

Which Equation Represents A Line That Passes Through (5,1) And Has A Slope Of 1/2

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When studying coordinate geometry, one common problem is to determine the equation of a line given specific information such as a point it passes through and its slope. In this article, we will explore how to find the equation of a line that passes through the point (5, 1) and has a slope of 1/2. Understanding this process involves familiarizing ourselves with the slope-intercept form, point-slope form, and the fundamental concepts behind linear equations. By the end, you'll be able to identify or derive the correct equation of any line given similar parameters.

Understanding the Basics: Slope and Point-Coordinates

Before diving into the specific problem, it's essential to understand some foundational concepts:

What Is the Slope?

- The slope of a line measures its steepness.
- It is usually denoted by m .
- Mathematically, the slope between two points (x_1, y_1) and (x_2, y_2) is calculated as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- A positive slope indicates the line rises from left to right, while a negative slope indicates it falls.

The Coordinates of a Point

- A point on a line is represented as (x, y) .
- In this problem, the point is $(5, 1)$.

The Equation of a Line: Forms and Their Uses

There are several ways to express the equation of a line, each suitable for different scenarios.

1. Slope-Intercept Form: $y = mx + b$

- Useful when the slope m and the y-intercept b are known.
- The y-intercept is where the line crosses the y-axis.

2. Point-Slope Form: $y - y_1 = m(x - x_1)$

- Ideal when you know one point (x_1, y_1) on the line and its slope m .

3. Standard Form: $Ax + By = C$

- Useful for algebraic manipulations and certain types of graphing.

In our case, since we are given a point and the slope, the point-slope form is the most straightforward starting point.

Deriving the Equation of the Line

Given:

- Point $(x_1, y_1) = (5, 1)$
- Slope $m = \frac{1}{2}$

Step 1: Use the Point-Slope Form

The point-slope form is:

$$y - y_1 = m(x - x_1)$$

Plugging in the known values:

$$y - 1 = \frac{1}{2}(x - 5)$$

Step 2: Simplify to Slope-Intercept Form

Distribute the slope:

$$y - 1 = \frac{1}{2}x - \frac{1}{2} \times 5$$

$$y - 1 = \frac{1}{2}x - \frac{5}{2}$$

Add 1 to both sides:

$$y = \frac{1}{2}x - \frac{5}{2} + 1$$

Express 1 as a fraction with denominator 2:

$$y = \frac{1}{2}x - \frac{5}{2} + \frac{2}{2}$$

Combine the constants:

$$y = \frac{1}{2}x - \frac{3}{2}$$

Final Equation:

$$\boxed{y = \frac{1}{2}x - \frac{3}{2}}$$

This is the slope-intercept form of the line passing through (5, 1) with a slope of 1/2.

Alternate Forms of the Equation

While the slope-intercept form is most common, sometimes you may need the equation in other forms.

Standard Form

Starting from:

$$y = \frac{1}{2}x - \frac{3}{2}$$

Multiply both sides by 2 to clear fractions:

$$2y = x - 3$$

Rearranged as:

$$x - 2y = 3$$

This is the standard form of the line.

Summary of the Equations

- Slope-Intercept Form: $y = \frac{1}{2}x - \frac{3}{2}$
- Standard Form: $x - 2y = 3$

Verifying the Equation

To ensure the equation is correct, verify that it passes through the point $(5, 1)$.

Substitute $x = 5$:

- Using slope-intercept form:

$$y = \frac{1}{2} \times 5 - \frac{3}{2} = \frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1$$

- Since $y = 1$, the point $(5, 1)$ lies on the line.

Next, check the slope:

- The coefficient of x in the slope-intercept form is $\frac{1}{2}$, confirming the slope.

Real-World Applications of the Equation

Understanding how to derive and interpret the equation of a line is crucial in various fields:

1. Physics

- Calculating velocity when given initial position and rate of change.
- Graphing motion over time.

