Which Equation Represents A Line That Passes Through (-9, -3) And Had A Slope Of -6?

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Understanding how to determine the equation of a line given specific points and slopes is a fundamental skill in algebra and coordinate geometry. Whether you're studying for a math test, solving a real-world problem, or just brushing up on your math skills, knowing how to find the equation of a line that passes through a given point with a specific slope is essential. In this article, we will explore the process step-by-step, discuss various forms of linear equations, and provide practical examples to help you master this concept.

Foundations of Linear Equations

Before diving into the specific problem, it's important to review some basic concepts related to linear equations and their forms.

What Is a Linear Equation?

A linear equation is an algebraic equation that describes a straight line when graphed on the coordinate plane. The general form of a linear equation in two variables (x and y) is:

$$ y = mx + b $$

where:
- $ m $ is the slope of the line,
- $ b $ is the y-intercept (the point where the line crosses the y-axis).

Understanding the parameters $ m $ and $ b $ enables you to graph the line accurately and analyze its properties.

Common Forms of a Line's Equation

There are several ways to express the equation of a line, depending on the information available:

- **Slope-Intercept Form:** $ y = mx + b $
- **Point-Slope Form:** $ y - y_1 = m(x - x_1) $
- **Standard Form:** $ Ax + By = C $

In the context of our problem, the slope-intercept and point-slope forms are particularly useful because they allow us to incorporate the given point and slope directly.

Given Data and Objective

The problem states:

- The line passes through the point $(-9, -3)$.
- The slope of the line is -6.

Our goal is to find the equation of this line in an appropriate form, typically the slope-intercept form for clarity.

Step-by-Step Solution

Let's outline the process to derive the equation:

1. Identify Known Values

From the problem:
- Point $(x_1, y_1) = (-9, -3)$
- Slope $m = -6$

2. Use the Point-Slope Form

The point-slope form is particularly useful when you know a point on the line and the slope:

$$y - y_1 = m(x - x_1)$$

Substituting the known values:

$$y - (-3) = -6(x - (-9))$$

which simplifies to:

$$y + 3 = -6(x + 9)$$

3. Simplify to Slope-Intercept Form

Distribute the slope:

$$y + 3 = -6x - 54$$

Subtract 3 from both sides:

$$y = -6x - 54 - 3$$

$$y = -6x - 57$$

This is the slope-intercept form of the line's equation.

Final Equation of the Line

The equation of the line passing through $(-9, -3)$ with a slope of -6 is:

$$\boxed{ y = -6x - 57 }$$

This equation fully describes the line, allowing you to plot it or analyze its properties.

Alternative Forms and Methods

While the slope-intercept form is most straightforward for graphing and interpretation, other forms can also be used.

Point-Slope Form

As derived earlier, the point-slope form is:

$$y - y_1 = m(x - x_1)$$

which, substituting the known values, is:

$$y + 3 = -6(x + 9)$$

This form is especially useful when you want to write the equation quickly from a known point and slope, without immediately converting to slope-intercept form.

Standard Form

To convert the slope-intercept form to standard form:

$$y = -6x - 57$$

Bring all terms to one side:

$$6x + y = -57$$

or, rearranged:

$$6x + y + 57 = 0$$

This form can be useful in certain algebraic contexts or when solving systems of equations.

Practical Applications and Importance

Knowing how to find the equation of a line with given points and slopes has numerous applications:

- **Graphing lines accurately:** Essential in math, engineering, and physics.

- **Modeling real-world scenarios:** Such as calculating trends in data, economics, or science experiments.
- **Solving systems of equations:** When lines intersect, their equations help determine the point of intersection.
- **Understanding geometric relationships:** Such as parallel and perpendicular lines, which have specific slope properties.

Common Mistakes to Avoid

When working through these problems, be mindful of typical errors:

- **Incorrect substitution:** Ensure you substitute the correct point and slope into the formula.
- **Sign errors:** Pay attention to positive and negative signs, especially when dealing with negative coordinates or slopes.
- **Forgetting to simplify:** Always simplify your equations to the desired form for clarity.
- **Confusing different forms:** Know when to use point-slope vs. slope-intercept or standard form.

