

What Is The Equation Of The Line That Passes Through The Points (0, 5) And (1, 2)

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Understanding the equation of a line is a fundamental concept in algebra and analytic geometry. It allows us to describe the relationship between two variables, often represented as points on a coordinate plane. When given two points, the task is to find the linear equation that passes through both, which provides a mathematical representation of the line connecting these points. This process involves calculating the slope of the line and then using point-slope or slope-intercept forms to derive the complete equation.

In this article, we will explore in detail how to determine the equation of the line passing through the specific points (0, 5) and (1, 2). We will cover the concepts of slope, point-slope form, slope-intercept form, and the step-by-step process to arrive at the final equation. Additionally, we will discuss practical applications, common mistakes, and tips to master this essential skill in algebra.

Understanding the Basics: Points and Lines in Coordinate Geometry

What Are Points and Lines?

In the Cartesian coordinate system, points are represented as ordered pairs (x, y) , where:

- x is the horizontal coordinate
- y is the vertical coordinate

A line is a one-dimensional figure that extends infinitely in both directions and can be defined mathematically by an equation relating x and y .

The Importance of Finding the Equation of a Line

Knowing the equation of a line enables:

- Prediction of y -values for given x -values
- Graphing the line accurately
- Solving real-world problems involving relationships between two quantities

Given Points: (0, 5) and (1, 2)

The problem involves two specific points:

- Point A: (0, 5)
- Point B: (1, 2)

These points are in the coordinate plane, and our goal is to find the linear equation that passes through both.

Step 1: Calculating the Slope of the Line

What Is the Slope?

The slope, denoted as m , measures the steepness of the line. It is calculated as the ratio of the change in y to the change in x between two points:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating the Slope for Our Points

Using the points (0, 5) and (1, 2):

$$- (x_1 = 0, y_1 = 5)$$

$$- (x_2 = 1, y_2 = 2)$$

Plugging these into the slope formula:

$$m = \frac{2 - 5}{1 - 0} = \frac{-3}{1} = -3$$

Result: The slope of the line is -3.

Step 2: Choosing the Equation Format

There are two common ways to express a line's equation:

$$- \text{Point-Slope Form: } (y - y_1 = m(x - x_1))$$

$$- \text{Slope-Intercept Form: } (y = mx + b)$$

Given the slope and a point, we can use either format. The slope-intercept form is often more

straightforward once the y-intercept b is known.

Step 3: Deriving the Equation of the Line

Using the Point-Slope Form

Select one of the points, for example, $(0, 5)$:

$$\begin{aligned} & \backslash \\ y - 5 &= -3(x - 0) \end{aligned}$$

$\backslash$

$$\begin{aligned} & \backslash \\ y - 5 &= -3x \end{aligned}$$

$\backslash$

$$\begin{aligned} & \backslash \\ y &= -3x + 5 \end{aligned}$$

$\backslash$

This is the equation in slope-intercept form.

Confirming the Equation with the Second Point

Verify the equation with point $(1, 2)$:

$$\begin{aligned} & \backslash \\ y &= -3(1) + 5 = -3 + 5 = 2 \end{aligned}$$

$\backslash$

Since the calculation matches the y-value of the second point, the equation is confirmed as correct.

Final Equation of the Line

The equation of the line passing through (0, 5) and (1, 2) is:

\[

$$\boxed{y = -3x + 5}$$

\]

Additional Insights and Applications

Graphing the Line

- The y-intercept is at (0, 5), meaning the line crosses the y-axis at 5.
- The slope of -3 indicates that for every 1 unit increase in x, y decreases by 3 units.
- Plotting these points and drawing the line is straightforward using the slope and intercept.

Real-World Applications

Understanding how to find this equation is vital in various fields:

- Physics: Describing velocity over time
- Economics: Modeling cost or revenue functions
- Engineering: Analyzing linear relationships in system behaviors
- Data Science: Fitting linear models to data points

Common Mistakes and Tips for Mastery

Common Mistakes

- Mixing up the order of points when calculating slope
- Incorrectly calculating the slope (e.g., sign errors)
- Forgetting to verify the equation with both points
- Confusing point-slope and slope-intercept forms

Tips for Success

- Always label your points clearly
- Double-check the slope calculation
- Use both points to verify the final equation
- Practice with different pairs of points to build confidence

Extended Example: Finding the Equation of a Line Through Any Two Points

Suppose you are given arbitrary points, say, (x_1, y_1) and (x_2, y_2) . The process remains the same:

1. Calculate the slope $(m = \frac{y_2 - y_1}{x_2 - x_1})$
2. Use the point-slope form with either point to find the line's equation
3. Simplify to slope-intercept form (if desired)

This method applies universally, making it a powerful tool in your algebra toolkit.