2. Economics

- Modeling supply and demand curves.
- Finding relationships between variables.

3. Engineering

- Designing systems with linear relationships.
- Analyzing data trends.

4. Data Science

- Performing linear regression to fit models to data points.

Common Mistakes to Avoid

- Using incorrect point or slope: Always double-check the given data.
- Mismanaging fractions: Multiply through appropriately to clear fractions when converting forms.
- Mixing up forms: Choose the proper form based on the problem requirements.
- Forgetting to verify: Always substitute back to confirm the equation passes through the given point.

Practice Problems to Reinforce Learning

Try solving these to solidify your understanding:

1. Find the equation of a line passing through $(2, 3)$ with a slope of $-\frac{1}{4}$.

2. Write the equation of a line with slope $\frac{1}{2}$ passing through $(0, -2)$.
3. Convert the equation $y = -2x + 5$ into standard form.

Conclusion

Determining the equation of a line based on a point and slope is a fundamental skill in algebra and coordinate geometry. By understanding the key forms—point-slope, slope-intercept, and standard—you can derive the equation efficiently. In the specific case of passing through $(5, 1)$ with a slope of $\frac{1}{2}$, the equation in slope-intercept form is $y = \frac{1}{2}x - \frac{3}{2}$, and in standard form, it is $x - 2y = 3$. Mastering these concepts allows for better problem-solving and application in various scientific and mathematical contexts.

Frequently Asked Questions

What is the equation of a line passing through the point $(5, 1)$ with a slope of $\frac{1}{2}$?

The equation is $y = \frac{1}{2}x - \frac{1}{2}$.

How do you find the equation of a line given a point and a slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form if needed.

What is the point-slope form of the line passing through $(5, 1)$ with slope $\frac{1}{2}$?

The point-slope form is $y - 1 = \frac{1}{2}(x - 5)$.

How do you convert the point-slope form to slope-intercept form?

Distribute the slope, then solve for y : $y - 1 = \frac{1}{2}x - \frac{5}{2}$, so $y = \frac{1}{2}x - \frac{5}{2} + 1$, which simplifies to $y = \frac{1}{2}x - \frac{3}{2}$.

Is the equation $y = \frac{1}{2}x - \frac{3}{2}$ correct for the given point and slope?

Yes, because substituting $x=5$ gives $y = \frac{1}{2}5 - \frac{3}{2} = 2.5 - 1.5 = 1$, which matches the point $(5, 1)$.

Can the line passing through (5, 1) with slope 1/2 be written in standard form?

Yes, the standard form is $x - 2y = 3$.

What is the significance of the slope 1/2 in the line's equation?

It indicates the line rises 1 unit for every 2 units it runs horizontally.

How does the point (5, 1) influence the line's equation?

It provides a specific point that the line must pass through, helping to determine the line's specific equation.

If the slope were different, how would the equation change?

Changing the slope would alter the coefficient in the equation, resulting in a different line passing through the same point or additional points.

What is the general method to find the equation of a line given a point and slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, then convert to slope-intercept or standard form as needed.

Additional Resources

Which Equation Represents A Line That Passes Through (5,1) And Has A Slope Of 1/2

When working with linear equations, understanding how to formulate the equation of a line given specific conditions is a fundamental skill in algebra and coordinate geometry. In particular, determining which equation represents a line that passes through the point (5,1) and has a slope of 1/2 involves applying the point-slope form of a linear equation. This process not only reinforces key concepts but also provides a clear step-by-step approach to translating real-world data points into an algebraic expression.

Understanding the Basics: Slope-Intercept and Point-Slope Forms

Before diving into the specific problem, it's essential to review two core forms of linear equations:

- Slope-Intercept Form:

$$y = mx + b$$

where m is the slope and b is the y-intercept.

- Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a known point on the line, and m is the slope.

