What Is The Eighth Term Of The Arithmetic Sequence Defined As: A(n) = 21 2(n - 1)

What Is The Eighth Term Of The Arithmetic Sequence Defined As: A(n) = 21 2(n - 1)

Understanding sequences, particularly arithmetic sequences, is fundamental in mathematics, as they form the basis for many advanced topics such as series, algebra, and calculus. The sequence in question is defined by the formula A(n) = 21 2(n - 1), which indicates a specific pattern and rule governing each term. This article will delve into analyzing this sequence, understanding its structure, and ultimately determining the eighth term.

Analyzing the Given Sequence Formula

Understanding the Formula Structure

The formula provided is:

A(n) = 21 2(n - 1)

At first glance, this appears to be a geometric sequence rather than an arithmetic one. Typically, an arithmetic sequence has a common difference, with each term increasing or decreasing by a fixed amount. Conversely, a geometric sequence involves each term being multiplied by a common ratio.

In this context, the formula suggests that:

- The initial term (when $n=1$) is $A(1) = 21\ 2(1 - 1) = 21\ 2^0 = 21\ 1 = 21$
- The subsequent terms grow by multiplying by 2 each time, indicating a geometric progression.

Therefore, despite the initial statement referring to an arithmetic sequence, the formula actually defines a geometric sequence with a first term of 21 and a common ratio of 2.

Clarifying the Sequence Type

Given the formula $A(n) = 21\ 2^{n-1}$, the sequence can be characterized as:

- First term, $A(1) = 21$
- Common ratio, $r = 2$

The sequence progresses as:
- $A(1) = 21$
- $A(2) = 21\ 2$
- $A(3) = 21\ 2^2$
- $A(4) = 21\ 2^3$
- and so on.

Note: If the sequence were truly arithmetic, the difference between consecutive terms would be constant. Here, the difference varies, confirming it's geometric.

Calculating the Eighth Term

Applying the Formula for the nth Term

Given the formula:

$$A(n) = 21 \ 2^{n - 1}$$

To find the eighth term, substitute $n=8$:

$$A(8) = 21 \ 2^{8 - 1} = 21 \ 2^7$$

Calculating 2^7

Recall that:

$$2^7 = 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 = 128$$

Therefore,

$$A(8) = 21 \ 128$$

Final Calculation

Multiply 21 by 128:

$$21 \ 128 = (20 + 1) \ 128 = (20 \ 128) + (1 \ 128) = 2560 + 128 = 2688$$

Result:

The eighth term, $A(8)$, of the sequence is 2688.

Understanding the Nature of the Sequence

Is It Arithmetic or Geometric?

Despite the initial description, the sequence defined by A(n) = 21 2^{n-1} is a geometric sequence, not an arithmetic one.

- Arithmetic sequence characteristics:
- Difference between consecutive terms is constant.
- Formula: A(n) = A(1) + (n - 1)d, where d is the common difference.
- Geometric sequence characteristics:
- Ratio between consecutive terms is constant.
- Formula: A(n) = A(1) r^{n - 1}, where r is the common ratio.

Since our formula matches the geometric sequence pattern, the correct classification is geometric.

Implications of the Sequence Type

Knowing the sequence is geometric helps in understanding its growth pattern:

- The terms increase exponentially.
- The sequence doubles each time, as indicated by the ratio r=2.
- The sequence grows rapidly, which is typical in exponential growth models such as population growth, radioactive decay, and financial interest calculations.

Additional Details and Applications

Visualizing the Sequence

Plotting the sequence shows rapid exponential growth, with the terms increasing by a factor of 2 each time:

n	Term A(n)
1	21
2	42
3	84
4	168
5	336
6	672
7	1344
8	2688

This table demonstrates the exponential pattern clearly.

Real-World Examples

Sequences like this are prevalent in various fields:

- Finance: Compound interest calculations.
- Biology: Population growth models.
- Physics: Decay or amplification processes.
- Computer Science: Algorithm complexity and data growth.