Conclusion

Determining the equation of a line passing through two points is a fundamental skill in algebra that combines understanding of slope, point-slope form, and slope-intercept form. For the specific points (0, 5) and (1, 2), the line's equation is $y = -3x + 5$. This equation provides a complete description of the line's behavior, including its steepness and intercepts.

Mastering this process enhances your ability to analyze linear relationships in mathematics and real-world scenarios, paving the way for success in higher-level algebra, calculus, and various applied fields. Practice with different sets of points to solidify your understanding and develop confidence in deriving line equations efficiently.

Remember: Always verify your equations by plugging in both points to ensure accuracy, and keep practicing with diverse examples to strengthen your skills in analytic geometry.

Frequently Asked Questions

How do you find the equation of a line passing through the points (0, 5) and (1, 2)?

First, calculate the slope using the formula $m = (y_2 - y_1) / (x_2 - x_1)$: $m = (2 - 5) / (1 - 0) = -3 / 1 = -3$. Then, use point-slope form with one point, for example (0, 5): $y - 5 = -3(x - 0)$. Simplifying gives $y =$

$-3x + 5$.

What is the slope of the line passing through (0, 5) and (1, 2)?

The slope is -3, calculated by $(2 - 5) / (1 - 0) = -3$.

What is the equation of the line passing through the points (0, 5) and (1, 2)?

The equation is $y = -3x + 5$.

Why does the point (0, 5) make the equation $y = -3x + 5$?

Because plugging in $x = 0$ gives $y = -3(0) + 5 = 5$, matching the point (0, 5).

Can the equation of the line be written in slope-intercept form? If so, what is it?

Yes, the line's equation in slope-intercept form is $y = -3x + 5$.

How do you verify that the line passes through both (0, 5) and (1, 2)?

Substitute $x = 0$ into $y = -3x + 5$, get $y = 5$, matching the first point. Substitute $x = 1$, $y = -3(1) + 5 = 2$, matching the second point.

What is the significance of point (0, 5) in the line's equation?

Point (0, 5) is the y-intercept of the line, indicating where the line crosses the y-axis.

Additional Resources

What Is The Equation Of The Line That Passes Through The Points (0, 5) And (1, 2)

Understanding the equation of a line that passes through specific points is fundamental in algebra and analytic geometry. This process not only solidifies comprehension of the slope-intercept form but also enhances problem-solving skills for more complex geometric and algebraic scenarios. In this article, we delve into the principles underlying the derivation of a line's equation passing through the points (0, 5) and (1, 2), exploring the concepts of slope, intercepts, and the algebraic steps involved. By the end, readers will have a clear, detailed understanding of how to determine the equation given two points, along with insights into broader applications and significance.

Foundational Concepts in Line Equations

Before tackling the specific problem, it is important to review some foundational concepts that underpin the process of deriving the equation of a line.

Coordinates and Points

In the Cartesian coordinate system, any point on a plane is represented by an ordered pair (x, y) , where:

- x-coordinate indicates the position along the horizontal axis (the x-axis).
- y-coordinate indicates the position along the vertical axis (the y-axis).

In our case, the points are:

- Point A: (0, 5)
- Point B: (1, 2)

Slope of a Line

The slope (denoted as m) measures the steepness and direction of a line. It is calculated as the ratio of the change in y-coordinates to the change in x-coordinates between two points:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

This value indicates how much y increases or decreases as x increases by a unit.

Equation of a Line

The general form of a line's equation can be expressed in several ways:

- Slope-intercept form: $y = mx + b$, where b is the y-intercept.
- Point-slope form: $y - y_1 = m(x - x_1)$, which uses a specific point and the slope.
- Standard form: $Ax + By + C = 0$.

Our goal is to determine the specific equation in slope-intercept form, using the given points.

Calculating the Slope Between the Points (0, 5) and (1, 2)

The first step in deriving the line's equation is to find its slope, as this characterizes the line's tilt.