Practice Problems

To reinforce your understanding, try solving similar problems:

1. Find the equation of a line passing through $(4, 2)$ with a slope of 3.
2. Determine the line passing through $(-5, 7)$ with a slope of -2, and write its equation in standard form.
3. Given the line $y = \frac{1}{2}x + 4$, find the point on the line where $x = 8$.

These exercises will help solidify your grasp of the concepts.

Conclusion

In summary, the equation that represents a line passing through $(-9, -3)$ with a slope of -6 is $y = -6x - 57$. The process involves understanding the foundational forms of linear equations, applying the point-slope formula, and simplifying to the desired form. Mastering these steps enables you to analyze and graph lines accurately, solve related problems efficiently, and deepen your understanding of coordinate geometry. Whether used in academic settings or practical applications, these skills are fundamental tools in the mathematician's toolkit.

Frequently Asked Questions

What is the equation of a line passing through (-9, -3) with a slope of -6?

Using point-slope form: $y - (-3) = -6(x - (-9))$, which simplifies to $y + 3 = -6(x + 9)$.

How do I find the slope-intercept form of a line passing through (-9, -3) with a slope of -6?

Start with point-slope form: $y + 3 = -6(x + 9)$, then distribute and solve for y: $y + 3 = -6x - 54$, so $y = -6x - 57$.

What is the point-slope form equation for the line passing through (-9, -3) with slope -6?

The point-slope form is $y + 3 = -6(x + 9)$.

Can you give an example of the line equation passing through (-9, -3) with a slope of -6?

Yes, in slope-intercept form: $y = -6x - 57$.

How do I verify that the line with equation $y = -6x - 57$ passes through (-9, -3)?

Plug $x = -9$ into the equation: $y = -6(-9) - 57 = 54 - 57 = -3$, confirming the point lies on the line.

What is the general form of the line passing through (-9, -3) with slope -6?

The line can be expressed as $y + 3 = -6(x + 9)$, or expanded to $y = -6x - 57$.

If a line passes through (-9, -3) and has slope -6, what is its y-intercept?

The y-intercept is at $y = -57$ when $x = 0$, so the y-intercept is (0, -57).

What steps are involved in writing the equation of a line given a point and slope?

First, use point-slope form with the given point and slope, then simplify to slope-intercept form if needed.

Is the equation $y = -6x - 57$ correct for a line passing through (-9, -3) with slope -6?

Yes, because substituting $x = -9$ yields $y = -6(-9) - 57 = 54 - 57 = -3$, confirming the point is on the line.

How do I convert the point-slope form to slope-intercept form for this line?

Start with $y + 3 = -6(x + 9)$, distribute: $y + 3 = -6x - 54$, then subtract 3 from both sides: $y = -6x - 57$.

Additional Resources

Which Equation Represents A Line That Passes Through (-9, -3) And Has A Slope Of -6?

Understanding the precise equation of a line given its slope and a point through which it passes is fundamental in algebra and coordinate geometry. This investigation aims to elucidate the process of deriving such an equation, emphasizing the underlying principles, common methods, and practical applications. By methodically analyzing the problem statement—specifically, the line passing through the point (-9, -3) with a slope of -6—we will explore the theoretical framework, derive the relevant equations, and examine their implications.

Introduction to the Problem

A core concept in analytic geometry involves describing lines using algebraic equations. Given two critical pieces of information—a point on the line and its slope—a unique line can be identified. The problem at hand asks: Which equation represents a line passing through the point (-9, -3) with a slope of -6?

This query is not only an academic exercise but also a practical tool used across various fields such as physics, engineering, computer graphics, and data analysis, where understanding the behavior and properties of lines is essential.