Summary and Conclusion

The sequence defined by $A(n) = 21\ 2^{n-1}$ is a geometric sequence with a first term of 21 and a common ratio of 2. The eighth term in this sequence is obtained by plugging $n=8$ into the formula:

$$A(8) = 21\ 2^{7} = 21\ 128 = 2688$$

Therefore, the eighth term of the sequence is 2688.

This calculation illustrates the power of understanding sequence formulas—whether arithmetic or geometric—and how to manipulate them to find specific terms. Recognizing the pattern type is crucial, as it dictates the method used for calculations and interpretations. Although the initial description referenced an arithmetic sequence, the formula itself reveals the sequence as geometric, emphasizing the importance of carefully analyzing the given formula to determine the correct sequence type and properties.

In conclusion, the eighth term of the sequence defined by $A(n) = 21\ 2^{n-1}$ is 2688, a result derived through understanding the sequence's exponential nature and applying the appropriate formula.

Frequently Asked Questions

What is the general formula for the nth term of the arithmetic sequence given as A(n) = 21 2(n – 1)?

The general formula is $A(n) = 21\ 2^{(n - 1)}$.

How do you find the eighth term (A(8)) of the sequence defined by A(n) = 21 2(n – 1)?

Substitute $n = 8$ into the formula: $A(8) = 21\ 2^{(8 - 1)} = 21\ 2^7 = 21\ 128 = 2688$.

Is the sequence A(n) = 21 2(n – 1) an arithmetic sequence?

No, it is a geometric sequence because each term is multiplied by 2 to get the next, not added by a constant difference.

What is the value of the eighth term in the sequence A(n) = 21 2(n –

1)?

The eighth term is 2688.

Can the formula A(n) = 21 2(n - 1) be used to find any term in the sequence?

Yes, by plugging in the value of n into the formula, you can find any term in the sequence.

What type of sequence is represented by A(n) = 21 2(n - 1)?

It is a geometric sequence due to the common ratio of 2 between terms.

How does the eighth term relate to the first term in this sequence?

The first term is A(1) = 21 2^0 = 21, and the eighth term is 2688, which is 128 times the first term, since 2^7 = 128.

Additional Resources

What Is The Eighth Term Of The Arithmetic Sequence Defined As: A(n) = 21 ▯ 2(n - 1)?

Introduction

In the realm of mathematics, sequences serve as fundamental building blocks for understanding patterns, relationships, and structures within numbers. Among these, arithmetic sequences are perhaps the most straightforward and widely studied. They form the backbone of many mathematical concepts, from basic algebra to advanced calculus.

This article delves into an intriguing problem involving an arithmetic sequence defined by the formula:

$$A(n) = 21 \times 2^{(n - 1)}$$

At first glance, this may seem like a simple formula, but it conceals a wealth of mathematical insights, especially when examining specific terms like the eighth element. Understanding what the eighth term of this sequence is requires a deep dive into the sequence's structure, properties, and implications.

Understanding the Sequence: Is It Truly Arithmetic?

Before proceeding to find the eighth term, it is crucial to clarify the nature of the sequence. The question arises: Is the sequence $A(n) = 21 \times 2^{(n - 1)}$ an arithmetic sequence?

Definition of an Arithmetic Sequence

An arithmetic sequence is a sequence of numbers where each term after the first is obtained by adding a constant difference, denoted as d:

$$A(n) = A(1) + (n - 1)d$$

for all $n \geq 1$.

Analyzing the Given Sequence

The sequence in question is:

$$A(n) = 21 \times 2^{(n - 1)}$$

This is a geometric sequence, not an arithmetic one, because each term is multiplied by a fixed ratio rather than increased by a fixed difference. The common ratio r is 2:

$$ r = \frac{A(n+1)}{A(n)} = \frac{21 \times 2^{n}}{21 \times 2^{n-1}} = 2 $$

Conclusion: The sequence $A(n) = 21 \times 2^{(n - 1)}$ is geometric, with the first term $A(1) = 21$, and common ratio $r = 2$.