Applying the Slope Formula

Using the points:

$$- \text{ } (x_1, y_1) = (0, 5) \text{ }$$

$$- \text{ } (x_2, y_2) = (1, 2) \text{ }$$

We compute:

$$[m = \frac{2 - 5}{1 - 0} = \frac{-3}{1} = -3]$$

This indicates that for every unit increase in x, y decreases by 3 units. The negative slope signifies a downward slant from left to right.

Implication: The line is decreasing, falling as we move from left to right, with a slope of -3.

Deriving the Equation of the Line

Having established the slope, the next step is to write the equation in a form suitable for analysis and application.

Using the Point-Slope Form

The point-slope form is particularly useful because it directly incorporates a known point and the slope:

$$[y - y_1 = m(x - x_1)]$$

Choosing the point $(0, 5)$:

$$[y - 5 = -3(x - 0)]$$

Simplifying:

$$\backslash y - 5 = -3x \backslash$$

$$\backslash y = -3x + 5 \backslash$$

This is the slope-intercept form of the line passing through (0, 5) and (1, 2).

Verifying with the Second Point

To ensure accuracy, substitute the second point $\backslash (1, 2) \backslash$ into the derived equation:

$$\backslash y = -3x + 5 \backslash$$

$$\backslash 2 = -3(1) + 5 \backslash$$

$$\backslash 2 = -3 + 5 \backslash$$

$$\backslash 2 = 2 \backslash$$

This confirms the equation correctly represents the line passing through both points.

Interpreting the Equation and Its Features

Understanding the derived equation's components provides deeper insight into the line's geometric characteristics.

The Slope ($m = -3$)

- Indicates the rate of change of y with respect to x .
- Negative value signifies a downward trend as x increases.
- Steepness is relatively significant; for each unit increase in x , y decreases by 3 units.

The Y-Intercept ($b = 5$)

- The point where the line crosses the y -axis.
- In this case, at $(0, 5)$, directly corresponding to the point given.

Graphical Representation

Plotting the points:

- Point A at $(0, 5)$: located on the y -axis.
- Point B at $(1, 2)$: one unit to the right and three units down from $(0, 5)$.

Drawing the line through these points confirms the slope and intercept, illustrating a decreasing line crossing the y -axis at 5 and passing through $(1, 2)$.

Broader Applications and Significance

While deriving the line's equation for these specific points may seem straightforward, the process exemplifies broader mathematical principles applicable in various fields.

In Physics

- Linear relationships between variables, such as speed versus time or force versus displacement, are modeled with similar equations.
- The slope indicates the rate of change, crucial in understanding physical phenomena.

In Economics

- Cost functions and demand curves often involve lines with specific slopes and intercepts.
- Understanding these relationships helps in making informed business decisions.

In Data Science and Engineering

- Linear regression relies on deriving the best-fit line through data points.
- The concepts of slope and intercept underpin predictive modeling and analysis.

Alternative Methods and Considerations

Besides the straightforward calculation, other approaches and considerations can be relevant.

Using the Two-Point Formula in General

The two-point form generalizes to any pair of points:

$$[y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)]$$

which ensures flexibility when points are known but the equation is not.

Standard Form Conversion

The slope-intercept form can be converted into standard form:

$$[y = -3x + 5 \rightarrow 3x + y = 5]$$

which might be preferred in certain algebraic contexts.

Limitations and Special Cases

- Vertical lines: When $(x_1 = x_2)$, the slope formula results in division by zero, indicating an undefined slope, and the equation takes the form $(x = a)$.

- Horizontal lines: When $(y_1 = y_2)$, the slope is zero, producing a line $(y = c)$.

In our specific problem, the slope is well-defined and neither of these special cases apply.

Conclusion: The Final Equation and Its Significance

In summary, the line passing through the points (0, 5) and (1, 2) has the equation:

$$[\boxed{ y = -3x + 5 }]$$

This equation encapsulates the geometric and algebraic relationship between these points, embodying the slope's rate of change and the y-intercept's position. Its derivation exemplifies core principles in

coordinate geometry, providing a foundation for more complex analytical tasks across scientific, economic, and engineering domains. Understanding how to move from points to an explicit line equation reinforces critical problem-solving skills and promotes a deeper appreciation for the elegant interconnectedness of algebraic concepts. As such, mastering this process is a vital step in mathematical literacy and applied analysis.

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