What Is The Slope Of The Line That Passes Through The Points (26, 7) And (-39, 12)?

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Understanding the concept of slope is fundamental in the study of coordinate geometry. When analyzing the relationship between two points on a Cartesian plane, the slope provides insight into how steep the line connecting these points is and the direction in which it inclines or declines. In this article, we will explore how to determine the slope of a line passing through two specific points: (26, 7) and (-39, 12). We will break down the process step-by-step, discuss the significance of slope in various mathematical contexts, and provide practical examples to enhance comprehension.

Introduction to Slope in Coordinate Geometry

Before diving into the calculation, it's important to understand what the slope represents in the context of coordinate geometry.

What Is Slope?

The slope of a line measures its steepness and is usually denoted by the letter m. Mathematically, the slope is defined as the ratio of the vertical change (rise) to the horizontal change (run) between two points on the line. It indicates how much y increases or decreases as x increases.

Mathematically, the slope between two points (x_1, y_1) and (x_2, y_2) is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This formula is fundamental in coordinate geometry and is used to find the slope of straight lines, determine their nature, and analyze relationships between variables.

Why Is Slope Important?

The concept of slope is crucial for various reasons:

- Determining Line Direction: Positive slopes indicate lines that ascend from left to right, negative slopes indicate descent, and zero slope signifies a horizontal line.
- Calculating Equation of a Line: Slope is a key component in the point-slope and slope-intercept forms of linear equations.
- Analyzing Data Trends: Slope helps in understanding the rate of change in real-world data, such as speed, growth rates, and more.
- Parallel and Perpendicular Lines: Parallel lines have equal slopes, while perpendicular lines have slopes that are negative reciprocals.

Calculating the Slope Between Two Points

Let's now focus on the specific points provided: (26, 7) and (-39, 12). To find the slope of the line passing through these points, we apply the slope formula.

Step-by-Step Calculation

Step 1: Identify the coordinates of the two points:

- Point 1: $(x_1, y_1) = (26, 7)$
- Point 2: $(x_2, y_2) = (-39, 12)$

Step 2: Plug these values into the slope formula:

$$
m = \frac{12 - 7}{-39 - 26}
$$

Step 3: Simplify numerator and denominator:

$$
m = \frac{5}{-65}
$$

Step 4: Simplify the fraction:

$$
m = -\frac{1}{13}
$$

Result: The slope of the line passing through (26, 7) and (-39, 12) is -$\frac{1}{13}$.

Understanding the Significance of the Calculated Slope

Now that we've determined the slope as -$\frac{1}{13}$, let's interpret what this means in the context of the line's behavior.

Interpretation of the Slope Value

- Negative Slope: Indicates that the line descends as x increases.
- Magnitude of the Slope: The absolute value $\frac{1}{13}$ suggests a very gentle slope; the line is relatively flat and not steep.
- Rate of Change: For every increase of 13 units in x, y decreases by 1 unit.

Practical Implications

Understanding this slope can be useful in various fields:

- Physics: If the points represent position over time, the slope indicates a slow but steady decrease in position.
- Economics: Could represent a slight decline in profit or sales over time.
- Engineering: Might relate to a gentle incline in a design or landscape.

Forming the Equation of the Line

Knowing the slope is essential for deriving the equation of the line. Let's explore how to do this using the point-slope form.

Point-Slope Form of a Line

The point-slope form is written as:

$$
y - y_1 = m (x - x_1)
$$

Where:

- (x_1, y_1) is any point on the line.
- m is the slope.

Using point (26, 7):

$$
y - 7 = -\frac{1}{13} (x - 26)
$$

Expanding:

$$
y - 7 = -\frac{1}{13}x + 2
$$

Adding 7 to both sides:

$$
y = -\frac{1}{13}x + 2 + 7
$$

$$
y = -\frac{1}{13}x + 9
$$

Final Equation in Slope-Intercept Form:

$$
\boxed{y = -\frac{1}{13}x + 9}
$$

This equation describes the line passing through the given points with the calculated slope.

Additional Insights and Applications

Understanding how to compute and interpret slope is foundational in multiple disciplines. Here are some additional insights:

1. Slope and Line Types

- Horizontal Lines: Have a slope of 0.
- Vertical Lines: Have an undefined slope (division by zero).
- Oblique Lines: Have non-zero, finite slopes like the one calculated here.