Clarification: Is the Sequence Arithmetic or Geometric?

Given the formula, the sequence is geometric, not arithmetic. The title's mention of "arithmetic sequence" appears to be a misnomer or a deliberate challenge. For clarity, let's explore the properties of this sequence and then address the specific question: What is the eighth term?

Calculating the Eighth Term

Given the sequence:

$$ A(n) = 21 \times 2^{(n - 1)} $$

the eighth term, $A(8)$, is:

$$ A(8) = 21 \times 2^{(8 - 1)} = 21 \times 2^{7} $$

Since $2^{7} = 128$, we have:

$$ A(8) = 21 \times 128 = 2688 $$

Answer: The eighth term of the sequence is 2,688.

Deep Dive: The Nature of the Sequence and Its Implications

While the initial question seems straightforward, exploring the properties and implications of such a sequence offers valuable insights.

---

1. Geometric Sequence Characteristics

- First Term ($A(1)$): 21
- Common Ratio (r): 2
- General term: $A(n) = 21 \times 2^{n-1}$
- Growth Pattern: Exponential increase; each term doubles the previous one.

This exponential growth model appears widely in contexts like population dynamics, radioactive decay, and computer science algorithms involving binary operations.

---

2. Visual Representation and Pattern Recognition

Plotting the sequence reveals an exponential curve, illustrating rapid growth. For the first ten terms:

n	$A(n)$
1	21
2	42

3	84
4	168
5	336
6	672
7	1344
8	2688
9	5376
10	10752

This table underscores the exponential acceleration as n increases.

Contextual Applications and Broader Significance

Understanding sequences like $A(n) = 21 \times 2^{n-1}$ has broad applications:

- Computer Science: Binary data structures, such as tree growth or memory allocation, often follow exponential patterns.
- Biology: Population models where each generation doubles, akin to bacteria proliferation.
- Finance: Compound interest calculations, where growth compounds exponentially.
- Physics: Radioactive decay or chain reactions, which follow exponential laws.

Clarifying the Misnomer: Is It Arithmetic or Geometric?

Given the formula, it is essential to resolve the initial ambiguity. The sequence, as defined, is geometric, not arithmetic, because of the multiplicative pattern.

If the original problem intended to examine an arithmetic sequence, it might have been a misstatement or a trick question. Alternatively, it suggests a broader discussion about sequence types and their properties.

Summary of Key Findings

- The sequence $A(n) = 21 \times 2^{n-1}$ is geometric with first term 21 and common ratio 2.
- The eighth term is 2,688.
- The sequence exhibits exponential growth, doubling each time.

Final Thoughts

Understanding the nature of sequences is foundational for advanced mathematics and real-world applications. Correctly identifying whether a sequence is arithmetic or geometric influences how we analyze and interpret data. In this case, despite initial assumptions, the sequence is geometric, emphasizing the importance of carefully examining sequence definitions before proceeding with calculations.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Rosen, K. H. (2012). Discrete Mathematics and Its Applications. McGraw-Hill.
- Weisstein, Eric W. "Sequence." From MathWorld--A Wolfram Web Resource. https://mathworld.wolfram.com/Sequence.html

---

In conclusion, the eighth term of the sequence $A(n) = 21 \times 2^{n-1}$ is 2,688. Recognizing the sequence's geometric nature is crucial for understanding its growth pattern and applications across various disciplines.

What Is The Eighth Term Of The Arithmetic Sequence Defined As A N 21 2 N 1

What Is The Eighth Term Of The Arithmetic Sequence Defined As A N 21 2 N 1

Related Articles

- Translate This Sentence Into An Equation. Craig's Savings Decreased By 23 Is .55
- In A Refined Grain, Which Part Remains?a) Germb) Branc) Endospermd) All Of The Above
- A Strategy Focuses On How To Execute And Implement A Marketingplan (True Or False)

Back to Home