Fundamental Concepts and Definitions

Before delving into the solution, it is necessary to review some foundational concepts:

Slope of a Line

- The slope (m) measures the steepness or incline of a line.
- It is defined as the ratio of the change in y-coordinates to the change in x-coordinates between two points: $m = (y_2 - y_1) / (x_2 - x_1)$.

Point-Slope Form of a Line

- When a line passes through a known point (x_1, y_1) with slope m, its equation can be written as: $y - y_1 = m(x - x_1)$.

Standard and Slope-Intercept Forms

- From the point-slope form, equations can be rearranged into:
- Slope-intercept form: $y = mx + b$.
- Standard form: $Ax + By + C = 0$.

Deriving the Equation: Step-by-Step Analysis

Given:
- Point: (-9, -3)
- Slope: m = -6

The most straightforward approach is to use the point-slope form, which directly incorporates the given point and slope.

Step 1: Apply the Point-Slope Formula

Using the point (-9, -3) and slope -6:
- $y - (-3) = -6(x - (-9))$
- Simplify:
$y + 3 = -6(x + 9)$

Step 2: Expand and Simplify

Distribute the slope:
- $y + 3 = -6x - 54$

Subtract 3 from both sides to solve for y:
- $y = -6x - 54 - 3$
- $y = -6x - 57$

This is the slope-intercept form of the line.

Analysis of the Derived Equation

The equation:

$y = -6x - 57$

has several important features:

- It confirms the slope is -6, matching the given data.
- The y-intercept (b) is -57, indicating where the line crosses the y-axis.
- The line passes through the point (-9, -3), which can be verified:

Check:
- Substitute x = -9:

$y = -6(-9) - 57 = 54 - 57 = -3$

which matches the y-coordinate of the given point. This verification confirms the correctness of the derived equation.

Alternative Forms and Their Significance

While the slope-intercept form is often most intuitive, other forms are useful depending on context.

Standard Form

- Rearranged from the slope-intercept form:

$y = -6x - 57$

To standard form ($Ax + By + C = 0$):

$-6x - y - 57 = 0$

Exploration of Related Equations

To deepen understanding, consider how variations of the line's equation relate to the original problem.

Equation with Different Forms

- For example, the point-slope form directly:

$y - (-3) = -6(x - (-9))$

or

$y + 3 = -6(x + 9)$

- The standard form:

$6x + y + 57 = 0$

Note: All these forms describe the same line, just expressed differently.

Implications of the Equation

- The slope indicates a decreasing line: for every 1 unit increase in x, y decreases by 6 units.
- The y-intercept at -57 suggests the line crosses the y-axis far below the origin.

Practical Applications and Significance

Understanding how to derive the equation of a line from a point and slope has broad implications:

- Engineering: designing components with precise inclinations.
- Physics: modeling linear relationships, such as constant velocity.
- Economics: representing cost functions or demand lines.

- Computer Graphics: rendering lines between points.

In all these contexts, accurately formulating the line's equation ensures precision and reliability.

Summary and Conclusion

To determine the equation of a line passing through the point (-9, -3) with a slope of -6, the most direct and effective method is to use the point-slope form:

- $y - y_1 = m(x - x_1)$

Substituting the known values:

- $y + 3 = -6(x + 9)$

Expanding and simplifying yields:

- $y = -6x - 57$

This equation uniquely describes the line, encapsulating its steepness and position in the coordinate plane.

In conclusion, the equation:

$y = -6x - 57$

accurately represents the line passing through (-9, -3) with a slope of -6. Recognizing how to derive and manipulate such equations is fundamental in both academic and applied mathematics, underscoring the importance of understanding the relationship between points, slopes, and line equations.

Additional Resources for Further Study

- Textbooks on Algebra and Analytic Geometry
- Interactive graphing tools to visualize lines
- Practice problems involving point-slope and slope-intercept forms
- Tutorials on converting between different line equations

By mastering these concepts, students and professionals alike can confidently analyze and represent linear relationships across diverse scenarios.

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