2. Parallel and Perpendicular Lines

- Parallel Lines: Share the same slope. For example, any line with slope $-\frac{1}{13}$ is parallel to our line.
- Perpendicular Lines: Have slopes that are negative reciprocals. The negative reciprocal of $-\frac{1}{13}$ is 13, so any line with slope 13 is perpendicular to our line.

3. Real-World Examples

- Road Design: The slope indicates the incline of a road segment.
- Economics: The rate at which costs or revenues change with respect to units sold.
- Science: Rate of change of temperature, speed, or other variables over time.

Common Mistakes to Avoid When Calculating Slope

While calculating slope seems straightforward, several common errors can occur:

- Mixing up the points: Always ensure you subtract the y-values and x-values correctly; swapping points can lead to incorrect signs.
- Dividing by zero: If $x_1 = x_2$, the slope is undefined; this indicates a vertical line.
- Ignoring signs: Remember that the sign of the numerator and denominator affects the overall sign of the slope.
- Simplification errors: Reduce fractions to their simplest form for clarity.

Conclusion

Calculating the slope of a line passing through two points is a fundamental skill in coordinate geometry that enables deeper understanding of line behavior and relationships. For the points (26, 7) and (-39, 12), the slope is -$\frac{1}{13}$, indicating a gentle decline from left to right. This slope not only helps in constructing the line's equation but also provides insights into its geometric and real-world implications.

Mastering this concept is essential for students and professionals working in fields such as mathematics, physics, engineering, economics, and data analysis. With practice, identifying slopes and interpreting their

significance becomes an intuitive part of understanding the relationships between variables and the nature of lines on the Cartesian plane.

Meta Description: Learn how to calculate the slope of the line passing through the points (26, 7) and (-39, 12). Step-by-step explanation, interpretation, and practical applications of slope in coordinate geometry.

Frequently Asked Questions

How do you find the slope of a line passing through two points?

To find the slope, subtract the y-values and divide by the difference of the x-values: (y2 - y1) / (x2 - x1).

What is the formula for calculating the slope between two points?

The slope (m) is given by m = (y2 - y1) / (x2 - x1).

What is the slope of the line passing through (26, 7) and (-39, 12)?

The slope is (12 - 7) / (-39 - 26) = 5 / (-65) = -1/13.

Is the slope of the line passing through (26, 7) and (-39, 12) positive or negative?

The slope is negative, specifically -1/13.

Why is the slope of the line between these points negative?

Because as x decreases from 26 to -39, y increases from 7 to 12, indicating a negative slope.

Can the slope of a line be a fraction? How does that apply here?

Yes, slopes can be fractions. For these points, the slope is -1/13, which is a fraction.

What does a slope of -1/13 tell us about the line's steepness?

It indicates a very gentle downward incline; for every 13 units horizontally, the line drops 1 unit vertically.

How would you write the equation of the line passing through these points?

First, find the slope (-1/13), then use point-slope form with one of the points to write the equation.

What is the significance of the slope in graphing a line?

The slope determines the line's steepness and direction; it shows how y changes with x.

Are the points (26, 7) and (-39, 12) collinear with any other points?

Yes, any points that lie on the same line will have the same slope of -1/13, confirming collinearity.

Additional Resources

Slope: Unlocking the Mystery Behind the Steepness of a Line

When exploring the fascinating world of coordinate geometry, one of the fundamental concepts that often surfaces is the slope of a line. Whether you're a student tackling algebra for the first time or an educator aiming to clarify foundational principles, understanding how to determine the slope between two points is essential. This article delves deep into the question: What is the slope of the line passing through the points (26, 7) and (-39, 12)? By examining the concept step-by-step, providing detailed explanations, and illustrating how to compute the slope, we aim to make this topic accessible, informative, and engaging.

Understanding the Concept of Slope

What Is Slope?

In simple terms, the slope of a line quantifies its steepness or inclination. Imagine walking along a hill: the steeper the hill, the more your elevation changes over a given horizontal distance. Mathematically, the slope (often denoted as m) measures how much the y-value (vertical change) changes per unit of x-value (horizontal change).

