $(m = \frac{4}{5})$.

Determining the Equation of the Line

Using the Point-Slope Form

The point-slope form of a line's equation is:

- $y - y_1 = m(x - x_1)$

Where:

- $((x_1, y_1))$ is a point on the line.

- (m) is the slope.

Choosing either point for substitution provides a straightforward route to the line's equation.

Applying the Point-Slope Formula

Let's select the point $((-5, -7))$ and the slope $(\frac{4}{5})$.

Substitute into the formula:

$$y - (-7) = (4/5)(x - (-5))$$

Simplify:

$$y + 7 = (4/5)(x + 5)$$

Expanding and Simplifying to Slope-Intercept Form

Distribute the slope:

$$y + 7 = (4/5)x + (4/5) \cdot 5$$

which simplifies to:

$$y + 7 = (4/5)x + 4$$

Subtract 7 from both sides:

$$y = (4/5)x + 4 - 7$$

resulting in:

$$y = (4/5)x - 3$$

This is the slope-intercept form of the line passing through (-5, -7) and (5, 1).

Verifying the Line Equation

Check with the Second Point

To confirm the accuracy, substitute the second point $((5, 1))$ into the equation:

$$y = (4/5)x - 3$$

Calculate:

$$y = (4/5)(5) - 3 = 4 - 3 = 1$$

Since the calculated y-value matches the point's y-coordinate, the line equation correctly passes through both points.

Graphical Interpretation

Plotting the points and the line on a coordinate plane provides a visual confirmation:

- The line crosses the y-axis at $(0, -3)$.
- It passes through both $(-5, -7)$ and $(5, 1)$.
- The slope of $\left(\frac{4}{5}\right)$ indicates that for every 5 units moved horizontally, the line rises 4 units vertically.

Understanding the Significance of the Line Equation

Applications of Line Equations

Line equations are fundamental in various fields, including:

- Engineering: designing structures and analyzing forces.
- Economics: modeling cost and revenue functions.
- Physics: describing motion and forces.
- Data Science: fitting linear models to data.

Interpreting the Slope and Y-Intercept

- The slope $\left(\frac{4}{5}\right)$ indicates a positive, moderate rate of increase.
- The y-intercept $(0, -3)$ shows where the line crosses the y-axis, serving as a baseline for the relationship.

Extensions and Variations

- Standard Form: $Ax + By = C$

- Point-Slope Form: useful when a point and slope are known.
- Two-Point Form: directly uses two points for the line's equation.

Summary and Key Takeaways

- To find the equation of a line passing through two points, first calculate the slope using the coordinate formula.
- Use the point-slope form to derive the line's equation, then simplify to slope-intercept form.
- Verify the line by substituting the second point.
- The final equation provides a complete algebraic description of the line's behavior.

Conclusion

Deriving the equation of a line that passes through two points is a vital skill in algebra. In the case of points $(-5, -7)$ and $(5, 1)$, we calculated the slope as $\left(\frac{4}{5}\right)$, then used point-slope form to find the line's equation as $\left(y = \frac{4}{5}x - 3\right)$. This comprehensive process illustrates how simple algebraic techniques enable us to understand and represent linear relationships accurately. Mastery of these concepts enhances problem-solving skills and prepares you for more advanced topics in mathematics, science, and engineering.

Meta Description: Learn how to find the equation of the line passing through points $(-5, -7)$ and $(5, 1)$. Step-by-step guide to calculating the slope, deriving the line equation, and understanding its significance in coordinate geometry.

Frequently Asked Questions

How do I find the equation of a line passing through points $(-5, -7)$ and $(5, 1)$?

First, calculate the slope using $(y_2 - y_1) / (x_2 - x_1)$. Then, use point-slope form with one of the points to find the equation.

What is the slope of the line passing through $(-5, -7)$ and $(5, 1)$?

The slope is $(1 - (-7)) / (5 - (-5)) = 8 / 10 = 0.8$.

How do I write the equation of a line given two points?

Calculate the slope, then use the point-slope form $y - y_1 = m(x - x_1)$, plugging in one of the points and the slope.

What is the equation of the line passing through $(-5, -7)$ and $(5, 1)$?

Using the slope 0.8 and point $(-5, -7)$, the equation in point-slope form is $y + 7 = 0.8(x + 5)$. Simplified, it becomes $y = 0.8x - 3$.

How can I verify that the points $(-5, -7)$ and $(5, 1)$ lie on the same line?

Substitute each point into the line's equation to see if both satisfy it. For $y = 0.8x - 3$, plugging in $x = -5$ and $x = 5$ confirms this.

What is the slope-intercept form of the line passing through $(-5, -7)$ and $(5, 1)$?

The slope-intercept form is $y = 0.8x - 3$.

Can I use the two-point form to find the line equation?

Yes, the two-point form is $y - y_1 = [(y_2 - y_1)/(x_2 - x_1)](x - x_1)$. Using this with your points yields the same equation.

Why is it important to find the equation of a line passing through two points?

It allows you to predict or analyze the relationship between variables, graph the line, and solve related problems efficiently.