Given the data:

- Point: $(5, 1)$

- Slope: $m = \frac{1}{2}$

the point-slope form provides a straightforward way to derive the line's equation.

Step-by-Step Guide to Find the Equation

Step 1: Identify the known values

- Point $(x_1, y_1) = (5, 1)$

- Slope $m = \frac{1}{2}$

Step 2: Write the point-slope formula

Plugging these into the point-slope form:

$$y - 1 = \frac{1}{2}(x - 5)$$

Step 3: Simplify to slope-intercept form (if desired)

Distribute the slope:

$$y - 1 = \frac{1}{2}x - \frac{5}{2}$$

Add 1 to both sides:

$$y = \frac{1}{2}x - \frac{5}{2} + 1$$

Express 1 as $\frac{2}{2}$:

$$y = \frac{1}{2}x - \frac{5}{2} + \frac{2}{2}$$

Combine the constants:

$$y = \frac{1}{2}x - \frac{3}{2}$$

Thus, the slope-intercept form of the line is:

$$\boxed{y = \frac{1}{2}x - \frac{3}{2}}$$

Verifying the Equation

To confirm this equation correctly represents the line passing through $((5, 1))$:

- Substitute $(x = 5)$:

$$y = \frac{1}{2}(5) - \frac{3}{2} = \frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1$$

which matches the given point $((5, 1))$, confirming the accuracy.

Alternative Forms and Multiple Choice Options

In many problems, you might be presented with multiple options, such as:

- $(y = \frac{1}{2}x - \frac{3}{2})$
- $(y = 2x + 1)$
- $(y = -\frac{1}{2}x + 3)$
- $(y = \frac{1}{2}x + 2)$

Identifying the correct equation involves:

1. Matching the slope: The correct equation must have a slope of $(\frac{1}{2})$.
2. Checking the point: The line must pass through $((5, 1))$.

Applying these checks:

- The first option $(y = \frac{1}{2}x - \frac{3}{2})$ matches our derived equation.
- The second $(y = 2x + 1)$ has a slope of 2, which is incorrect.
- The third $(y = -\frac{1}{2}x + 3)$ has an incorrect slope and y-intercept.
- The fourth $(y = \frac{1}{2}x + 2)$ has the correct slope but a different y-intercept, so check if it passes through $(5, 1)$:

$$y = \frac{1}{2}(5) + 2 = \frac{5}{2} + 2 = 2.5 + 2 = 4.5 \neq 1$$

So, it does not pass through (5, 1). Therefore, the first option is the correct.

General Approach for Similar Problems

When faced with the question "which equation represents a line passing through a given point with a specified slope," follow these steps:

1. Identify the known point and the slope.
2. Use the point-slope form: $(y - y_1 = m(x - x_1))$.
3. Simplify to slope-intercept form (optional but helpful).
4. Verify the equation by substituting the point to ensure correctness.
5. Compare with multiple options to select the correct one based on slope and point inclusion.

Additional Tips and Common Pitfalls

- Watch for signs: When substituting into the point-slope form, carefully handle negative signs.
- Constants in fractions: Simplify constants to make comparisons easier.
- Check the point: Always verify the final equation by plugging in the original point.
- Understand the purpose of each form: Point-slope is useful for passing through a known point; slope-intercept is more straightforward for graphing.

Real-World Applications

Understanding how to find the equation of a line passing through a specific point with a given slope is essential across various fields:

- Physics: Calculating the trajectory of objects.
- Economics: Modeling cost or revenue functions.
- Engineering: Designing components with specific linear relationships.
- Data Analysis: Creating trend lines that fit data points.

Conclusion

Determining which equation represents a line passing through a specific point with a given slope is a fundamental skill in algebra. By mastering the use of the point-slope form and understanding how to manipulate and verify linear equations, you gain a powerful tool for analyzing and interpreting linear relationships. Remember to verify your solutions and understand the significance of each component of the equation, ensuring accurate and meaningful mathematical communication.

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