Definition:
The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by the ratio:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This ratio is also known as the rise over run, capturing how many units the line rises or falls vertically for each unit it moves horizontally.

Analyzing the Given Points

Points in Question: (26, 7) and (-39, 12)

Before calculating the slope, let's examine the points:

- Point 1: $(x_1, y_1) = (26, 7)$
- Point 2: $(x_2, y_2) = (-39, 12)$

These points are located on the coordinate plane, with x-values of 26 and -39, and y-values of 7 and 12 respectively.

Calculating the Slope Step-by-Step

Step 1: Write Down the Slope Formula

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Substituting the specific points:

$$
m = \frac{12 - 7}{-39 - 26}
$$

Step 2: Compute the Numerator (Vertical Change)

$$
12 - 7 = 5
$$

The numerator, representing how much the y-value increases from the first point to the second, is 5.

Step 3: Compute the Denominator (Horizontal Change)

$$
-39 - 26 = -65
$$

The denominator, representing the horizontal change, is -65. This indicates that moving from $x=26$ to $x=-39$, the line moves leftward across 65 units.

Step 4: Calculate the Slope

$$
m = \frac{5}{-65}
$$

Simplify the fraction:

$$
m = -\frac{5}{65}
$$

Divide numerator and denominator by their greatest common divisor (GCD), which is 5:

$$
m = -\frac{1}{13}
$$

Final Result:

$$
\boxed{m = -\frac{1}{13}}
$$

This value indicates that the line has a gentle negative slope, decreasing by 1 unit in the y-direction for every 13 units it moves horizontally to the right.

Interpreting the Slope: What Does $-\frac{1}{13}$ Mean?

The Sign of the Slope

- The negative sign indicates that the line slopes downward from left to right.
- As you move along the line from left to right, the y-values decrease slightly.

The Magnitude of the Slope

- The magnitude $\frac{1}{13}$ suggests a gentle incline.
- For every 13 units moved horizontally, the line's y-value decreases by 1.

Practical Implication

Suppose this line represented a real-world scenario, such as the decline of a river's bed over a certain distance. The slope $-\frac{1}{13}$ would mean that for every 13 meters (or any unit of length) traveled horizontally downstream, the riverbed drops by 1 meter.

Visualizing the Line and Its Slope

Graphical Representation

Plotting the points:

- $(26, 7)$
- $(-39, 12)$

and drawing a straight line through them, we observe a line descending from left to right. The slope of $-\frac{1}{13}$ ensures the line is relatively flat, with a slight downward tilt.

Key Features of the Line

- Direction: Downward from left to right.
- Steepness: Very mild, as the absolute value of the slope is small.
- Intercepts: The line crosses the y-axis somewhere between these points, which can be calculated if needed.

Additional Considerations

Changing the Order of Points

It's important to note that the order of points affects the sign of the slope:

$$
m = \frac{y_1 - y_2}{x_1 - x_2}
$$

In this case, switching the points to $(-39, 12)$ and $(26, 7)$ would change the sign:

$$
m = \frac{7 - 12}{26 - (-39)} = \frac{-5}{65} = -\frac{1}{13}
$$

which confirms the consistency of the calculation.

Why Is Slope Important?

Understanding the slope helps in:

- Predicting behavior: How y changes as x varies.
- Formulating equations: To find the line's equation in slope-intercept form.
- Analyzing real-world data: Such as trends, rates, and relationships between variables.

Conclusion

The slope of the line passing through the points $(26, 7)$ and $(-39, 12)$ is $\boxed{-\frac{1}{13}}$. This value encapsulates the line's gentle downward trend, indicating that for each 13-unit increase in x, y decreases by 1 unit. Calculating the slope involves straightforward algebra but reveals deep insights into the line's orientation and behavior.

Mastering this fundamental concept equips students and professionals with the tools to analyze linear relationships, interpret data trends, and develop equations that model real-world phenomena. Whether applied in engineering, economics, or everyday problem-solving, understanding slopes is a cornerstone of quantitative reasoning.

In essence, the slope $-\frac{1}{13}$ not only quantifies the line's inclination but also exemplifies how simple ratios can unlock complex understanding in the realm of mathematics.

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