Additional Resources

What Is An Equation Of The Line That Passes Through The Points (-5, -7) And (5, 1)

When analyzing relationships between two variables, mathematicians often seek to describe their connection using a straight line. Such a line can be expressed mathematically through an equation, which encapsulates all possible points lying along the line. In particular, understanding how to determine the equation of a line passing through specific points is foundational in algebra, geometry, and numerous applied fields like engineering and data science. Today, we will explore how to find the equation of a line that passes through the points (-5, -7) and (5, 1), unraveling the process step by step in a clear, accessible manner.

Understanding the Basics: What Is a Line Equation?

Before diving into calculations, it's important to understand what a line equation represents. In a two-dimensional coordinate system, any straight line can be described by an algebraic equation. The most

common form is the slope-intercept form:

$$y = mx + b$$

- m is the slope of the line, indicating its steepness.
- b is the y-intercept, the point where the line crosses the y-axis (when $x=0$).

Alternatively, lines can be represented in point-slope form:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line, and m is the slope.

To find the specific equation of the line passing through two points, the key steps involve calculating the slope and then using one of the points to find the equation.

Step 1: Calculating the Slope (m)

The slope of a line quantifies its incline or decline. It is calculated based on the change in y-values relative to the change in x-values between the two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given the points:

- $(-5, -7)$ as (x_1, y_1)
- $(5, 1)$ as (x_2, y_2)

Plugging into the formula:

$$\left[m = \frac{1 - (-7)}{5 - (-5)} = \frac{1 + 7}{5 + 5} = \frac{8}{10} = \frac{4}{5} \right]$$

So, the slope of the line passing through these points is $\left(\frac{4}{5} \right)$.

Step 2: Using the Point-Slope Form to Find the Equation

With the slope known, the next step is to write the equation in point-slope form, choosing either of the two points. Let's select $((-5, -7))$.

Plugging into $(y - y_1 = m(x - x_1))$:

$$\left[y - (-7) = \frac{4}{5}(x - (-5)) \right]$$

Simplify:

$$\left[y + 7 = \frac{4}{5}(x + 5) \right]$$

This is the point-slope form, which clearly indicates the point and the slope.

Step 3: Converting to Slope-Intercept Form

To make the line's equation more familiar and practical, especially for graphing, it's useful to convert it into slope-intercept form $(y = mx + b)$.

Starting from:

$$y + 7 = \frac{4}{5}(x + 5)$$

Distribute the slope:

$$y + 7 = \frac{4}{5}x + \frac{4}{5} \times 5$$

$$y + 7 = \frac{4}{5}x + 4$$

Subtract 7 from both sides:

$$y = \frac{4}{5}x + 4 - 7$$

$$y = \frac{4}{5}x - 3$$

This reveals the final equation of the line:

$$y = \frac{4}{5}x - 3$$

Interpreting the Line Equation

Understanding the components of this equation provides insights into the line's behavior:

- Slope ($\frac{4}{5}$): The line rises 4 units vertically for every 5 units it moves horizontally. This positive slope indicates an upward trend from left to right.
- Y-intercept (-3): The line crosses the y-axis at $(0, -3)$. When $x=0$, $y=-3$.

Verifying the Equation with the Original Points

To ensure our derived equation is accurate, let's verify that both original points satisfy it.

1. For $((-5, -7))$:

$$[y = \frac{4}{5}x - 3]$$

$$[y = \frac{4}{5} \times (-5) - 3 = -4 - 3 = -7]$$

Match confirmed.

2. For $((5, 1))$:

$$[y = \frac{4}{5} \times 5 - 3 = 4 - 3 = 1]$$

Match confirmed.

This verification confirms that $(y = \frac{4}{5}x - 3)$ is the correct equation for the line passing through the two points.

Additional Considerations and Applications

Why is this important? Understanding how to find a line's equation from two points is fundamental in

various fields:

- Data analysis: Regression lines approximate relationships between variables.
- Physics: Describing motion or relationships between quantities.
- Engineering: Designing components or systems based on linear relationships.
- Navigation and mapping: Plotting courses or paths.

Furthermore, this process exemplifies the general method used to derive line equations when dealing with real-world data points, highlighting the importance of the slope and intercept in understanding linear relationships.

Summary: The Complete Process

To summarize, finding the line passing through two points involves:

1. Calculating the slope using the difference in y-values divided by the difference in x-values.
2. Choosing one of the points and applying the point-slope form.
3. Simplifying to slope-intercept form for clarity and practical use.
4. Verifying the equation with the original points.

The line passing through $(-5, -7)$ and $(5, 1)$ is:

$$[y = \frac{4}{5}x - 3]$$

This straightforward yet powerful process provides a foundation for more advanced exploration of linear equations, enabling students and professionals alike to analyze and interpret relationships within data and systems effectively.

In conclusion, understanding how to derive the equation of a line through given points is a vital skill in mathematics, with broad applications across science, technology, engineering, and beyond. With practice, identifying slopes, applying algebraic transformations, and verifying your solutions becomes an accessible and invaluable part of your analytical toolkit.

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